EE 330
Lecture 34

- High Gain Amplifiers
- Cascode and Cascade Configurations
Exam Schedule

Exam 1  Friday  Sept 24
Exam 2  Friday  Oct 22
Exam 3  Friday  Nov 19
Final   Tues Dec 14  12:00 p.m.
As a courtesy to fellow classmates, TAs, and the instructor, wearing of masks during lectures and in the laboratories for this course would be appreciated irrespective of vaccination status.
Current Sources/Mirrors

- Termed a “current mirror”
- Output current linearly dependent on $I_{in}$
- Serves as a current amplifier
- Widely used circuit

But $I_{in}$ and $I_{out}$ must be positive!
Review from Last Lecture

Current Sources/Mirrors

Multiple-Output Bipolar Current Source and Sink

\[
I_{nk} = \left[ \frac{A_{Enk}}{A_{E0}} \right] I_0
\]

\[
I_{pk} = \left[ \frac{A_{En1}}{A_{E0}} \right] \left[ \frac{A_{Epk}}{A_{Ep0}} \right] I_0
\]
Current Sources/Mirrors

Amplifiers both positive and negative currents (provided $i_{in}>-I_{BS}$)

Current amplifiers are easy to build!!

Current gain can be accurately controlled with appropriate layout!!

Review from Last Lecture

npn current mirror amplifier

$$i_{out} = \begin{bmatrix} \frac{A_{E1}}{A_{E0}} \end{bmatrix} i_{in}$$
Current Sources/Mirrors Summary

Review from Last Lecture

**n-channel Current Mirror**

\[ I_{out} = \left[ \frac{A_{E1}}{A_{E0}} \right] I_{in} \]

**npn Current Mirror**

\[ I_{out} = \left[ \frac{W_1}{W_0} \right] \frac{L_0}{L_1} I_{in} \]
Layout of Current Mirrors

Example with $M = 2$

Standard layout

$$M = \begin{bmatrix} W_2 & L_1 \\ W_1 & L_2 \end{bmatrix}$$

Better Layout

$$M = \begin{bmatrix} \frac{2W_1 + 4\Delta W}{W_1 + 2\Delta W} \cdot \frac{L_1 + 2\Delta L}{L_1 + 2\Delta L} \end{bmatrix} = 2$$
Centroid and Common Centroid

Denotes Geometric Centroid

Review from Last Lecture
Centroid and Common Centroid

Geometric Centroids of Channel

M_1

M_2

Review from Last Lecture
Review from Last Lecture

Centroid and Common Centroid

M_1

M_2

M_1

M_2
Centroid and Common Centroid

Common Centroid for Matched Devices

Review from Last Lecture
Review from Last Lecture

Centroid and Common Centroid

Common Centroid for Matched Devices
Review from Last Lecture

Centroid and Common Centroid

Common Centroid for Ratioed Devices

\[ M = \frac{W_2 L_1}{W_1 L_2} = 2 \]
If the threshold voltage of a transistor changes with position, it can be reasonably accurately modeled with an “equivalent” threshold voltage.

For linear gradient, $V_{\text{THEQ}} = V_{\text{TH}}(X_C, Y_C)$

$\times : (X_C, Y_C)$
Review from Last Lecture

Layout of Current Mirrors

Example with $M = 2$

Standard layout

Better Layout

Even Better Layout

This is termed a common-centroid layout

Linear gradient mismatch eliminated with common-centroid layout!

\[
M = \begin{bmatrix} W_2 & L_1 \\ W_1 & L_2 \end{bmatrix}
\]

\[
M = \left[ \frac{2W_1 + 4\Delta W}{W_1 + 2\Delta W} \cdot \frac{L_1 + 2\Delta L}{L_1 + 2\Delta L} \right] = 2
\]
n-channel current mirror current amplifier

Amplifies both positive and negative currents
Current Sources/Mirrors

\[ I_k = \begin{bmatrix} \frac{W_k}{W_0} & L_0 \end{bmatrix} I_0 \]

multiple output n-channel current sink array

multiple output p-channel current source array
Current Sources/Mirrors

multiple sourcing and sinking current outputs

\[ I_{pj} = \frac{W_{pj} \cdot L_{p0}}{W_{p0}} \cdot M_{I0} \]

\[ M = \frac{W_{n0} \cdot L_{0}}{L_{n0j} \cdot W_{0}} \]

\[ I_{nj} = \frac{W_{nj} \cdot L_{0}}{L_{nj} \cdot W_{0}} \cdot I_{0} \]

\[ M_{0} = \frac{W_{0} \cdot L_{0}}{W_{0} \cdot L_{n0}} \]

\[ W_{0, L_{0}} \]

\[ M_{n0} = \frac{W_{n0} \cdot L_{n0}}{W_{n0} \cdot L_{n0}} \]

\[ W_{n0, L_{n0}} \]

\[ I_{n0} \]

\[ I_{n1} \]

\[ I_{n2} \]

\[ I_{nk} \]

\[ W_{nk, L_{nk}} \]

\[ I_{0} \]

\[ I_{n0} \]

\[ I_{p0} \]

\[ I_{p1} \]

\[ I_{p2} \]

\[ I_{pm} \]

\[ V_{DD} \]

\[ M_{p0} \]

\[ M_{p1} \]

\[ M_{p2} \]

\[ M_{pm} \]

\[ W_{p0, L_{p0}} \]

\[ W_{p1, L_{p1}} \]

\[ W_{p2, L_{p2}} \]

\[ W_{pm, L_{pm}} \]

\[ M_{0} \]

\[ W_{0, L_{0}} \]

\[ M_{n0} \]

\[ W_{n0, L_{n0}} \]

\[ W_{n1, L_{n1}} \]

\[ W_{n2, L_{n2}} \]

\[ W_{nk, L_{nk}} \]

\[ m \text{ and } k \text{ may be different} \]

Often \( M = 1 \)
Why are we interested in high-gain amplifiers?

- High gain amplifiers typically have some very undesirable properties
  Nonlinear, gain highly dependent upon process variations and temperature, frequency response poor, noisy, 
  ....
- So we can build feedback amplifiers !!
How can we build the current source?

What is the small-signal model of an actual current source?
Basic Current Sources and Sinks

Bipolar Mirror-Based Current Sink

Biasing circuit uses same $V_{CC}$ as amplifier and no other independent sources
High-gain amplifier

- Bias circuitry requires only a single independent dc voltage source!
- **Incremental** overhead is only one transistor, $Q_B$
Basic Current Sources and Sinks

Basic Bipolar Current Sinks

\[ I_x = J_A e^{V_{xx}/V_t} \]

\[ I_x \approx \frac{V_{CC} - 0.6V}{R} \]

Basic Bipolar Current Sources

\[ V_{CC} \]

\[ V_{YY} \]

\[ V_{CC} \]

- Very practical methods for biasing the BJTs (or MOSFETs) can be used
- Current Mirrors often used for generating sourcing and sinking currents
- Can think of biasing transistors with \( V_{XX} \) and \( V_{YY} \) in these current sources
How can we build the current source?

What is the small-signal model of an actual current source?
Basic Current Sources and Sinks

Small-signal Model of BJT Current Sinks and Sources

Small-signal model of all other BJT Sinks and Sources introduced so far are the same
Basic Current Sources and Sinks

Small-signal Model of MOS Current Sinks and Sources

Small-signal model of all other MOS Sinks and Sources introduced thus far are the same
High-gain amplifier

\[ AV = \frac{-g_m}{g_0} \]

\[ AV = \frac{-g_{m1}}{g_{01} + g_{02}} \approx \frac{-g_{m1}}{2g_{01}} \]
High-gain amplifier

\[
A_V = \frac{-g_m}{g_0} \approx -8000
\]

- Nonideal current source decreased the gain by a factor of 2
- But the voltage gain is still quite large (-4000)

Can the gain be made even larger?
High-gain amplifier
Can the gain be made even larger?

The Cascode Configuration

Discuss
The Cascode Amplifier (consider npn BJT version)

• Actually a cascade of a CE stage followed by a CB stage but usually viewed as a “single-stage” structure

• Cascode structure is widely used
Basic Amplifier Structures

1. Common Emitter/Common Source
2. Common Collector/Common Drain
3. Common Base/Common Gate
4. Common Emitter with $R_E$ / Common Source with $R_S$
5. Cascode (actually CE:CB or CS:CD cascade)
6. Darlington (special CE:CE or CS:CS cascade)

The first 4 are most popular
(V_X + V_2)g_{o2} + V_2g_{m2} = I_X
V_1 g_{m1} - V_2(g_{o1} + g_{\pi2}) = I_X

Observing V_1 = V_{IN} and eliminating V_2 between these two equations, we obtain

V_{IN} = I_1 \cdot \frac{1}{g_{\pi1}}

and

V_X = I_X \cdot \left[ \frac{g_{o1} + g_{o2} + g_{\pi2} + g_{m2}}{g_{o2}(g_{o1} + g_{\pi2})} \right] - V_{IN} \cdot \left[ \frac{g_{m1}(g_{o2} + g_{m2})}{g_{o2}(g_{\pi2} + g_{o1})} \right]
Cascode Configuration

Two-port model of cascode amplifier

V_X = I_X \cdot \left[ \frac{g_{01} + g_{02} + g_{\pi2} + g_{m2}}{g_{02} (g_{01} + g_{\pi2})} \right] - V_{IN} \cdot \left[ \frac{g_{m1} (g_{02} + g_{m2})}{g_{02} (g_{\pi2} + g_{01})} \right]

V_{IN} = I_1 \cdot \frac{1}{g_{\pi1}}

It thus follows for the npn bipolar structure that:

\[ A_{VCC} = \frac{g_{m1} (g_{02} + g_{m2})}{g_{02} (g_{\pi2} + g_{01})} \approx \frac{g_{m1} g_{m2}}{g_{02} g_{\pi2}} \]

\[ g_{0CC} = \frac{g_{02} (g_{01} + g_{\pi2})}{g_{01} + g_{02} + g_{\pi2} + g_{m2}} \approx \frac{g_{02} g_{\pi2}}{g_{m2}} \]

\[ g_{\piCC} = g_{\pi1} \]
Cascode Configuration

Voltage gain is a factor of $\beta$ larger than that of the CE amplifier with current source load.

Output impedance is a factor of $\beta$ larger than that of the CE amplifier.

$$A_{VCC} \approx -\frac{g_{m1}g_{m2}}{g_{02}g_{\pi2}}$$

$$g_{0CC} \approx \frac{g_{02}g_{\pi2}}{g_{m2}}$$

$$g_{\pi CC} = g_{\pi 1}$$

$$A_{VCC} \approx -\left[ \frac{g_{m1}}{g_{02}} \right] \beta \approx -\left[ \frac{g_{m1}}{g_{01}} \right] \beta$$

$$g_{0CC} \approx \frac{g_{01}}{\beta}$$
Cascode Configuration

\[ A_{VCC} \approx - \left[ \frac{g_{m1}}{g_{02}} \right] \beta \approx - \left[ \frac{g_{m1}}{g_{01}} \right] \beta \]

\[ g_{0CC} \approx \frac{g_{02}}{\beta} \]

\[ A_{VCC} \approx - \left[ \frac{g_{m1}}{g_{01}} \right] \beta = \left[ \frac{2V_{AF}}{V_t} \right] \beta = [-8000]100 \]

\[ A_{VCC} \approx -800,000 \]

This gain is very large and only requires two transistors!

What happens to the gain if a transistor-level current source is used for \( I_B \)?
Cascode Configuration

\[ V_{XX} \]
\[ Q_1 \]
\[ Q_2 \]
\[ V_{IN} \]
\[ V_{SS} \]
\[ V_{OUT} \]
\[ I_B \]
\[ V_{CC} \]
Cascode Configuration
High-gain amplifier comparisons

It thus follows that

$$A_V = A_{VCC} \left[ \frac{g_{0CC}}{g_{03} + g_{0CC}} \right]$$

But $$g_{0CC} \approx g_{03}/\beta$$

$$A_V \approx A_{VCC} \left[ \frac{g_{0CC}}{g_{03}} \right] \approx \frac{A_{VCC}}{\beta}$$

This is a dramatic reduction in gain compared to what the ideal current source biasing provided.
Cascode Configuration

\[ A_V \approx A_{VCC} \left[ \frac{g_{0CC}}{g_{03}} \right] \approx \frac{A_{VCC}}{\beta} \]

But recall

\[ A_{VCC} \approx -\left[ \frac{g_{m1}}{g_{01}} \right] \beta \]

Thus

\[ A_V \approx -\left[ \frac{g_{m1}}{g_{01}} \right] \]

\[ A_V \approx -\left[ \frac{I_{CQ}}{V_t} \right] = -\left[ \frac{V_{AF}}{V_t} \right] \approx -8000 \]

- This is still a factor of 2 better than that of the CE amplifier with transistor current source \( A_{VCE} \approx -\left[ \frac{g_{m1}}{2g_{01}} \right] \)
- It only requires one additional transistor
- But its not nearly as good as the gain the cascode circuit seemed to provide
Cascode Configuration Comparisons

Gain limited by output impedance of current source!!
Can we design a better current source?
In particular, one with a higher output impedance?
Better current sources

Need a higher output impedance than $g_o$.

The output impedance of the cascode circuit itself was very large!

$$g_{0CC} \approx \frac{g_{01}}{\beta}$$

Can a current source be built with the cascode circuit?
Cascode current sources

Discuss
Cascode current sources

All have the same small-signal model

\[ g_{0CC} = \frac{g_{02}(g_{01} + g_{\pi2})}{g_{01} + g_{02} + g_{\pi2} + g_{m2}} \]
This gain is very large and is a factor of 2 below that obtained with an ideal current source biasing. Although the factor of 2 is not desired, the performance of this circuit is still very good. This factor of 2 gain reduction is the same as was observed for the CE amplifier when a transistor-level current source was used.
Can we use more cascoding to further increase the gain?
High Gain Amplifiers Seldom Used Open Loop

\[ A_V = \frac{-g_m}{g_0} \]

\[ A_V = -8,000 \]

\[ A_V = -400,000 \]

If \( A_V = -400,000 \) and \( V_{IN} \) increases by 1mV, what would happen at the output?

\( V_{OUT} \) would decrease by \( 400,000 \times 1 \text{mV} = -400 \text{V} \)
The Cascode Amplifier (consider n-ch MOS version)

\[ V_{CC} \]

\[ V_{IN} \]

\[ V_{SS} \]

\[ V_{OUT} \]

\[ V_{XX} \]

\[ I_B \]

\[ g_{m1}g_{m2} \]

\[ \frac{g_{01}g_{02}}{g_{m2}} \]

\[ A_{VCC} \approx - \begin{bmatrix} g_{m1}g_{m2} \\ \frac{g_{01}g_{02}}{g_{m2}} \end{bmatrix} \]

Same issues for biasing with current source as for BJT case

With cascode current source for \( I_B \), gain only drops by a factor of 2 from value with ideal current source
The Cascode Amplifier (consider n-ch MOS version)

\[ A_{VCC} \approx -\frac{g_{m1}g_{m2}}{g_{01}g_{02}} \]

\[ A_{VCC} \approx -\frac{g_{m1}}{g_{01}} \]

\[ A_{VCC} \approx -\frac{1}{2} \left[ \frac{g_{m1}g_{m2}}{g_{01}g_{02}} \right] \]
Current Source Summary (BJT)

**Basic**

- $V_{YY}$
- $V_{SS}$
- $Q_1$
- $I_x$

- $V_{CC}$
- $V_{YY}$
- $Q_1$
- $I_x$

- $g_0 \approx g_{01}$

**Cascode**

- $V_{XX}$
- $V_{YY}$
- $Q_2$
- $Q_1$
- $I_x$

- $V_{CC}$
- $V_{YY}$
- $Q_1$
- $I_x$

- $g_{0CC} \approx \frac{g_{01}}{\beta}$
Current Source Summary (MOS)

**Basic**

- $V_{YY}$
- $M_1$
- $V_{SS}$

- $V_{DD}$
- $M_1$

$$g_0 \approx g_{01}$$

**Cascode**

- $V_{XX}$
- $M_2$
- $V_{SS}$

- $V_{YY}$
- $M_1$
- $V_{ZZ}$
- $M_2$
- $V_{DD}$

$$g_0 \approx g_{01} \frac{g_{02}}{g_{m2}}$$
High Gain Amplifier Comparisons (n-ch MOS)

\[ A_V \approx -\left( \frac{g_{m1}}{g_{01}} \right) \]

\[ A_V \approx -\frac{1}{2} \left( \frac{g_{m1}}{g_{01}} \right) \]

\[ A_{\text{VCC}} \approx -\left( \frac{g_{m1}g_{m2}}{g_{01}g_{02}} \right) \]

\[ A_{\text{VCC}} \approx -\left( \frac{g_{m1}}{g_{01}} \right) \]

\[ A_{\text{VCC}} \approx -\frac{1}{2} \left( \frac{g_{m1}g_{m2}}{g_{01}g_{02}} \right) \]
High Gain Amplifier Comparisons (BJT)

\[ A_V = \frac{-g_m}{g_0} \]

\[ A_V \approx -\frac{1}{2} \frac{g_m1}{g_01} \]

\[ A_V \approx -\left[ \frac{g_m1}{g_01} \right] \beta \]

- Single-ended high-gain amplifiers inherently difficult to bias (because of the high gain)
- Biasing becomes practical when used in differential applications
- These structures are widely used but usually with differential inputs
The Cascode Amplifier

- Operational amplifiers often built with basic cascode configuration
- CMFB used to address the biasing problem
- Usually configured as a differential structure when building op amps
- Have high output impedance (but can be buffered)

- Terms “telescopic cascode”, “folded-cascode”, and “regulated cascode” often refer to op amps based upon the cascode configuration
Cascade Configurations

Two-stage CE:CE or CS:CS Cascade

\[ A_{VCB} = ? \]

\[ A_{VCM} = ? \]
Cascade Configurations

Two-stage CE:CE or CS:CS Cascade

\[ A_{VCB} \approx \left[ \frac{-g_{m1}}{g_{01}+g_{\pi2}} \right] \left[ \frac{-g_{m2}}{g_{02}} \right] \approx \frac{g_{m1}g_{m2}}{g_{\pi2}g_{02}} = \beta \frac{g_{m1}}{g_{02}} \]

\[ A_{VCM} = \left[ \frac{-g_{m1}}{g_{01}} \right] \left[ \frac{-g_{m2}}{g_{02}} \right] = \frac{g_{m1}g_{m2}}{g_{01}g_{02}} \]

- Significant increase in gain
- Gain is noninverting
- Comparable to that obtained with the cascode but noninverting
Cascade Configurations

Two-stage CE:CE or CS:CS Cascade

\[ A_{VCB} \approx \left[ \frac{-g_{m1}}{g_{01} + g_{03} + g_{\pi2}} \right] \left[ \frac{-g_{m2}}{g_{02} + g_{04}} \right] \approx \frac{g_{m1}g_{m2}}{2g_{\pi}g_{02}} = \beta \frac{g_{m1}}{2g_{02}} \]

\[ A_{VCM} = \left[ \frac{-g_{m1}}{g_{01} + g_{03}} \right] \left[ \frac{-g_{m2}}{g_{02} + g_{04}} \right] = \frac{g_{m1}g_{m2}}{4g_{01}g_{02}} \]

Note factor or 2 and 4 reduction in gain due to actual current source bias
Two-stage CE Cascade

- Large gains can be obtained by cascading
- Gains are multiplicative (when loading is included)
- Large gains used to build “Op Amps” and feedback used to control gain value
- Some attention is needed for biasing but it is manageable
- Minor variant of the two-stage cascade often used to build Op Amps
- Compensation of two-stage cascade needed if feedback is applied to maintain stability
- For many years three or more stages were seldom cascaded because of challenges in compensation to maintain stability though recently some industrial adoptions

Three-stage CE Cascade
Differential Amplifiers

Basic operational amplifier circuit
Stay Safe and Stay Healthy!