EE 330
Lecture 32

Two-Port Amplifier Models

Basic amplifier architectures

- Common Emitter/Source
- Common Collector/Drain
- Common Base/Gate
Exam 3
Friday November 22
Review from Previous Lecture

Two-port representation of amplifiers

Unilateral amplifiers:

\[ R_{\text{IN}}, A_V, \text{ and } R_{\text{OUT}} \text{ often used to characterize the two-port of amplifiers} \]

Unilateral amplifier in terms of “amplifier” parameters

\[ R_{\text{IN}} = \frac{1}{y_{11}}, \quad A_V = -\frac{y_{21}}{y_{22}}, \quad R_{\text{OUT}} = \frac{1}{y_{22}} \]
Relationship with Dependent Sources?

Two Port (Thevenin)

Dependent sources from EE 201

Example showing two dependent sources
Relationship with Dependent Sources?

`\[ V_s = \mu V_x \]

`\[ I_s = \alpha V_x \]

`\[ V_s = \rho I_x \]

`\[ I_s = \beta I_x \]

Dependent sources from EE 201
Relationship with Dependent Sources?

It follows that

\[ V_s = \mu V_x \]

Voltage dependent voltage source is a unilateral floating two-port voltage amplifier with \( R_{IN} = \infty \) and \( R_{OUT} = 0 \)
Relationship with Dependent Sources?

It follows that

\[ V_s = \rho I_x \]

Current dependent voltage source is a unilateral floating two-port transresistance amplifier with \( R_{IN}=0 \) and \( R_{OUT}=0 \)

Review From Previous Lecture
It follows that

\[ I_S = \beta I_x \]

Current dependent current source is a floating unilateral two-port current amplifier with \( R_{IN} = 0 \) and \( R_{OUT} = \infty \)
It follows that

$$I_s = \alpha V_x$$

Voltage dependent current source is a floating unilateral two-port transconductance amplifier with $R_{IN}=\infty$ and $R_{OUT}=\infty$
Dependent sources are unilateral two-port amplifiers with ideal input and output impedances

Dependent sources do not exist as basic circuit elements but amplifiers can be designed to perform approximately like a dependent source

- Practical dependent sources typically are not floating on input or output
- One terminal is usually grounded
- Input and output impedances of realistic structures are usually not ideal

Why were “dependent sources” introduced as basic circuit elements instead of two-port amplifiers in the basic circuits courses???
Why was the concept of “dependent sources” not discussed in the basic electronics courses???
Basic Amplifier Structures

- MOS and Bipolar Transistors both have 3 primary terminals
- MOS transistor has a fourth terminal that is generally considered a parasitic terminal

Transistors as 3-terminal Devices

Small Signal Transistor Models as 3-terminal Devices
Basic Amplifier Structures

Observation:

These circuits considered previously have a terminal (emitter or source) common to the input and output in the small-signal equivalent circuit.

For BJT, E is common, input on B, output on C  
Termed “Common Emitter”

For MOSFET, S is common, input on G, output on D  
Termed “Common Source”
Basic Amplifier Structures

Amplifiers using these devices generally have one terminal common and use remaining terminals as input and output.

Since devices are nearly unilateral, designation of input and output terminals is uniquely determined.

Three different ways to designate the common terminal:

- Source or Emitter termed Common Source or Common Emitter
- Gate or Base termed Common Gate or Common Base
- Drain or Collector termed Common Drain or Common Collector
Basic Amplifier Structures

Identification of Input and Output Terminals is not arbitrary

It will be shown that all 3 of the basic amplifiers are useful!
Basic Amplifier Structures

Common Source or Common Emitter
Common Gate or Common Base
Common Drain or Common Collector

Objectives in Study of Basic Amplifier Structures

1. Obtain key properties of each basic amplifier
2. Develop method of designing amplifiers with specific characteristics using basic amplifier structures
Characterization of Basic Amplifier Structures

- Observe that the small-signal equivalent of any 3-terminal network is a two-port.
- Thus, to characterize any of the 3 basic amplifier structures, it suffices to determine the two-port equivalent network.
- Since small signal model when expressed in terms of small-signal parameters of BJT and MOSFET differ only in the presence/absence of $g_{\pi}$ term, can analyze the BJT structures and then obtain characteristics of corresponding MOS structure by setting $g_{\pi}=0$. 
The three basic amplifier types for both MOS and bipolar processes

Common Emitter

Common Base

Common Collector

Common Source

Common Gate

Common Drain

Will focus on the performance of the bipolar structures and then obtain performance of the MOS structures by observation
The three basic amplifier types for both MOS and bipolar processes

- **Common Emitter**
  - $V_{in}$
  - $V_{OUT}$
  - $R_L$

- **Common Base**
  - $V_{in}$
  - $V_{OUT}$
  - $R_L$

- **Common Collector**
  - $V_{in}$
  - $V_{OUT}$
  - $R_L$

- $v_{OUT} = -g_m R_L v_{be}$
- $v_{IN} = v_{be}$
- $A_V = \frac{v_{OUT}}{v_{IN}} = -g_m R_L$
- \( v_{OUT} = (g_m + g_\pi) v_{be} R_L \)
- $v_{IN} = v_{be} + (g_m + g_\pi) v_{be} R_L$
- $A_V = \frac{v_{OUT}}{v_{IN}} = \frac{(g_m + g_\pi) R_L}{1 + (g_m + g_\pi) R_L} \approx 1$

- Significantly different gain characteristics for the three basic amplifiers
- There are other significant differences too ($R_{IN}$, $R_{OUT}$, ...) as well
The three basic amplifier types for both MOS and bipolar processes

Common Emitter
Common Base
Common Collector
Common Source
Common Gate
Common Drain

More general models are needed to accommodate biasing, understand performance capabilities, and include effects of loading of the basic structures

Two-port models are useful for characterizing the basic amplifier structures

How can the two-port parameters be obtained for these or any other linear two-port networks?
Two-Port Models of Basic Amplifiers widely used for Analysis and Design of Amplifier Circuits

Methods of Obtaining Amplifier Two-Port Network

1. \( \mathbf{v}_{\text{TEST}} : \mathbf{i}_{\text{TEST}} \) Method (considered in a previous lecture)

2. Write \( \mathbf{v}_1 : \mathbf{v}_2 \) equations in standard form

\[
\mathbf{v}_1 = \mathbf{i}_1 R_{\text{IN}} + A_{\text{VR}} \mathbf{v}_2 \\
\mathbf{v}_2 = \mathbf{i}_2 R_{\text{O}} + A_{\text{V0}} \mathbf{v}_1
\]

3. Thevenin-Norton Transformations

4. Ad Hoc Approaches

Any of these methods can be used to obtain the two-port model.
$V_{\text{test}} : i_{\text{test}}$ Method for Obtaining Two-Port Amplifier Parameters

**SUMMARY from PREVIOUS LECTURE**

If Unilateral $A_{VR} = 0$

$A_{V0} = \frac{V_{out-test}}{V_{test}}$

$R_{in} = \frac{V_{test}}{i_{test}}$

$R_{0} = \frac{V_{test}}{i_{test}}$

$A_{VR} = \frac{V_{out-test}}{V_{test}}$
Will now develop two-port model for each of the three basic amplifiers and look at one widely used application of each
Consider Common Emitter/Common Source Two-port Models

Will focus on Bipolar Circuit since MOS counterpart is a special case obtained by setting $g_{\pi}=0$
Basic CE/CS Amplifier Structures

Can include or exclude $R$ and $R_1$ in two-port models (of course they are different circuits)

The CE and CS amplifiers are themselves two-ports!
Two-port model for Common Emitter Configuration

\[ \begin{align*}
V_{be} & : \text{Voltage at the base-emitter junction} \\
g_T & : \text{Transconductance} \\
g_m & : \text{Transconductance} \\
g_O & : \text{Output conductance} \\
I_1 & : \text{Input current} \\
V_1 & : \text{Input voltage} \\
A_{V0}V_1 & : \text{Output voltage} \\
I_2 & : \text{Output current} \\
V_2 & : \text{Output voltage} \\
R_i & : \text{Input resistance} \\
R_0 & : \text{Output resistance} \\
\end{align*} \]
Two-Port Models of Basic Amplifiers widely used for Analysis and Design of Amplifier Circuits

Methods of Obtaining Amplifier Two-Port Network

1. $V_{\text{TEST}} : i_{\text{TEST}}$ Method

2. Write $V_1 : V_2$ equations in standard form

   $V_1 = i_1 R_{\text{IN}} + A_{\text{VR}} V_2$

   $V_2 = i_2 R_{\text{O}} + A_{V0} V_1$

3. Thevenin-Norton Transformations

4. Ad Hoc Approaches
Two-port model for Common Emitter Configuration

By Thevenin : Norton Transformations

\[ R_{in} = \frac{1}{g_{\pi}} \]
\[ A_{V0} = -\frac{g_m}{g_0} \]
\[ R_0 = \frac{1}{g_0} \]
\[ A_{VR} = 0 \]
Two-Port Models of Basic Amplifiers widely used for Analysis and Design of Amplifier Circuits

Methods of Obtaining Amplifier Two-Port Network

1. $\mathbf{v}_{\text{TEST}} : \mathbf{i}_{\text{TEST}}$ method

2. Write $\mathbf{v}_1 : \mathbf{v}_2$ equations in standard form

   $\mathbf{v}_1 = i_1 R_{\text{IN}} + A_{\text{VR}} \mathbf{v}_2$

   $\mathbf{v}_2 = i_2 R_{\text{O}} + A_{\text{VO}} \mathbf{v}_1$

3. Thevenin-Norton Transformations

4. Ad Hoc Approaches
Two-port model for Common Emitter Configuration

Alternately, by $\mathcal{V}_{\text{TEST}} : i_{\text{TEST}}$ Method

To obtain $R_{\text{in}}$

\[
\begin{align*}
    i_1 & \quad + \\
    R_{\text{in}} & \quad A_{V0}v_1 \\
    v_1 & \quad - \\

    i_2 & \quad + \\
    R_0 & \quad + \\
    v_2 & \quad - \\
\end{align*}
\]

\[
R_{\text{in}} = \frac{\mathcal{V}_{\text{test}}}{i_{\text{test}}}
\]

\[
R_{\text{in}} = \frac{1}{g_{\pi}}
\]

\{R_{\text{in}}, A_{V0} \text{ and } R_0\}
Two-port model for Common Emitter Configuration

Alternately, by $v_{\text{TEST}} : i_{\text{TEST}}$ Method

To obtain $A_{V0}$

\[
A_{V0} = \frac{v_{\text{out-test}}}{v_{\text{test}}} \\
v_{\text{out-test}} = v_{\text{test}} \left( -\frac{g_m}{g_0} \right) \\
A_{V0} = -\frac{g_m}{g_0}
\]

\{R_{\text{in}}, A_{V0} \text{ and } R_{0}\}
Two-port model for Common Emitter Configuration

Alternately, by $v_{\text{TEST}} : i_{\text{TEST}}$ Method

To obtain $g_0$

\[
\{R_{\text{in}}, A_{V0} \text{ and } R_0\}
\]
Two-port model for Common Emitter Configuration

In terms of small signal model parameters:

\[ R_{\text{in}} = \frac{1}{g_{\pi}} \quad A_{V0} = -\frac{g_{m}}{g_{0}} \quad R_{0} = \frac{1}{g_{0}} \quad A_{VR} = 0 \]

In terms of operating point and model parameters:

\[ R_{i} = \frac{\beta V_{t}}{I_{CQ}} \quad A_{V0} = -\frac{V_{\text{AF}}}{V_{t}} \quad R_{0} = \frac{V_{\text{AF}}}{I_{CQ}} \quad A_{VR} = 0 \]

Characteristics:

- Input impedance is mid-range
- Voltage Gain is Large and Inverting
- Output impedance is large
- Unilateral
- Widely used to build voltage amplifiers
Common Emitter Configuration

Consider the following CE application

\[ V_{out} \left( g_C + g_0 \right) = g_0 A V_0 V_{in} \]

\[ A_{VC} = \frac{V_{out}}{V_{in}} = \frac{g_0 A V_0}{g_0 + g_C} = \frac{-g_m}{g_0 + g_C} \approx -g_m R_C \]

\[ R_{inC} = R_{in} = r_{\pi} \]

\[ R_{outC} = R_o // R_C = \frac{1}{g_0 + g_C} \approx R_C \]
Common Emitter Configuration

Consider the following CE application

This circuit can also be analyzed directly without using 2-port model for CE configuration

\[ \mathbf{A}_V = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = -\left( \frac{g_m}{g_0 + g_C} \right) \]  
\[ \mathbf{R}_{in} = r_{\pi} \]  
\[ \mathbf{R}_{out} = \frac{1}{g_0 + g_C} \approx R_C \]  
\[ g_0 << g_C \]
Common Emitter Configuration

Consider the following CE application

(this is also a two-port model for this CE application)

**Small signal parameter domain**

\[
A_v \approx -g_m R_C \quad g_o << g_c
\]

\[
R_{out} = \frac{1}{g_o + g_C} \approx R_C
\]

\[
R_{in} = r_{\pi}
\]

\[
A_{VR} = 0
\]

**Operating point and model parameter domain**

\[
A_v \approx -\frac{I_{CQ} R_C}{V_t}
\]

\[
R_{out} \approx R_C
\]

\[
R_{in} = \frac{\beta V_t}{I_{CQ}}
\]

**Characteristics:**
- Input impedance is mid-range
- Voltage Gain is large and Inverting
- Output impedance is mid-range
- Unilateral
- Widely used as a voltage amplifier
Common Source/ Common Emitter Configurations

In terms of operating point and model parameters:

- Input impedance is mid-range (infinite for MOS)
- Voltage Gain is Large and Inverting
- Output impedance is large
- Unilateral
- Widely used to build voltage amplifiers
Common Source/Common Emitter Configuration

**Common Emitter inc R_C**

\[ R_{out} = \frac{1}{g_0 + g_C} \approx \frac{g_o}{g_o + g_C} \geq R_C \]

\[ A_v \approx -g_m R_C \]

\[ R_{in} = r_{\pi} \]

\[ A_{VR} = 0 \]

**Common Source inc R_D**

\[ R_{out} = \frac{1}{g_0 + g_D} \approx \frac{g_o}{g_o + g_D} \geq R_D \]

\[ A_v \approx -g_m R_D \]

\[ R_{in} = \infty \]

**Characteristics:**

- Input impedance is mid-range (infinite for MOS)
- Voltage Gain is Large and Inverting
- Output impedance is mid-range
- Unilateral
- Widely used as a voltage amplifier
Consider Common Collector/Common Drain Two-port Models

Will focus on Bipolar Circuit since MOS counterpart is a special case obtained by setting $g_{\pi}=0$.
Two-port model for Common Collector Configuration

\[ \begin{align*}
V_{be} & \quad g_{\Pi} \quad g_{m} V_{be} \quad g_{O} \\
\text{Common Collector Configuration} & \quad \text{Common Collector} \\
\end{align*} \]

\[ \begin{align*}
V_1 & \quad A_{V0r} V_2 \\
A_{V0} V_1 & \quad R_{iX} \quad R_{0X} \\
\end{align*} \]

\( \{ R_{iX}, A_{V0}, A_{V0r} \text{ and } R_{0X} \} \)
Two-Port Models of Basic Amplifiers widely used for Analysis and Design of Amplifier Circuits

Methods of Obtaining Amplifier Two-Port Network

1. \( v_{\text{TEST}} : i_{\text{TEST}} \) Method

2. Write \( v_1 : v_2 \) equations in standard form
   \[
   v_1 = i_1 R_{\text{IN}} + A_{\text{VR}} v_2 \\
   v_2 = i_2 R_{\text{O}} + A_{\text{V}_0} v_1
   \]

3. Thevenin-Norton Transformations

4. Ad Hoc Approaches
Two-port model for Common Collector Configuration

Applying KCL at the input and output node, obtain

\[ i_1 = (v_1 - v_2) g_\pi \]
\[ i_2 = (g_m + g_\pi + g_o) v_2 - (g_m + g_\pi) v_1 \]

These can be rewritten as

\[ v_1 = i_1 r_\pi + v_2 \]
\[ v_2 = \left( \frac{1}{g_m + g_\pi + g_o} \right) i_2 + \left( \frac{g_m + g_\pi}{g_m + g_\pi + g_o} \right) v_1 \]

It thus follows that

\[ R_{iX} = r_\pi \quad A_{VOR} = 1 \]
\[ R_{0X} = \left( \frac{1}{g_m + g_\pi + g_o} \right) \]
\[ A_{V0} = \left( \frac{g_m + g_\pi}{g_m + g_\pi + g_o} \right) \]
Two-port model for Common Collector Configuration

\[ V_{be} \]

\[ g_m \]

\[ V_1 \]

\[ i_1 \]

\[ v_1 \]

\[ i_2 \]

\[ v_2 \]

Two-port Common Collector Model

\[ R_{ix} = r_\pi \]

\[ A_{v0r} = 1 \]

\[ R_{0X} = \left( \frac{1}{g_m + g_\pi + g_o} \right) \approx \frac{1}{g_m} \]

\[ A_{v0} = \left( \frac{g_m + g_\pi}{g_m + g_\pi + g_o} \right) \approx 1 \]
Common Collector Configuration

Consider the following CC application

Determine $R_{in}$, $R_0$, and $A_V$

(this is not asking for a two-port model for the CC application – $R_{in}$ and $A_V$ defined for no additional load on output, $R_0$ defined for short-circuit input)

$$A_V = A_{V0} \frac{g_{ox}}{g_{ox} + g_E} = \frac{g_m + g_{\pi}}{g_{ox} + g_E} \left( \frac{g_m + g_{\pi} + g_o}{g_m + g_{\pi} + g_o + g_E} \right) = \frac{g_m + g_{\pi}}{g_m + g_{\pi} + g_o + g_E} \approx \frac{g_m}{g_m + g_E} \approx 1$$

$$v_{in} = i_1 R_{ix} + A_{V0} v_1$$

$$R_{in} = \frac{r_{\pi}}{1 - \frac{g_{ox}}{g_{m} + g_{\pi}} \frac{g_{m} + g_{\pi}}{g_{ox} + g_E}} = r_{\pi} \frac{g_{m} + g_{\pi} + g_o + g_E}{g_o + g_E} \approx r_{\pi} + \beta R_E$$

$$R_0 \approx \frac{1}{g_{m} + g_E + g_o + g_{\pi}} = \frac{1}{g_{m} + g_E} = \frac{R_E}{1 + g_m R_E} \approx \frac{1}{g_m}$$
Consider the following CC application

(this is not asking for a two-port model for the CC application, \( R_{in} \) and \( A_V \) defined for no additional load on output, \( R_o \) defined for short-circuit input)

Alternately, this circuit can also be analyzed directly

\[
\begin{align*}
V_{out} &= g_E + g_o + g_{\pi} = V_{in}g_{\pi} + g_{m}V_1 \\
V_{in} &= V_1 + V_{out} \\
i_{in} &= g_{\pi}(V_{in} - V_{out}) \\
V_{out} &= V_{in}(g_{\pi} + g_{m}) \\
A_V &= \frac{g_{\pi} + g_{m}}{g_{m} + g_E + g_o + g_{\pi}} \approx \frac{g_{m}}{g_{m} + g_E} = \frac{I_{CQ}R_E}{I_{CQ}R_E + V_t} \\
i_{in} &= g_{\pi}\left(V_{in} - V_{out}\right) = g_{\pi}V_{in}(g_{E} + g_0) \\
R_{in} &= \frac{g_{m} + g_{\pi} + g_o + g_E}{g_o + g_E} \approx \frac{g_{\pi}g_o}{g_o + g_E} \\
&= r_{\pi} + \beta R_E
\end{align*}
\]
Common Collector Configuration

Consider the following CC application

(this is not asking for a two-port model for the CC application, \(- R_{in} \) and \(A_v \) defined for no additional load on output, \(R_o \) defined for short-circuit input -)

To obtain \(R_0\), set \(V_{in} = 0\)

\[
i_{out} = V_{out} \left( g_E + g_0 + g_\pi \right) - g_m \left( -V_{out} \right)
\]

\[
R_{out} = \frac{1}{g_m + g_\pi + g_o + g_E} \quad g_s \ll g_e \quad \approx \quad \frac{1}{g_m}
\]
Consider the following CC application

\[ A_V = \frac{g_{\pi} + g_m}{g_m + g_E + g_0 + g_{\pi}} \approx \frac{g_m}{g_m + g_E} = \frac{I_{CQ}R_E}{I_{CQ}R_E + V_t} \approx 1 \]

\[ R_{in} = r_{\pi} \frac{g_m + g_{\pi} + g_o + g_E}{g_o + g_E} \approx r_{\pi} + \beta R_E \]

\[ R_{out} = \frac{1}{g_m + g_{\pi} + g_o + g_E} \approx \frac{1}{g_m} \]

Question: Why are these not the two-port parameters of this circuit?

- \( R_{in} \) defined for open-circuit on output instead of short-circuit (see previous slide: -2 slides)
- \( A_{V0r} \neq 0 \)
Common Collector Configuration

For this CC application

(this is not a two-port model for this CC application)

Small signal parameter domain

\[ A_V = \frac{g_\pi + g_m}{g_m + g_E + g_0 + g_\pi} \quad \text{if} \quad g_s >> g_e \]

\[ R_{in} \approx r_\pi + \beta R_E \]

\[ R_0 \approx \frac{R_E}{1 + g_m R_E} \quad g_s R_e >> 1 \quad \approx \frac{1}{g_m} \]

Operating point and model parameter domain

\[ A_V \approx \frac{I_{CQ} R_E}{I_{CQ} R_E + V_t} \quad \approx \frac{I_{CQ} R_E >> V_t}{1} \]

\[ R_{in} \approx \beta R_E \]

\[ R_0 \approx \frac{I_{CQ} R_E >> V_t}{V_t \quad I_{CQ}} \]

Characteristics:

- Output impedance is low
- \( A_{V0} \) is positive and near 1
- Input impedance is very large
- Widely used as a buffer
- Not completely unilateral but output-input transconductance (or \( A_{Vr} \)) is small and effects are generally negligible though magnitude same as \( A_V \)
Common Collector/Common Drain Configurations

For these CC/CD applications (not two-port models for these applications)

\[ A_V = \frac{g_m + g_\pi + g_m}{g_m + g_E + g_0 + g_\pi} \approx \frac{1}{\beta R_E} \]

\[ R_{in} \approx r_{nE} + \beta R_E \]

\[ R_0 \approx \frac{R_E}{1 + g_m R_E} \approx \frac{1}{g_m} \]

In terms of operating point and model parameters:

\[ A_V \approx \frac{l_{CQ} R_E}{l_{CQ} R_E + V_t} \approx \frac{l_{CQ} R_E}{l_{CQ}} \]

\[ R_{in} \approx \beta R_E \]

- Output impedance is low
- \( A_{V0} \) is positive and near 1
- Input impedance is very large

- Widely used as a buffer
- Not completely unilateral but output-input transconductance is small
Consider Common Base/Common Gate Two-port Models

Will focus on Bipolar Circuit since MOS counterpart is a special case obtained by setting $g_{\pi}=0$. 
Two-port model for Common Base Configuration

\[ V_{be}, g_{m}, V_{be}, g_{o} \]

Common Base

\[ \begin{align*}
V_{1} & = A_{v0} V_{2} \\
A_{v0} V_{1} & = V_{2}
\end{align*} \]

\[ \{ R_{ix}, A_{v0}, A_{v0r}, and R_{ox} \} \]
Two-Port Models of Basic Amplifiers widely used for Analysis and Design of Amplifier Circuits

Methods of Obtaining Amplifier Two-Port Network

1. \( V_{\text{TEST}} : i_{\text{TEST}} \) Method

2. Write \( V_1 : V_2 \) equations in standard form

\[
V_1 = i_1 R_{\text{IN}} + A_{VR} V_2
\]

\[
V_2 = i_2 R_{O} + A_{V0} V_1
\]

3. Thevenin-Norton Transformations

4. Ad Hoc Approaches
Two-port model for Common Base Configuration

From KCL

\[
\begin{align*}
i_1 &= V_1 g_\pi + (V_1 - V_2) g_0 + g_m V_1 \\
i_2 &= (V_2 - V_1) g_0 - g_m V_1
\end{align*}
\]

These can be rewritten as

\[
\begin{align*}
V_1 &= \left(\frac{1}{g_m + g_\pi + g_0}\right)i_1 + \left(\frac{g_0}{g_m + g_\pi + g_0}\right)V_2 \\
V_2 &= \left(\frac{1}{g_0}\right)i_2 + \left(1 + \frac{g_m}{g_0}\right)V_1
\end{align*}
\]

It thus follows that:

\[
\begin{align*}
R_{ix} &= \frac{1}{g_m + g_\pi + g_0} \approx \frac{1}{g_m} \\
A_{VOr} &= \frac{g_0}{g_m + g_\pi + g_0} \\
A_{V0} &= 1 + \frac{g_m}{g_0} \approx \frac{g_m}{g_0} \\
R_{ox} &= \frac{1}{g_0}
\end{align*}
\]
Two-port model for Common Base Configuration

Two-port Common Base Model

\[ R_{iX} = \frac{1}{g_m + g_\pi + g_0} \approx \frac{1}{g_m} \]

\[ A_{V0r} = \frac{g_0}{g_m + g_\pi + g_0} \approx \frac{g_0}{g_m} \]

\[ A_{V0} = 1 + \frac{g_m}{g_0} \approx \frac{g_m}{g_0} \]

\[ R_{oX} = \frac{1}{g_0} \]
Common Base Configuration

Consider the following CB application

(two-load model for this CB application – $R_{in}$ and $A_v$ defined for no load on output, $R_o$ defined for short-circuit input)

$$A_V = A_{V0} \frac{R_C}{R_C + R_{0X}} = \left( \frac{g_m + g_0}{g_0} \right) \left( \frac{g_0}{g_C + g_0} \right) = \frac{g_m + g_0}{g_C + g_0} \approx g_m R_C$$

$$R_{in} = \frac{V_{in}}{i_1} = \frac{i_1 R_{iX} + A_{V0r} V_{out}}{i_1}$$

$$R_{out} = \frac{R_C}{1 + g_0 R_C}$$
Consider the following CB application

\[ (g_C + g_0)v_0 = (g_m + g_0)v_{\text{in}} \]

\[ A_V = \frac{g_m + g_0}{g_C + g_0} \approx g_m R_C \]

By KCL at the output node, obtain

\[ R_{\text{in}} = \frac{g_0 + g_C}{g_C (g_m + g_{\pi} + g_0) + g_{\pi} g_0} \approx \frac{1}{g_m} \]

\[ R_{\text{out}} = R_C // r_0 \]

Alternately, this circuit can also be analyzed directly

By KCL at the emitter node, obtain

\[ R_{\text{in}} = \frac{R_C}{1 + g_0 R_C} \approx R_C \]
### Common Base Application

(this is not a two-port model for this CB application)

**Characteristics:**

- Output impedance is mid-range
- \( A_{V0} \) is large and positive (equal in mag to that to CE)
- Input impedance is very low
- Not completely unilateral but output-input transconductance is small
Common Base/Common Gate Application

(output impedance is mid-range)

- $A_V \approx g_m R_C$
- $R_{in} \approx \frac{1}{g_m}$
- $R_{out} \approx R_C$
- $A_V \approx g_m R_D$
- $R_{in} \approx \frac{1}{g_m}$
- $R_{out} \approx R_D$

In terms of operating point and model parameters:

- $A_V \approx \frac{I_{CQ} R_C}{V_t}$
- $R_{in} \approx \frac{V_t}{I_{CQ}}$
- $R_{out} \approx \frac{I_{bo} R_C}{V_{EBQ}}$
- $A_V \approx \frac{2I_{DQ} R_D}{V_{EBQ}}$
- $R_{in} \approx \frac{V_{EBQ}}{2I_{DQ}}$
- $R_{out} \approx \frac{I_{bo} R_D}{V_{EBQ}}$

Characteristics:

- Output impedance is mid-range
- $A_{V0}$ is large and positive (equal in mag to that to CE)
- Input impedance is very low
- Not completely unilateral but output-input transconductance is small
The three basic amplifier types for both MOS and bipolar processes

Common Emitter
Common Base
Common Collector
Common Source
Common Gate
Common Drain

- Have developed both two-ports and a widely used application of all 6
- A fourth structure (two additional applications) is also quite common so will be added to list of basic applications
Common Emitter with Emitter Resistor Configuration Application

(this is not a two-port model for this CE with \( R_E \) application)

By KCL at two non-grounded nodes

\[
\begin{align*}
\nu_{out} (g_C + g_0) + (\nu_{in} - \nu_E) g_m &= g_0 \nu_E \\
\nu_E (g_E + g_0 + g_\pi) - (\nu_{in} - \nu_E) g_m &= g_0 \nu_{out} + g_\pi \nu_{in}
\end{align*}
\]

\[
A_V = \frac{\nu_{out}}{\nu_{in}} = \frac{-g_m g_E + g_0 g_\pi}{g_C g_m + g_C (g_0 + g_\pi + g_E) + g_0 (g_\pi + g_E)} \approx -\frac{R_C}{R_E}
\]
Common Emitter with Emitter Resistor Configuration Application

(it is not a two-port model for this CE with $R_E$ application)

It can also be shown that

$$A_V \approx -\frac{R_C}{R_E}$$

$$R_{in} \approx r_{\pi} + \beta R_E$$

$$R_{out} \approx R_C$$

Nearly unilateral (is unilateral if $g_o=0$)
Common Emitter with Emitter Resistor Configuration Application

(this is not a two-port model for this CE with $R_E$ application)

$A_v \approx -\frac{R_C}{R_E}$

$R_{in} \approx r_T + \beta R_E$

$R_{out} \approx R_C$

(this is not a two-port model)

Characteristics:

- Analysis would simplify if $g_0$ were set to 0 in model
- Gain can be accurately controlled with resistor ratios
- Useful for reasonably accurate low gains
- Input impedance is high
Basic Amplifier Gain Table

<table>
<thead>
<tr>
<th></th>
<th>CE/CS</th>
<th>CC/CD</th>
<th>CB/CG</th>
<th>CEwRE/CSwRS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BJT</td>
<td>MOS</td>
<td>BJT</td>
<td>MOS</td>
</tr>
<tr>
<td><strong>A_V</strong></td>
<td><img src="image1.png" alt="BJT" /></td>
<td><img src="image2.png" alt="MOS" /></td>
<td><img src="image3.png" alt="BJT" /></td>
<td><img src="image4.png" alt="MOS" /></td>
</tr>
<tr>
<td></td>
<td>$-g_m R_C$</td>
<td>$\frac{g_m}{g_m + g_E}$</td>
<td>$\frac{g_m R_C}{2l_{DQ} R_E}$</td>
<td>$\frac{g_m}{g_m + g_E}$</td>
</tr>
<tr>
<td><strong>R_{in}</strong></td>
<td>$\frac{\beta V_t}{I_C}$</td>
<td>$\frac{V_{EB}}{V_t}$</td>
<td>$\frac{V_{EB}}{2l_{DQ}}$</td>
<td>$\frac{V_{EB}}{2l_{DQ}}$</td>
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<td></td>
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<td>$\frac{V_{EB}}{2l_{DQ}}$</td>
</tr>
<tr>
<td><strong>R_{out}</strong></td>
<td>$R_C$</td>
<td>$g_m$</td>
<td>$R_C$</td>
<td>$R_C$</td>
</tr>
</tbody>
</table>

(not two-port models for the four structures)
Can use these equations only when small signal circuit is EXACTLY like that shown!!

<table>
<thead>
<tr>
<th>Basic Amplifier Gain Table</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>AV</strong></td>
</tr>
<tr>
<td>BJT CE/CS</td>
</tr>
<tr>
<td>- $g_m R_C$</td>
</tr>
<tr>
<td>$\frac{l_{CG} R_C}{V_t}$</td>
</tr>
<tr>
<td>$- \frac{2I_{DQ} R_D}{V_{EB}}$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>BJT CC/CD</td>
</tr>
<tr>
<td>$\frac{g_m}{g_m + g_E}$</td>
</tr>
<tr>
<td>$\frac{l_{CG} R_E}{l_{CG} R_E + V_t}$</td>
</tr>
<tr>
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<td>$g_m R_C$</td>
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</tr>
<tr>
<td></td>
</tr>
<tr>
<td>BJT CEwRE/CSwRS</td>
</tr>
<tr>
<td>$- \frac{R_C}{R_E}$</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

| **R_in**                   |
| BJT CE/CS                  |
| $r_{TT}$                   |
| $\frac{\beta V_t}{l_{CQ}}$ |
| $\infty$                  |
|  |
| BJT CC/CD                  |
| $r_m + \beta R_E$          |
| $\beta \left( \frac{V_t}{l_{CQ}} + R_E \right)$ |
| $\infty$                  |
|  |
| BJT CB/CG                  |
| $g_m^{-1}$                 |
| $\frac{V_t}{l_{CQ}}$       |
| $\frac{V_{EB}}{2I_{DQ}}$   |
|  |
| BJT CEwRE/CSwRS            |
| $r_m + \beta R_E$          |
| $\beta \left( \frac{V_t}{l_{CQ}} + R_E \right)$ |
| $\infty$                  |

| **R_out**                  |
| BJT CE/CS                  |
| $R_C$                      |
| $\frac{V_t}{l_{CQ}}$       |
| $\frac{V_{EB}}{2I_{DQ}}$   |
|  |
| BJT CC/CD                  |
| $g_m^{-1}$                 |
| $\frac{V_t}{l_{CQ}}$       |
| $\frac{V_{EB}}{2I_{DQ}}$   |
|  |
| BJT CB/CG                  |
| $R_C$                      |
| $\frac{V_t}{l_{CQ}}$       |
| $\frac{V_{EB}}{2I_{DQ}}$   |
|  |
| BJT CEwRE/CSwRS            |
| $R_C$                      |
| $\frac{V_t}{l_{CQ}}$       |
| $\frac{V_{EB}}{2I_{DQ}}$   |
|  |
End of Lecture 32