EE 330
Lecture 34

• High Gain Amplifiers
• Current Source Biasing
• Current Sources and Mirrors
• Other Special Configurations
• Amplifier Biasing
Formalization of cascade circuit analysis working from load to input: (when stages are unilateral or not unilateral)

\[ \frac{V_{OUT}}{V_{IN}} = \frac{V_1}{V_{IN}} \frac{V_2}{V_1} \frac{V_3}{V_2} \frac{V_{OUT}}{V_3} \]

This was the approach used in analyzing the previous cascaded amplifier.
Example: \( A_v = \frac{V_{out}}{V_{in}} = ? \) Express in terms of small-signal parameters.
Review from Last Lecture

Example:

\[
A_V = \frac{v_{out}}{v_2} \frac{v_2}{v_1} \frac{v_1}{v_{in}} \approx \left[ -g_{m4} \left( R_D / R_L \right) \right][1] \left[ \frac{-g_{m1}}{g_{m2} + \left( \beta_3 \left( R_{B1} / / R_{B2} \right) \right)^{-1}} \right]
\]
Review from Last Lecture

High-gain amplifier

This gain is very large!

Too good to be true!

Need better model of MOS device!

\[ A_V = \frac{-g_m}{0} = -\infty \]
High-gain amplifier

This gain is very large (but realistic)!

And no design parameters affect the gain

But how can we make a current source?
Current Sources/Mirrors

Review from Last Lecture
Review from Last Lecture

Current Sources/Mirrors

Multiple-Output Bipolar Current Source and Sink

\[ I_{nk} = \frac{A_{Enk}}{A_{E0}} I_0 \]

\[ I_{pk} = \frac{A_{En1}}{A_{E0}} \left[ \frac{A_{Epk}}{A_{Ep0}} \right] I_0 \]
Current Sources/Mirrors

- Termed a “current mirror”
- Output current linearly dependent on $I_{in}$
- Small-signal and large-signal relationships the same since linear
- Serves as a current amplifier
- Widely used circuit

But $I_{in}$ must be positive!
Current Sources/Mirrors

\[ i_{\text{out}} = \text{?} \]

\[ \frac{i_{\text{OUT}} + MI_{BS}}{i_{\text{in}} + I_{BS}} = M \]

\[ i_{\text{OUT}} + MI_{BS} = M \left( i_{\text{in}} + I_{BS} \right) \]

\[ i_{\text{OUT}} + MI_{BS} = M \left( i_{\text{in}} + I_{BS} \right) \]

\[ \frac{i_{\text{OUT}}}{i_{\text{in}}} = M \]

But \( I_{BS} + i_{\text{in}} > 0 \)!
Current Sources/Mirrors

npn current mirror amplifier

\[ i_{\text{out}} = \left[ \frac{A_{E1}}{A_{E0}} \right] i_{\text{in}} \]

Amplifiers both positive and negative currents (provided \( i_{\text{in}} > -I_{BS} \))

Current amplifiers are easy to build!!

Current gain can be accurately controlled with appropriate layout!!
Current Sources/Mirrors

npn Current Mirror

I_0

Q_0

A_{E_0}

Q_1

A_{E_1}

n-channel Current Mirror

I_{in}

M_0

W_{0,L_0}

M_1

W_{1,L_1}

I_{out} = ?
Current Sources/Mirrors

n-channel Current Mirror

\[ I_{in} = \frac{\mu C_{OX} W_0}{2L_0} (V_{GS0} - V_{T0})^2 \]
\[ I_{out} = \frac{\mu C_{OX} W_1}{2L_1} (V_{GS1} - V_{T1})^2 \]

If process parameters are matched, it follows that

\[ I_{out} = \left[ \frac{W_1}{W_0} \frac{L_0}{L_1} \right] I_{in} \]

- Current mirror gain can be accurately controlled!
- Layout is important to get accurate gain (for both MOS and BJT)
Current Sources/Mirrors Summary

**npn Current Mirror**

\[
I_{\text{out}} = \left[ \frac{A_{E1}}{A_{E0}} \right] I_{\text{in}}
\]

**n-channel Current Mirror**

\[
I_{\text{out}} = \left[ \frac{W_1}{W_0} \frac{L_0}{L_1} \right] I_{\text{in}}
\]
Layout of Current Mirrors

Example with $M = 2$

Standard layout

Gate area after fabrication depicted

$$M = \left[ \begin{array}{c} W_2 \\ L_1 \end{array} \right]$$

$$M = \left[ \begin{array}{cc} \frac{W_2 + 2\Delta W}{W_1 + 2\Delta W} & \frac{L_1 + 2\Delta L}{L_2 + 2\Delta L} \\ \frac{2W_1 + 2\Delta W}{W_1 + 2\Delta W} & \frac{L_1 + 2\Delta L}{L_1 + 2\Delta L} \end{array} \right] \neq 2$$
Layout of Current Mirrors

Example with $M = 2$

Standard layout

Better layout

$$M = \begin{bmatrix} W_2 & L_1 \\ W_1 & L_2 \end{bmatrix}$$

$$M = \left[ \frac{2W_1 + 2\Delta W}{W_1 + 2\Delta W} \cdot \frac{L_1 + 2\Delta L}{L_1 + 2\Delta L} \right] \neq 2$$

$$M = \left[ \frac{2W_1 + 4\Delta W}{W_1 + 2\Delta W} \cdot \frac{L_1 + 2\Delta L}{L_1 + 2\Delta L} \right] = 2$$
**Layout of Current Mirrors**

Example with $M = 2$

![Standard layout](image)

$$M = \begin{bmatrix} W_2 & L_1 \\ W_1 & L_2 \end{bmatrix}$$

$$M = \left[ \frac{2W_1 + 4\Delta W}{W_1 + 2\Delta W} \cdot \frac{L_1 + 2\Delta L}{L_1 + 2\Delta L} \right] = 2$$

Even Better Layout

This is termed a common-centroid layout
n-channel current mirror current amplifier

Amplifies both positive and negative currents

\[ i_{\text{out}} = \begin{bmatrix} \frac{W_2}{W_1} & \frac{L_1}{L_2} \end{bmatrix} i_{\text{in}} \]
Current Sources/Mirrors

\[ I_k = \begin{bmatrix} \frac{W_k}{W_0} & L_0 \\ W_0 & L_k \end{bmatrix} I_0 \]

multiple output n-channel current sink array

multiple output p-channel current source array
Current Sources/Mirrors

multiple sourcing and sinking current outputs

\[ I_p_j = \left[ \frac{W_{p_j}}{L_{p_j}} \cdot \frac{L_{p_0}}{W_{p_0}} \right] M I_0 \]

\[ M = \left[ \frac{W_{n_0}}{L_{n_0}} \cdot \frac{L_0}{W_0} \right] \]

\[ I_{n_j} = \left[ \frac{W_{n_j}}{L_{n_j}} \cdot \frac{L_0}{W_0} \right] I_0 \]

\[ M_n_0, M_n_1, M_n_2, \ldots, M_n_k \]

\[ W_{n_0}, L_{n_0}, W_{n_1}, L_{n_1}, W_{n_2}, L_{n_2}, \ldots, W_{n_k}, L_{n_k} \]

m and k may be different
Often M=1
High-gain amplifier

How can we build the current source?

What is the small-signal model of an actual current source?
Basic Current Sources and Sinks

Bipolar Mirror-Based Current Sink

Bipolar Mirror-Based Current Source

Biasing circuit uses same $V_{CC}$ as amplifier and no other independent sources.
High-gain amplifier

- Bias circuitry requires only a single independent dc voltage source!
- Incremental overhead is only one transistor, $Q_B$
Basic Current Sources and Sinks

Basic Bipolar Current Sinks

\[ I_X = J_S A_\text{Ee} \frac{V_{XX}}{V_t} \]

\[ I_X \approx \frac{V_{CC} - 0.6V}{R} \]

Basic Bipolar Current Sources

- Very practical methods for biasing the BJTs (or MOSFETs) can be used
- Current Mirrors often used for generating sourcing and sinking currents
- Can think of biasing transistors with \( V_{XX} \) and \( V_{YY} \) in these current sources
How can we build the current source?

What is the small-signal model of an actual current source?
Basic Current Sources and Sinks

Small-signal Model of BJT Current Sinks and Sources

Small-signal model of all other BJT Sinks and Sources introduced so far are the same

Not Diode Connected!
Basic Current Sources and Sinks

Small-signal Model of MOS Current Sinks and Sources

Small-signal model of all other MOS Sinks and Sources introduced thus far are the same
High-gain amplifier

\[ A_V = \frac{-g_m}{g_0} \]

\[ A_V \approx \frac{-g_{m1}}{2g_{01}} \]
High-gain amplifier

\[ g_m = \frac{V_{AF}}{V_t} \approx 8000 \]

\[ A_V = \frac{-g_m}{g_0} \]

- Nonideal current source decreased the gain by a factor of 2
- But the voltage gain is still quite large (-4000)

Can the gain be made even larger?
High-gain amplifier
Can the gain be made even larger?

The Cascode Configuration

Discuss
The Cascode Amplifier (consider npn BJT version)

- Actually a cascade of a CE stage followed by a CB stage but usually viewed as a "single-stage" structure
- Cascode structure is widely used
Basic Amplifier Structures

1. Common Emitter/Common Source
2. Common Collector/Common Drain
3. Common Base/Common Gate
4. Common Emitter with \( R_E \)/ Common Source with \( R_S \)

5. Cascode (actually CE:CB or CS:CD cascade)
6. Darlington (special CE:CE or CS:CS cascade)

The first 4 are most popular
(V_X + V_2)g_{02} + V_2g_{m_2} = I_X
V_1 g_{m_1} - V_2(g_{01} + g_{\pi_2}) = I_X

Observing V_1 = V_{IN} and eliminating V_2 between these two equations, we obtain

V_{IN} = I_1 \cdot \frac{1}{g_{\pi_1}}

and

V_X = I_X \cdot \left[ \frac{g_{01} + g_{02} + g_{\pi_2} + g_{m_2}}{g_{02}(g_{01} + g_{\pi_2})} \right] - V_{IN} \cdot \left[ \frac{g_{m_1}(g_{02} + g_{m_2})}{g_{02}(g_{\pi_2} + g_{01})} \right]

Standard Form for Amplifier Two-Port

v_1 = i_1 R_{IN} + A_{VR} v_2
v_2 = i_2 R_O + A_{VO} v_1

Discuss
Cascode Configuration

Two-port model of cascode amplifier

\[ V_X = I_X \cdot \left[ \frac{g_{01} + g_{02} + g_{\pi 2} + g_{m2}}{g_{02} \left( g_{01} + g_{\pi 2} \right)} \right] - V_{IN} \cdot \left[ \frac{g_{m1} \left( g_{02} + g_{m2} \right)}{g_{02} \left( g_{\pi 2} + g_{01} \right)} \right] \]

\[ V_{IN} = I_1 \cdot \frac{1}{g_{\pi 1}} \]

It thus follows for the npn bipolar structure that:

\[ A_{VCC} = - \frac{g_{m1} \left( g_{02} + g_{m2} \right)}{g_{02} \left( g_{\pi 2} + g_{01} \right)} \approx - \frac{g_{m1} g_{m2}}{g_{02} g_{\pi 2}} \]

\[ g_{0CC} = \left[ \frac{g_{02} \left( g_{01} + g_{\pi 2} \right)}{g_{01} + g_{02} + g_{\pi 2} + g_{m2}} \right] \approx \left[ \frac{g_{02} g_{\pi 2}}{g_{m2}} \right] \]

\[ g_{\pi CC} = g_{\pi 1} \]
Cascode Configuration

Voltage gain is a factor of $\beta$ larger than that of the CE amplifier with current source load.

Output impedance is a factor of $\beta$ larger than that of the CE amplifier.

$$A_{VCC} \approx -\frac{g_{m1} g_{m2}}{g_{02} g_{\pi2}}$$

$$g_{0CC} \approx \frac{g_{02} g_{\pi2}}{g_{m2}}$$

$$g_{\pi CC} = g_{\pi1}$$

$$A_{VCC} \approx -\left[ \frac{g_{m1}}{g_{02}} \right] \beta \approx -\left[ \frac{g_{m1}}{g_{01}} \right] \beta$$

$$g_{0CC} \approx \frac{g_{01}}{\beta}$$
Cascode Configuration

\[ AV_{CC} \approx -\left[ \frac{g_{m1}}{g_{02}} \right] \beta \approx -\left[ \frac{g_{m1}}{g_{01}} \right] \beta \]

\[ g_{0CC} \approx \frac{g_{02}}{\beta} \]

\[ AV_{CC} \approx -\left[ \frac{g_{m1}}{g_{01}} \right] \beta = \left[ \frac{2V_{AF}}{V_t} \right] \beta = [-8000]100 \]

\[ AV_{CC} \approx -800,000 \]

This gain is very large and only requires two transistors!

What happens to the gain if a transistor-level current source is used for \( I_B \)?
Cascode Configuration

![Cascode Configuration Diagram]
Cascode Configuration

\[ V_{CC} \rightarrow Q_3 \rightarrow V_{YY} \]
\[ V_{XX} \rightarrow Q_2 \rightarrow Q_1 \]
\[ V_{IN} \rightarrow V_{SS} \]

\[ V_{OUT} \]

\[ V_{IN} \rightarrow V_{OUT} \]
High-gain amplifier comparisons

$V_{IN}$ $V_{OUT}$

It thus follows that

$$A_V = A_{VCC} \left[ \frac{g_{0CC}}{g_{03} + g_{0CC}} \right]$$

But $g_{0CC} \approx g_{03}/\beta$

$$A_V \approx A_{VCC} \left[ \frac{g_{0CC}}{g_{03}} \right] \approx \frac{A_{VCC}}{\beta}$$

This is a dramatic reduction in gain compared to what the ideal current source biasing provided.
Cascode Configuration

\[ A_V \approx A_{VCC} \left[ \frac{g_{0CC}}{g_{03}} \right] \approx \frac{A_{VCC}}{\beta} \]

But recall

\[ A_{VCC} \approx -\left[ \frac{g_{m1}}{g_{01}} \right] \beta \]

Thus

\[ A_V \approx -\left[ \frac{g_{m1}}{g_{01}} \right] \]

\[ A_V \approx -\left[ \frac{l_{CQ}}{V_t} \right] = -\left[ \frac{V_{AF}}{V_t} \right] \approx -8000 \]

- This is still a factor of 2 better than that of the CE amplifier with transistor current source \( A_{VCE} \approx -\left[ \frac{g_{m1}}{2g_{01}} \right] \)
- It only requires one additional transistor
- But its not nearly as good as the gain the cascode circuit seemed to provide
Gain limited by output impedance of current source!!

Can we design a better current source?

In particular, one with a higher output impedance?
Better current sources

Need a higher output impedance than $g_o$.

The output impedance of the cascode circuit itself was very large!

$$g_{0CC} \approx \frac{g_{01}}{\beta}$$

Can a current source be built with the cascode circuit?
Cascode current sources

Discuss
This gain is very large and is a factor of 2 below that obtained with an ideal current source biasing.

Although the factor of 2 is not desired, the performance of this circuit is still very good.

This factor of 2 gain reduction is the same as was observed for the CE amplifier when a transistor-level current source was used.
Can we use more cascoding to further increase the gain?
High Gain Amplifiers Seldom Used Open Loop

\[ A_V = \frac{-g_m}{g_0} \]

\( A_V = -8,000 \)

\[ A_V = -\left[ \frac{g_{m1}}{g_{01}} \right]^{\frac{1}{2}} \]

\( A_V = -400,000 \)

If \( A_V = -400,000 \) and \( V_{IN} \) increases by 1mV, what would happen at the output?

\[ V_{OUT} \text{ would decrease by } 400,000 \times 1 \text{mV} = -400 \text{V} \]
The Cascode Amplifier (consider n-ch MOS version)

Same issues for biasing with current source as for BJT case

With cascode current source, gain only drops by a factor of 2
The Cascode Amplifier (consider n-ch MOS version)

\[ A_{VCC} \approx -\frac{g_{m1}g_{m2}}{g_{01}g_{02}} \]

\[ A_{VCC} \approx -\frac{g_{m1}}{g_{01}} \]

\[ A_{VCC} \approx -\frac{1}{2}\left[ \frac{g_{m1}g_{m2}}{g_{01}g_{02}} \right] \]
Current Source Summary (BJT)

**Basic**

\[ V_{YY} \]
\[ V_{SS} \]
\[ Q_1 \]
\[ I_x \]

\[ g_0 \approx g_{01} \]

**Cascode**

\[ V_{XX} \]
\[ V_{YY} \]
\[ Q_2 \]
\[ Q_1 \]

\[ V_{CC} \]
\[ V_{SS} \]
\[ I_x \]

\[ g_{01/\beta} \]

\[ g_{0CC} \approx \frac{g_{01}}{\beta} \]
Current Source Summary (MOS)

**Basic**

- \( V_{YY} \) to \( M_1 \)
- \( V_{SS} \)
- \( I_X \)

**Cascode**

- \( V_{XX} \) to \( M_2 \)
- \( M_1 \) to \( V_{YY} \)
- \( V_{SS} \)
- \( I_X \)

\[ g_0 \approx g_{01} \]

\[ g_0 \approx g_{01} \frac{g_{02}}{g_{m2}} \]
High Gain Amplifier Comparisons (n-ch MOS)

\[ A_V \approx -\frac{g_{m1}}{g_{01}} \]

\[ A_V \approx -\frac{1}{2} \left[ \frac{g_{m1}}{g_{01}} \right] \]

\[ A_{VCC} \approx -\frac{g_{m1}g_{m2}}{g_{01}g_{02}} \]

\[ A_{VCC} \approx -\frac{1}{2} \left[ \frac{g_{m1}g_{m2}}{g_{01}g_{02}} \right] \]
High Gain Amplifier Comparisons (BJT)

$$A_V = \frac{-g_m}{g_0}$$

$$A_V \approx -\frac{1}{2} \frac{g_m 1}{g_{01}}$$

$$A_V \approx -\left[ \frac{g_m 1}{g_{01}} \right] \beta$$

- Single-ended high-gain amplifiers inherently difficult to bias (because of the high gain)
- Biasing becomes practical when used in differential applications
- These structures are widely used but usually with differential inputs
The Cascode Amplifier

- Operational amplifiers often built with basic cascode configuration
- CMFB used to address the biasing problem
- Usually configured as a differential structure when building op amps
- Have high output impedance (but can be buffered)
- Terms “telescopic cascode”, “folded-cascode”, and “regulated cascode” often refer to op amps based upon the cascode configuration
Cascade Configurations

Two-stage CE:CE or CS:CS Cascade

A_{VCB} = ?

A_{VCM} = ?
Cascade Configurations

Two-stage CE:CE or CS:CS Cascade

\[ A_{V_{CB}} \approx \left[ \frac{-g_{m1}}{g_{01}+g_{\pi2}} \right] \left[ \frac{-g_{m2}}{g_{02}} \right] \approx \frac{g_{m1}g_{m2}}{g_{\pi2}g_{02}} = \beta \frac{g_{m1}}{g_{02}} \]

\[ A_{V_{CM}} = \left[ \frac{-g_{m1}}{g_{01}} \right] \left[ \frac{-g_{m2}}{g_{02}} \right] = \frac{g_{m1}g_{m2}}{g_{01}g_{02}} \]

- Significant increase in gain
- Gain is noninverting
- Comparable to that obtained with the cascode but noninverting
Cascade Configurations

Two-stage CE:CE or CS:CS Cascade

\[ A_{V_{CB}} = \left[ \frac{-g_{m1}}{g_{01} + g_{03} + g_{\pi 2}} \right] \left[ \frac{-g_{m2}}{g_{02} + g_{04}} \right] \approx \frac{g_{m1}g_{m2}}{2g_{\pi 2}g_{02}} = \beta \frac{g_{m1}}{2g_{02}} \]

\[ A_{V_{CM}} = \left[ \frac{-g_{m1}}{g_{01} + g_{03}} \right] \left[ \frac{-g_{m2}}{g_{02} + g_{04}} \right] = \frac{g_{m1}g_{m2}}{4g_{01}g_{02}} \]

Note factor or 2 and 4 reduction in gain due to actual current source bias
Cascade Configurations

Two-stage CE Cascade

- Large gains can be obtained by cascading
- Gains are multiplicative (when loading is included)
- Large gains used to build “Op Amps” and feedback used to control gain value
- Some attention is needed for biasing but it is manageable
- Minor variant of the two-stage cascade often used to built Op Amps
- Compensation of two-stage cascade needed if feedback is applied to maintain stability

Three-stage CE Cascade

- For many years three or more stages were seldom cascaded because of challenges in compensation to maintain stability though recently some industrial adoptions
Differential Amplifiers

Basic operational amplifier circuit
Amplifier Biasing

Amplifier biasing is that part of the design of a circuit that establishes the desired operating point (or Q-point)

Goal is to invariably minimize the impact the biasing circuit has on the small-signal performance of a circuit

Usually at most 2 dc power supplies are available and these are often fixed in value by system requirements – this restriction is cost driven

Discrete amplifiers invariably involve adding biasing resistors and use capacitor coupling and bypassing

Integrated amplifiers often use current sources which can be used in very large numbers and are very inexpensive
Amplifier Biasing

Example:

\[ A_V = -g_m R_L \]

Desired small-signal circuit
Common Emitter Amplifier

Actual small-signal circuit

\[ A_V = -g_m \left( R_L // R_{C1} \right) \]
Amplifier Biasing

Example:

Vin
\vdash\text{\raggedright Biased small-signal circuit}

Desired small-signal circuit
Common Emitter Amplifier

Vin
\vdash V_{\text{out}}

biasing components shown in blue

Biased small-signal circuit
Amplifier Biasing

Example:

Desired small-signal circuit
Common Collector Amplifier

Biased circuit

Biasing components shown in blue
Amplifier Biasing

Example:

Desired small-signal circuit
Inverting Feedback Amplifier

Biasing components shown in blue

Biased circuit
Other Basic Configurations

Darlington Configuration

- Current gain is approximately $\beta^2$
- Two diode drop between $B_{\text{eff}}$ and $E_{\text{eff}}$
Other Basic Configurations

Sziklai Pair

- Same basic structure as Darlington Pair
- Current gain is approximately $\beta_n \beta_p$
- Current gain will not be as large when $\beta_p < \beta_n$
- Only one diode drop between $B_{\text{eff}}$ and $E_{\text{eff}}$
Other Basic Configurations

Buffer and Super Buffer

- Voltage shift varies with $V_{IN}$ in buffer
- Current through shift transistor is constant for Super Buffer as $V_{IN}$ changes so voltage shift does not change with $V_{IN}$
- Same nominal voltage shift
Other Basic Configurations

Low offset buffers

- Actually a CC-CC or a CD-CD cascade
- Significant drop in offset between input and output
- Biasing with DC current sources
- Can Add Super Buffer to Output
Other Basic Configurations

- Attenuation factor is quite accurate (Determined by geometry)
- Infinite input impedance
- $M_1$ in triode, $M_2$ in saturation
- Actually can be a channel-tapped structure
End of Lecture 34