High Gain Amplifiers
Current Source Biasing
Current Sources and Mirrors
The Cascode Configuration
The Differential Amplifier
Can use these equations only when small signal circuit is EXACTLY like that shown!!

**Basic Amplifier Gain Table**

<table>
<thead>
<tr>
<th></th>
<th>CE/CS</th>
<th>CC/CD</th>
<th>CB/CG</th>
<th>CEwRE/CSwRS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>AV</strong></td>
<td>$-g_mR_C$</td>
<td>$\frac{g_m}{g_m + g_E}$</td>
<td>$g_mR_C$</td>
<td>$-\frac{R_C}{R_E}$</td>
</tr>
<tr>
<td><strong>R_in</strong></td>
<td>$\frac{\beta V_t}{I_C Q}$</td>
<td>$\infty$</td>
<td>$\beta \left( \frac{V_t}{I_C Q} + R_E \right)$</td>
<td>$\infty$</td>
</tr>
<tr>
<td><strong>R_out</strong></td>
<td>$R_C$</td>
<td>$g_m^{-1}$</td>
<td>$R_C$</td>
<td>$R_C$</td>
</tr>
</tbody>
</table>

Review from Last Lecture
Review from Last Lecture

Basic Amplifier Characteristics Summary

**CE/CS**
- Large inverting gain
- Moderate input impedance
- Moderate (or high) output impedance
- Widely used as the basic high gain inverting amplifier

**CC/CD**
- Gain very close to +1 (little less)
- High input impedance for BJT (high for MOS)
- Low output impedance
- Widely used as a buffer

**CB/CG**
- Large noninverting gain
- Low input impedance
- Moderate (or high) output impedance
- Used more as current amplifier or, in conjunction with CD/CS to form two-stage cascode

**CEwRE/CSwRS**
- Reasonably accurate but somewhat small gain (resistor ratio)
- High input impedance
- Moderate output impedance
- Used when more accurate gain is required
Review from Last Lecture

Example:

Will calculate $A_V$ by determining the three ratios (not voltage gains of dependent source):

$$A_V = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_B} \frac{V_B}{V_A} \frac{V_A}{V_{in}} = A_{V2}A_{V1}A_{V0}$$
Review from Last Lecture

Example:

\[ A_{v2} = \frac{v_{\text{out}}}{v_{B}} \approx -\frac{R_6//R_8}{R_7} \]

\[ R_{in2} \approx \beta R_7 \]
Review from Last Lecture

Example:

\[ R_{\text{in2}} \approx \beta R_7 \]
Review from Last Lecture

Example:

\[ A_{V2} = \frac{V_{out}}{V_B} \approx -\frac{R_6//R_8}{R_7} \]

\[ R_{in2} \approx \beta R_7 \]
Example:

\[ A_{V1} = \frac{v_B}{v_A} \approx -g_m \left( R_3 // R_5 // R_{in2} \right) \]

\[ R_{in1} \approx r_{\pi 1} \]
Example:

\[ V_B \]

\[ R_{in1} \]

Review from Last Lecture
Review from Last Lecture

Example:

\[ AV_0 = \frac{\nu_A}{\nu_{in}} \approx \frac{R_1/R_2 // R_{in1}}{R_S + R_1/R_2 // R_{in1}} \]
Review from Last Lecture

Example:

Thus we have

\[ A_V = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_B} \cdot \frac{V_B}{V_A} \cdot \frac{V_A}{V_{in}} \]

where

\[ \frac{V_{out}}{V_B} \approx -\frac{R_6//R_8}{R_7} \]

\[ \frac{V_B}{V_A} \approx -g_{m1}(R_3//R_5//R_{in2}) \]

\[ \frac{V_A}{V_{in}} \approx \frac{R_1//R_2}{R_S + R_1//R_2 // R_{in1}} \]

\[ R_{in2} \approx \beta R_7 \]

\[ R_{in1} \approx r_{\pi 1} \]
Example:

Observation: By working from the output back to the input we were able to create a sequence of steps where the circuit at each step looked EXACTLY like one of the four basic amplifiers even though stages were not unilateral. Engineers often follow a design approach that uses a cascade of the basic amplifiers and that is why it is often possible to follow this approach to analysis.

Will formalize what we have done (consider 3-stage unilateral cascade) Exactly same approach when not unilateral
Formalization of cascade circuit analysis working from load to input: (consider 3-stage unilateral cascade)
Formalization of cascade circuit analysis working from load to input: (consider 3-stage unilateral cascade)

Starting at output stage obtain:

\[
\frac{V_{OUT}}{V_{IN}} = \frac{V_1}{V_{IN}} \frac{V_2}{V_1} \frac{V_3}{V_2} \frac{V_{OUT}}{V_3}
\]
Formalization of cascade circuit analysis working from load to input: (consider 3-stage unilateral cascade)

Replace Stage 3 with $R_{IN3}$

Analyze Stage 2:
Formalization of cascade circuit analysis working from load to input: (consider 3-stage unilateral cascade)

- Replace Stage 2 with $R_{IN2}$
- Analyze Stage 1:

$$\frac{V_{OUT}}{V_{IN}} = \frac{V_1}{V_{IN}} \frac{V_2}{V_1} \frac{V_3}{V_2} \frac{V_{OUT}}{V_3}$$
Formalization of cascade circuit analysis working from load to input: (consider 3-stage unilateral cascade)

Replace Stage 1 with $R_{IN1}$

Analyze Stage 0:

$$
\frac{V_{OUT}}{V_{IN}} = \frac{V_{1}}{V_{IN}V_{2}} \frac{V_{3}}{V_{OUT}}
$$
Example:

Observation: By working from the output back to the input we were able to create a sequence of steps where the circuit at each step looked EXACTLY like one of the four basic amplifiers. Engineers often follow a design approach that uses a cascade of the basic amplifiers and that is why it is often possible to follow this approach to analysis.

Two other methods could have been used to analyze this circuit.

What are they?
Two other methods could have been used to analyze this circuit

1. Create a two-port model of the two stages

(for this example, since the first-stage is unilateral, it can be shown that )

\[
A_V = \frac{v_{out}}{v_{in}} = \left( \frac{R_{iX_1}}{R_{iX_1} + R_S} \right) A_{V01} \left( \frac{R_{L1}/R_{iX_2}}{R_{L1}/R_{iX_2} + R_{0X_1}} \right) A_{V02} \left( \frac{R_L}{R_L + R_{0X_2}} \right)
\]

\(A_{V01}\) and \(A_{V02}\) are open-loop two-port gain and are different than what used above
Example:

Two other methods could have been used to analyze this circuit

2. Put in small-signal model for $Q_1$ and $Q_2$ and solve resultant circuit
   (not too difficult for this specific example)
Example: \[ A_v = \frac{v_{out}}{v_{in}} = ? \] Express in terms of small-signal parameters
Example:

Note: Even though the second stage has a resistor in the collector, the gain expressions developed for the common collector amplifier still apply.

\[
A_v = \frac{V_{out}}{V_2} \frac{V_2}{V_1} \frac{V_1}{V_{in}} \equiv \left[-g_{m4} \left(\frac{R_D}{R_L}\right)\right][1] \left[\frac{-g_{m1}}{g_{m2} + \left(\beta_3 \left(\frac{R_{B1}}{R_{B2}}\right)\right)^{-1}}\right]
\]
High-gain BJT amplifier

\[ A_V = \frac{-g_m}{g_0 + G_C} \approx -g_m R_C \]

To make the gain large, it appears that all one needs to do is make \( R_C \) large!

\[ A_V \approx -g_m R_C = \frac{-I_{CQ} R_C}{V_t} \]

But \( V_t \) is fixed at approx 25mV and for good signal swing, \( I_{CQ} R_C < (V_{DD} - V_{EE})/2 \)

\[ |A_V| < \frac{V_{DD} - V_{EE}}{2V_t} \]

If \( V_{DD} - V_{EE} = 5V \),

\[ |A_V| < \frac{5V}{2 \cdot 25mV} = 100 \]

- Gain is practically limited with this supply voltage to around 100
- And in extreme case, limited to 200 with this supply voltage with very small signal swing
High-gain MOS amplifier

\[ A_V = \frac{-g_m}{g_0 + G_D} \approx -g_m R_D \]

To make the gain large, it appears that all one needs to do is make \( R_D \) large!

\[ A_V \approx -g_m R_D = \frac{-2I_{DQ} R_D}{V_{EB}} \]

But \( V_{EB} \) is practically limited to around 100mV and for good signal swing, \( I_{DQ} R_D < (V_{DD} - V_{SS})/2 \)

\[ |A_V| < \frac{V_{DD} - V_{SS}}{V_{EB}} \]

If \( V_{DD} - V_{SS} = 5V \) and \( V_{EB} = 100mV \),

\[ |A_V| < \frac{5V}{100mV} = 50 \]

Gain is practically limited with this supply voltage to around 50

Are these fundamental limits on the gain of the BJT and MOS Amplifiers?
High-gain amplifier

This gain is very large!

Too good to be true!

Need better model of MOS device!

\[ A_V = \frac{-g_m}{0} = -\infty \]
High-gain amplifier

This gain is very large (but realistic)!
And no design parameters affect the gain
But how can we make a current source?
High-gain amplifier

\[ A_V \approx -8000 \]

How can we build the ideal current source?

What is the small-signal model of an actual current source?
Before addressing the issue of how a current source is designed, will consider another circuit that uses current source biasing.

The Basic Differential Amplifier

$$V_{OUT} = A_V(V_2 - V_1)$$

If $A_V$ is large

Operational Amplifier (Op Amp)
Example: Determine the voltage gain of the following circuit

\[ V_{\text{out}} = V_{E} \]

\[ V_{\text{in}} = \pm V_{E} \]

\[ i_{B1} = i_{B2} \]

\[ g_{m1} = g_{m2} = \frac{I_{EE}}{2V_{t}} \]

Since symmetric when \( V_{\text{IN}} = 0 \)

\[ I_{C1} = I_{C2} = \frac{I_{EE}}{2} \]
Example: Determine the voltage gain of the following circuit

\[
\begin{align*}
V_{out} &= V_E \\
\end{align*}
\]

\[
V_E (g_{\pi 1} + g_{\pi 1}) = g_{\pi 1} V_{IN} + g_{m1} (V_{IN} - V_E) + g_{m2} (-V_E) \\
V_{OUT} &= -R_{C1} g_{m1} (V_{IN} - V_E)
\]

\[
\begin{align*}
V_E (g_{\pi 1} + g_{\pi 2} + g_{m1} + g_{m2}) &= V_{IN} (g_{m1} + g_{\pi 1}) \\
V_E &= \frac{(g_{m1} + g_{\pi 1})}{(g_{\pi 1} + g_{\pi 2} + g_{m1} + g_{m2})} V_{IN} \\
V_{OUT} &= -R_{C1} g_{m1} V_{IN} \left[ 1 - \frac{(g_{m1} + g_{\pi 1})}{(g_{\pi 1} + g_{\pi 2} + g_{m1} + g_{m2})} \right] \\
V_{OUT} &= -R_{C1} g_{m1} V_{IN} \left[ \frac{g_{\pi 1} + g_{\pi 2} + g_{m1} + g_{m2} - (g_{m1} + g_{\pi 1})}{(g_{\pi 1} + g_{\pi 2} + g_{m1} + g_{m2})} \right]
\]
Example: Determine the voltage gain of the following circuit

\[ V_{OUT} = -R_C g_m V_{IN} \left[ \frac{g_{m2}}{g_{m1} + g_{m2}} \right] \]

\[ V_{OUT} \approx -R_C g_m V_{IN} \left[ \frac{g_{m2}}{g_{m1} + g_{m2}} \right] \]

\[ V_{OUT} \approx - \frac{R_C g_m}{2} V_{IN} \]

\[ V_{OUT2} \approx \frac{R_C g_m}{2} V_{IN} \]
Differential amplifier

\[
\begin{align*}
V_{OUT1} & \approx -\left[ \frac{R_{C1}g_{m1}}{2} \right] \left( V_{IN1} - V_{IN2} \right) \\
V_{OUT2} & \approx \left[ \frac{R_{C1}g_{m1}}{2} \right] \left( V_{IN1} - V_{IN2} \right)
\end{align*}
\]

- Very useful circuit
- This is a basic Op Amp
- Uses a current source and \( V_{DD} \) for biasing (no biasing resistors or caps!)
- But – needs a dc current source !!!!
High-gain amplifier

$$A_V \approx -8000$$

How can we build the dc current source?

What is the small-signal model of an actual current source?
Model of Current Source

“Reasonable Current Source”

$I_1$ independent of $V_1$ and $R_S$ large

Small-signal model of current source

want $R_{IN}$ large

Ideal Current Source

$I_1$ independent of $V_1$
Current Sources/Mirrors

Will show circuit in red behaves as a current source
Current Sources/Mirrors

If the base currents are neglected

\[ I_0 \approx \frac{(V_{CC} - 0.6V)}{R} \]
Current Sources/Mirrors

\[ I_0 \approx \frac{(V_{CC} - 0.6V)}{R} \]

If the base currents are neglected

\[ I_0 = J_S A_{E0} e^{\frac{V_{BE0}}{V_t}} \]
\[ I_1 = J_S A_{E1} e^{\frac{V_{BE1}}{V_t}} \]

since \( V_{BE1} = V_{BE2} \)

\[ I_1 \approx \left( \frac{A_{E1}}{A_{E0}} \right) I_0 = \left( \frac{A_{E1}}{A_{E0}} \right) \frac{V_{CC} - 0.6V}{R} \]

Note \( I_1 \) is not a function of \( V_1 \)

Behaves as a current source! So is ideal with this model!!

Actually termed a “sink” current since coming out of load

And does not require an additional dc voltage source!!!
Current Sources/Mirrors

- Multiple Outputs Possible
- Can be built for sourcing or sinking currents
- Also useful as a current amplifier
- MOS counterparts work very well and are not plagued by base current
Current Sources/Mirrors

Biasing Circuit

Key Block

Current Sink
End of Lecture 32