EE 330
Lecture 35

• Cascaded Amplifier Analysis and Design
• High Gain Amplifiers
• Current Source Biasing
• Current Sources and Mirrors
Exam 3  Friday April  13
Can use these equations only when small signal circuit is EXACTLY like that shown!!

Review from Last Lecture

<table>
<thead>
<tr>
<th></th>
<th>CE/CS</th>
<th>CC/CD</th>
<th>CB/CG</th>
<th>CEwRE/CSwRS</th>
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</thead>
<tbody>
<tr>
<td><strong>Av</strong></td>
<td><img src="image" alt="BJT" /> - ( g_m R_C ) ( \frac{I_{CQ} R_C}{V_t} )</td>
<td><img src="image" alt="BJT" /> ( \frac{g_m}{g_m + g_E} ) ( \frac{I_{CQ} R_E}{I_{CQ} R_E + V_t} )</td>
<td><img src="image" alt="BJT" /> ( g_m R_C ) ( \frac{I_{CQ} R_C}{V_t} )</td>
<td><img src="image" alt="BJT" /> - ( \frac{R_C}{R_E} )</td>
</tr>
<tr>
<td><strong>R\text{in}</strong></td>
<td>( \frac{\beta V_t}{I_C Q} ) ( \beta \frac{V_t}{I_C Q} ) ( \frac{V_t}{I_C Q} )</td>
<td>( \frac{r_n + \beta R_E}{2I_{DQ}} )</td>
<td>( \beta \frac{V_t}{I_C Q} ) ( \frac{V_E B}{2I_{DQ}} )</td>
<td>( \frac{r_n + \beta R_E}{2I_{DQ}} )</td>
</tr>
<tr>
<td><strong>R\text{out}</strong></td>
<td>( R_C )</td>
<td>( g_m ) ( \frac{V_t}{I_C Q} )</td>
<td>( R_C )</td>
<td>( R_C )</td>
</tr>
</tbody>
</table>
Why are we focusing on these basic circuits?

1. So that we can develop analytical skills
2. So that we can design a circuit
3. So that we can get the insight needed to design a circuit

Which is the most important?
Why are we focusing on these basic circuits?

1. So that we can develop analytical skills
2. So that we can design a circuit
3. So that we can get the insight needed to design a circuit

Which is the most important?

1. *So that we can get the insight needed to design a circuit*
2. *So that we can design a circuit*
3. *So that we can develop analytical skills*
Properties/Use of Basic Amplifiers

CE and CS

- More practical biasing circuits usually used
- \( R_C \) or \( R_D \) may (or may not) be load

- Large inverting gain
- Moderate input impedance for BJT (high for MOS)
- Moderate output impedance
- Most widely used amplifier structure
Properties/Use of Basic Amplifiers

CC and CD
(emitter follower or source follower)

- More practical biasing circuits usually used
- $R_E$ or $R_S$ may (or may not) be load

- Gain very close to $+1$ (little less)
- High input impedance for BJT (high for MOS)
- Low output impedance
- Widely used as a buffer
Properties/Use of Basic Amplifiers

CB and CG

- More practical biasing circuits usually used
- RC or RD may (or may not) be load

- Large noninverting gain
- Low input impedance
- Moderate (or high) output impedance
- Used more as current amplifier or, in conjunction with CD/CS to form two-stage cascode
Properties/Use of Basic Amplifiers

CEwRE or CSwRS

• More practical biasing circuits usually used
• $R_C$ or $R_D$ may (or may not) be load

• Accurate but small gain given by resistor ratio
• High input impedance
• Moderate output impedance
• Used when more accurate gain is required
Basic Amplifier Characteristics Summary

CE/CS

- Large inverting gain
- Moderate input impedance
- Moderate (or high) output impedance
- Widely used as the basic high gain inverting amplifier

CC/CD

- Gain very close to +1 (little less)
- High input impedance for BJT (high for MOS)
- Low output impedance
- Widely used as a buffer

CB/CG

- Large noninverting gain
- Low input impedance
- Moderate (or high) output impedance
- Used more as current amplifier or, in conjunction with CD/CS to form two-stage cascode

CEwRE/CSwRS

- Reasonably accurate but somewhat small gain (resistor ratio)
- High input impedance
- Moderate output impedance
- Used when more accurate gain is required
Cascaded Amplifiers

Amplifier cascading widely used to enhance gain
Amplifier cascading widely used to enhance other characteristics and/or alter functionality as well

\[ A_V = \frac{V_{out}}{V_{in}} = \left( \frac{R_{iX1}}{R_{iX1}+R_S} \right) A_{V01} \left( \frac{R_{iX2}}{R_{iX2}+R_{0X1}} \right) A_{V02} \left( \frac{R_L}{R_L+R_{0X2}} \right) \]

If \( R_o << R_i \) \hspace{1cm} \( R_S << R_i \) \hspace{1cm} \( R_o << R_L \)

\[ A_V \approx A_{V01}A_{V02} \]

- Amplifier cascading widely used to enhance gain
- Amplifier cascading widely used to enhance other characteristics and/or alter functionality as well
  e.g. (\( R_{iN} \), BW, Power, \( R_O \), Linearity, Impedance Conversion.. )
Cascaded Amplifier Analysis and Operation

Adjacent Stage Coupling Only

- Systematic Methods of Analysis/Design will be Developed

One or more couplings of nonadjacent stages

- Less Common
- Analysis Generally Much More Involved, Use Basic Circuit Analysis Methods
Cascaded Amplifier Analysis and Operation

Adjacent Stage Coupling Only

- Systematic Methods of Analysis/Design will be Developed

Case 1: All stages Unilateral

Case 2: One or more stages are not unilateral
Repeat from earlier discussions on amplifiers

Cascaded Amplifier Analysis and Operation

Case 1: All stages Unilateral

\[
A_V = \frac{V_{out}}{V_{in}} = \left( \frac{R_{iX1}}{R_{iX1} + R_S} \right) A_{V01} \left( \frac{R_{L1} / R_{iX2}}{R_{L1} / R_{iX2} + R_{0X1}} \right) A_{V02} \left( \frac{R_L}{R_L + R_{0X2}} \right)
\]

Accounts for all loading between stages!
Cascaded Amplifier Analysis and Operation

Case 2: One or more stages are not unilateral

- Standard two-port cascade

Analysis by creating new two-port of entire amplifier quite tedious because of the reverse-gain elements

- Right-to-left nested $R_{\text{inx}}, A_{\text{VKX}}$ approach

- $R_{\text{inx}}$ includes effects of all loading
- $A_{\text{VKX}}$ is the voltage ratio from input to output of a stage
- $A_{\text{VKX}}$’s include all loading
- Can not change any loading without recalculating everything!
Example: Goal: Determine the voltage gain of the following circuit in terms of the small-signal parameters of the transistors.

Q-point: 1) Assume Q₁ and Q₂ in Forward Active; 2) Open all caps, C₁ to C₄; 3) replace Q₁ and Q₂ by simple model; 4) compute all V and I; 5) verify Q₁ and Q₂ in F.A.

Assume I_B very small.
Example:

Ground $V_{CC}$
Short coupling caps

Will calculate $A_V$ by determining the three ratios (not voltage gains of dependent source):

$$A_V = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_B} \cdot \frac{V_B}{V_A} \cdot \frac{V_A}{V_{in}} = A_{V2}A_{V1}A_{V0}$$
Example:

\[
\frac{v_{\text{out}}}{v_B} \approx - \frac{R_6 // R_8}{R_7}
\]

\[
R_{\text{in2}} \approx \beta R_7
\]
Example:

\[ R_{\text{in2}} \approx \beta R_7 \]
Example:

\[ \frac{v_{out}}{v_B} \approx -\frac{R_6//R_8}{R_7} \]

\[ R_{in2} \approx \beta R_7 \]
Example:

\[ \frac{v_B}{v_A} \approx -g_{m1}(R_3//R_5//R_{in2}) \]

\[ R_{in1} \approx r_{\pi 1} \]
Example:
Example:

\[
\frac{V_A}{V_{in}} \approx \frac{R_1//R_2//R_{in}}{R_S + R_1//R_2//R_{in1}}
\]
Example:

Thus we have

\[ A_V = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{V_{\text{out}}}{V_B} \cdot \frac{V_B}{V_A} \cdot \frac{V_A}{V_{\text{in}}} \]

where

\[ \frac{V_{\text{out}}}{V_B} \approx -\frac{R_6}{R_7} \]

\[ \frac{V_B}{V_A} \approx -g_{m1} \left( \frac{R_3}{R_5} \right) \]

\[ \frac{V_A}{V_{\text{in}}} \approx \frac{R_1}{R_S + \frac{R_1}{R_2} + \frac{1}{R_{in1}}} \]

\[ R_{in2} \approx \beta R_7 \]

\[ R_{in1} \approx r_{\pi1} \]
Formalization of cascade circuit analysis working from load to input:

(when stages are unilateral or not unilateral)

\[ \frac{V_{OUT}}{V_{IN}} = \frac{V_1}{V_{IN}} \cdot \frac{V_2}{V_1} \cdot \frac{V_3}{V_2} \cdot \frac{V_{OUT}}{V_3} \]

\( R_{ink} \) includes effects of all loading
Must recalculate if any change in loading
Analysis systematic and rather simple
Example:

Observation: By working from the output back to the input we were able to create a sequence of steps where the circuit at each step looked EXACTLY like one of the four basic amplifiers. Engineers often follow a design approach that uses a cascade of the basic amplifiers and that is why it is often possible to follow this approach to analysis.

Two other methods could have been used to analyze this circuit.

What are they?
Two other methods could have been used to analyze this circuit

1. Create a two-port model of the two stages
   (for this example, since the first-stage is unilateral, it can be shown that )

$$A_V = \frac{v_{out}}{v_{in}} = \frac{v_A}{v_{in}} \frac{v_B}{v_{out}}$$
Two other methods could have been used to analyze this circuit

2. Put in small-signal model for $Q_1$ and $Q_2$ and solve resultant circuit

   (not too difficult for this specific example but time consuming)
Example: \[ A_V = \frac{V_{\text{out}}}{V_{\text{in}}} = ? \] Express in terms of small-signal parameters.
Example: \( A_v = \frac{V_{out}}{V_{in}} = ? \)

Express in terms of small-signal parameters.
Example:

\[
A_v = \frac{v_{out}}{v_2} \frac{v_2}{v_1} \frac{v_1}{v_{in}} \approx \left[ -g_{m4} \left( R_D / R_L \right) \right][1] \left[ \frac{-g_{m1}}{g_{m2} + \left( \beta_3 \left( R_{B1} / R_{B2} \right) \right)^{-1}} \right]
\]
High-gain BJT amplifier

\[ A_V = \frac{-g_m}{g_0 + G_C} \approx -g_m R_C \]

To make the gain large, it appears that all one needs to do is make \( R_C \) large!

\[ A_V \approx -g_m R_C = \frac{-I_{CQ} R_C}{V_t} \]

But \( V_t \) is fixed at approx 25mV and for good signal swing, \( I_{CQ} R_C < (V_{DD} - V_{EE})/2 \)

\[ |A_V| < \frac{V_{DD} - V_{EE}}{2V_t} \]

If \( V_{DD} - V_{EE} = 1.8V \),

\[ |A_V| < \frac{1.8V}{2 \cdot 25mV} = 36 \]

- Gain is practically limited with this supply voltage to 10’s
High-gain MOS amplifier

\[ AV = \frac{-g_m}{g_0 + G_D} \approx -g_m R_D \]

To make the gain large, it appears that all one needs to do is make \( R_D \) large!

\[ AV \approx -g_m R_D = \frac{-2I_{DQ} R_D}{V_{EB}} \]

But \( V_{EB} \) is practically limited to around 100mV and for good signal swing, \( I_{DQ} R_D < (V_{DD} - V_{SS})/2 \)

\[ |AV| < \frac{V_{DD} - V_{SS}}{V_{EB}} \]

If \( V_{DD} - V_{SS} = 1.8V \) and \( V_{EB} = 200mV \),

\[ |AV| < \frac{1.8V}{200mV} = 9 \]

Gain is limited by supply voltage and \( V_{EB} \) to around 10

Are these fundamental limits on the gain of the BJT and MOS Amplifiers?
High-gain amplifier

This gain is very large!
And no design parameters affect the gain.

\[ A_V = \frac{-g_m}{g_0} = -\frac{V_{AF}}{V_t} \]

\[ A_V = -\frac{V_{AF}}{V_t} \approx \frac{200V}{25mV} = -8000 \]
High-gain amplifier

\[ A_V \approx -8000 \]

How can we build ideal dc current sources?

What is the small-signal model of an actual current source?
Model of dc Current Source

Ideal dc Current Source

$I_{XX}$ independent of $V_1$ and $t$

“Realistic dc Current Source”

$I_{XX}$ independent of $V_1$ and $t$, $R_S$ large

Small-signal model of dc current source

want $R_{IN}$ large
Current Sources/Mirrors

Will show circuit in red behaves as a current source
Current Sources/Mirrors

\[ I_0 \approx \frac{(V_{CC} - 0.6V)}{R} \]

If the base currents are neglected

\[ I_0 = J_S A_{E0} e^{\frac{V_{BE0}}{V_t}} \]

\[ I_1 = J_S A_{E1} e^{\frac{V_{BE1}}{V_t}} \]

since \( V_{BE1} = V_{BE0} \)

\[ I_1 \approx \left( \frac{A_{E1}}{A_{E0}} \right) I_0 = \left( \frac{A_{E1}}{A_{E0}} \right) \frac{V_{CC} - 0.6V}{R} \]

Note \( I_1 \) is not a function of \( V_1 \)

Behaves as a current source! So is ideal with this model!!

Actually termed a “sink” current since coming out of load

And does not require an additional dc voltage source!!!
Current Sources/Mirrors

- Multiple Outputs Possible
- Can be built for sourcing or sinking currents
- Also useful as a current amplifier
- MOS counterparts work very well and are not plagued by base current
Current Sources/Mirrors

Biasing Circuit

Key Block

Current Sink
Current Sources/Mirrors

Multiple-Output Bipolar Current Sink

\[ I_k = \left[ \frac{A_{E_k}}{A_{E0}} \right] I_0 \]
Current Sources/Mirrors

Multiple-Output Bipolar Current Source

\[ I_k = \left[ \frac{A_{E_k}}{A_{E_0}} \right] I_0 \]
Current Sources/Mirrors

Multiple-Output Bipolar Current Source and Sink

\[ I_{nk} = ? \quad I_{pk} = ? \]
Multiple-Output Bipolar Current Source and Sink

\[ I_{nk} = \begin{bmatrix} \frac{A_{Enk}}{A_{E0}} \end{bmatrix} I_0 \]

\[ I_{pk} = \begin{bmatrix} \frac{A_{En1}}{A_{E0}} & \frac{A_{Epk}}{A_{Ep0}} \end{bmatrix} I_0 \]
Current Sources/Mirrors

- Termed a “current mirror”
- Output current linearly dependent on $I_{in}$
- Serves as a current amplifier
- Widely used circuit

But $I_{in}$ and $I_{out}$ must be positive!
**Current Sources/Mirrors**

npn current mirror amplifier

\[ \frac{i_{OUT} + MI_{BS}}{i_{in} + I_{BS}} = M \]

\[ i_{OUT} + MI_{BS} = M \left( i_{in} + I_{BS} \right) \]

\[ i_{OUT} + MI'_{BS} = M \left( i_{in} + I'_{BS} \right) \]

\[ \frac{i_{OUT}}{i_{in}} = M \]

But \( I_{BS} + i_{in} > 0 \)!
Current Sources/Mirrors

Amplifiers both positive and negative currents (provided $i_{in} > -I_{BS}$)

Current amplifiers are easy to build!!

Current gain can be accurately controlled with appropriate layout!!

npn current mirror amplifier

$$i_{out} = \begin{bmatrix} A_{E1} \\ A_{E0} \end{bmatrix} i_{in}$$
Current Sources/Mirrors

n-p-n Current Mirror

n-channel Current Mirror

$I_{out} =$?
Current Sources/Mirrors

If process parameters are matched, it follows that

\[
\begin{align*}
I_{\text{in}} &= \frac{\mu C_{\text{OX}} W_0}{2L_0} (V_{\text{GS0}} - V_{T0})^2 \\
I_{\text{out}} &= \frac{\mu C_{\text{OX}} W_1}{2L_1} (V_{\text{GS1}} - V_{T1})^2
\end{align*}
\]

If process parameters are matched, it follows that

\[
I_{\text{out}} = \left[ \frac{W_1}{W_0} \frac{L_0}{L_1} \right] I_{\text{in}}
\]

- Current mirror gain can be accurately controlled!
- Layout is important to get accurate gain (for both MOS and BJT)
Current Sources/MIRRORS

\[ I_{\text{out}} = \left[ \frac{A_{E1}}{A_{E0}} \right] I_{\text{in}} \]

\[ I_{\text{out}} = \left[ \frac{W_1}{W_0} \frac{L_0}{L_1} \right] I_{\text{in}} \]
Layout of Current Mirrors

Example with $M = 2$

<table>
<thead>
<tr>
<th>$W_1$</th>
<th>$L_1$</th>
<th>$W_2$</th>
<th>$L_1$</th>
</tr>
</thead>
</table>

$M = \left[ \begin{array}{c} \frac{W_2}{W_1} \\ \frac{L_1}{L_2} \end{array} \right]$  

Standard layout

$M = \left[ \begin{array}{c} \frac{W_2 + 2\Delta W}{W_1 + 2\Delta W} \\ \frac{L_1 + 2\Delta L}{L_2 + 2\Delta L} \end{array} \right]$  

Gate area after fabrication depicted

$M = \left[ \begin{array}{c} \frac{2W_1 + 2\Delta W}{W_1 + 2\Delta W} \\ \frac{L_1 + 2\Delta L}{L_1 + 2\Delta L} \end{array} \right] \neq 2$
Layout of Current Mirrors

Example with $M = 2$

Standard layout

Better Layout

$M = \begin{bmatrix} W_2 & L_1 \\ W_1 & L_2 \end{bmatrix}$

$M = \begin{bmatrix} \frac{2W_1 + 2\Delta W}{W_1 + 2\Delta W} & \frac{L_1 + 2\Delta L}{L_1 + 2\Delta L} \end{bmatrix} \neq 2$

$M = \begin{bmatrix} \frac{2W_1 + 4\Delta W}{W_1 + 2\Delta W} & \frac{L_1 + 2\Delta L}{L_1 + 2\Delta L} \end{bmatrix} = 2$
Layout of Current Mirrors

Example with $M = 2$

Standard layout

Better Layout

Even Better Layout

This is termed a common-centroid layout
n-channel current mirror current amplifier

Amplifies both positive and negative currents
Current Sources/Mirrors

\[ I_k = \begin{bmatrix} \frac{W_k}{W_0} \\ L_0 \end{bmatrix} I_0 \]

multiple output n-channel current sink array

multiple output p-channel current source array
Current Sources/Mirrors

multiple sourcing and sinking current outputs

\[ M = \left[ \begin{array}{c} W_{nj} \\ L_{nj} \\ W_0 \\ L_0 \end{array} \right] I_0 \]

\[ I_{nj} = \left[ \begin{array}{c} W_{nj} \\ L_{nj} \\ W_0 \\ L_0 \end{array} \right] I_0 \]

\[ M_p = \left[ \begin{array}{c} W_{pj} \\ L_{pj} \\ W_0 \\ L_0 \end{array} \right] M I_0 \]

m and k may be different
Often M = 1
High-gain amplifier

\[ A_V \approx -8000 \]

How can we build the current source?

What is the small-signal model of an actual current source?
Basic Current Sources and Sinks

Bipolar Mirror-Based Current Sink

Bipolar Mirror-Based Current Source

Biasing circuit uses same $V_{CC}$ as amplifier and no other independent sources
High-gain amplifier

- Bias circuitry requires only a single independent dc voltage source!
- Incremental overhead is only one transistor, $Q_B$
Basic Current Sources and Sinks

- Very practical methods for biasing the BJTs (or MOSFETs) can be used.
- Current Mirrors often used for generating sourcing and sinking currents.
- Can think of biasing transistors with $V_{xx}$ and $V_{yy}$ in these current sources.
How can we build the current source?

What is the small-signal model of an actual current source?
**Basic Current Sources and Sinks**

Small-signal Model of BJT Current Sinks and Sources

Small-signal model of all other BJT Sinks and Sources introduced so far are the same
Basic Current Sources and Sinks

Small-signal Model of MOS Current Sinks and Sources

Small-signal model of all other MOS Sinks and Sources introduced thus far are the same
High-gain amplifier

$$A_V = \frac{-g_m}{g_0}$$

$$A_V = \frac{-g_{m1}}{g_{01} + g_{02}} \approx \frac{-g_{m1}}{2g_{01}}$$
High-gain amplifier

\[
g_m = \frac{V_{AF}}{g_0} \approx 8000
\]

\[
A_V = \frac{-g_m}{g_0}
\]

- Nonideal current source decreased the gain by a factor of 2
- But the voltage gain is still quite large (-4000)

Can the gain be made even larger?
End of Lecture 35