Complete Logic Family

Other Logic Styles
  − complex logic gates
  − pass transistor logic

Improved Device Models
Review from Last Time

The key patents that revolutionized the electronics field:

Jack Kilby (34 years old at invention)  
 patented: 3,138,743  
 Filed  Feb 6, 1959  
 Issued  June 23, 1964

Robert Noyce (31 years old at invention)  
 patented: 2,981,877  
 Filed  July 30, 1959  
 Issued  April 25, 1961
MOS Transistor
Qualitative Discussion of n-channel Operation

Cross-Sectional View

Top View

Designer always works with top view
Complete Symmetry in construction between Drain and Source

Review from Last Time
MOS Transistor

Qualitative Discussion of n-channel Operation

Review from Last Time

Equivalent Circuit for n-channel MOSFET

- Source assumed connected (or close to) ground
- Boolean G at gate is relative to ground potential

This is the first model we have for the n-channel MOSFET!
Review from Last Time

MOS Transistor MODEL

Equivalent Circuit for n-channel MOSFET

Mathematically:

\[ I_D = 0 \quad \text{if} \quad V_{GS} \quad \text{is low} \]
\[ V_{DS} = 0 \quad \text{if} \quad V_{GS} \quad \text{is high} \]
MOS Transistor
Qualitative Discussion of p-channel Operation

Equivalent Circuit for p-channel MOSFET

Source assumed connected to (or close to) positive $V_{DD}$ and Boolean $G$ at gate is relative to ground potential

This is the first model we have for the p-channel MOSFET!
Review from Last Time

MOS Transistor MODEL

Equivalent Circuit for p-channel MOSFET

Mathematically:

\[ I_D = 0 \quad \text{if } V_G \text{ is high } \left( |V_{Gsp}| \text{ is small} \right) \]

\[ V_{DS} = 0 \quad \text{if } V_G \text{ is low } \left( |V_{Gsp}| \text{ is large} \right) \]
MOS Transistor

Comparison of Operation

Review from Last Time

Source assumed connected to (or close to) ground

Source assumed connected to $V_{DD}$ and Boolean G at gate is relative to ground
A model for the n-channel and p-channel transistor has been developed

- *Termed the ideal switch-level model*
- *Several other models will be developed later*
- *Invariably use simplest model that is justifiable*
- *Will introduce better models only when needed*

Symbols have been introduced for the two basic transistors

- *Other symbols will be introduced later*
Logic Circuits

Circuit Behaves as a Boolean Inverter
Logic Circuits

Inverter

Truth Table

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<th>A</th>
<th>B</th>
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Logic Circuits

A = 0
B = 0
C = 1
V_{DD}
Logic Circuits

\[
\begin{align*}
A &= 1 \\
B &= 0 \\
C &= 0
\end{align*}
\]
Logic Circuits

A = 0
B = 1
C = 0

V_{DD}

A
B
C

V_{DD}
Logic Circuits

A = 1
B = 1
A = 1
B = 1
C = 0
C = 0
Logic Circuits

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Logic Circuits

Approach can be extended to arbitrary number of inputs

n-input NOR gate

n-input NAND gate
Complete Logic Family

Family of n-input NOR gates forms a complete logic family

Family of n-input NAND gates forms a complete logic family

Having both NAND and NOR gates available is a luxury

Can now implement any combinational logic function !!

If add one flip flop, can implement any Boolean system !!

Flip flops easy to design but will discuss sequential logic systems later
Other logic circuits

• Other methods for designing logic circuits exist

• Insight will be provided on how other logic circuits evolve

• Several different types of logic circuits are often used simultaneously in any circuit design
PU network comprised of p-channel device and “tries” to pull B to VDD when conducting
PD network comprised of n-channel device and “tries to pull B to GND when conducting
One and only one of these networks is conducting at the same time
Pull-up and Pull-down Networks

PU network comprised of p-channel devices
PD network comprised of n-channel devices
One and only one of these networks is conducting at the same time
Pull-up and Pull-down Networks

PU network comprised of p-channel devices
PD network comprised of n-channel devices
One and only one of these networks is conducting at the same time
Pull-up and Pull-down Networks

In these circuits, the PUN and PDN have the 3 interesting characteristics

1. PU network comprised of p-channel devices
2. PD network comprised of n-channel devices
3. One and only one of these networks is conducting at the same time

What are $V_H$ and $V_L$?
What is the power dissipation?
How fast are these logic circuits?
What are $V_H$ and $V_L$?
What is the power dissipation?
How fast are these logic circuits?

Consider the inverter

Use switch-level model for MOS devices
What are $V_H$ and $V_L$?  
What is the power dissipation?  
How fast are these logic circuits?  

Consider the inverter  
Use switch-level model for MOS devices  

$V_H = V_{DD}$  
$V_L = 0$  

$I_D = 0$ thus $P_H = P_L = 0$  
$t_{HL} = t_{LH} = 0$  

(too good to be true?)
Pull-up and Pull-down Networks

For these circuits, the PUN and PDN have 3 interesting characteristics

Three key characteristics of these Static CMOS Gates
1. PU network comprised of p-channel devices
2. PD network comprised of n-channel devices
3. One and only one of these networks is conducting at the same time

Three key properties of these Static CMOS Gates
1. What are $V_H$ and $V_L$?
   \[ V_H = V_{DD}, \quad V_L = 0 \] (too good to be true?)
2. What is the power dissipation?
   \[ P_H = P_L = 0 \] (too good to be true?)
3. How fast are these logic circuits?
   \[ t_{HL} = t_{LH} = 0 \] (too good to be true?)

These 3 properties are inherent in all Boolean circuits that have these 3 characteristics !!!
Pull-up and Pull-down Networks

Three key characteristics of Static CMOS Gates:
1. PU network comprised of p-channel devices
2. PD network comprised of n-channel devices
3. One and only one of these networks is conducting at the same time

Three properties of Static CMOS Gates (based upon simple switch-level model):
1. \( V_H = V_{DD}, \ V_L = 0 \) (too good to be true?)
2. \( P_H = P_L = 0 \) (too good to be true?)
3. \( t_{HL} = t_{LH} = 0 \) (too good to be true?)

These 3 properties are inherent in Boolean circuits with these 3 characteristics
Pull-up and Pull-down Networks

Concept can be extended to arbitrary number of inputs

n-input NOR gate

n-input NAND gate
Pull-up and Pull-down Networks

- PU network comprised of p-channel devices
- PD network comprised of n-channel devices
- One and only one of these networks is conducting at the same time

Concept can be extended to arbitrary number of inputs

n-input NOR gate

n-input NAND gate

1. PU network comprised of p-channel devices
2. PD network comprised of n-channel devices
3. One and only one of these networks is conducting at the same time
Pull-up and Pull-down Networks

1. PU network comprised of p-channel devices
2. PD network comprised of n-channel devices
3. One and only one of these networks is conducting at the same time

\[ V_{H} = V_{DD}, \quad V_{L} = 0 \]

\[ P_{H} = P_{L} = 0 \]

\[ t_{HL} = t_{LH} = 0 \]
In this class, logic circuits that are implemented by interconnecting multiple-input NAND and NOR gates will be referred to as “Static CMOS Logic”.

Since the set of NAND gates is complete, any combinational logic function can be realized with the NAND circuit structures considered thus far.

Since the set NOR gates is complete, any combinational logic function can be realized with the NOR circuit structures considered thus far.

Many logic functions are realized with “Static CMOS Logic” and this is probably the dominant design style used today!
Example 1:

How many transistors are required to realize the function

\[ F = A \cdot \overline{B} + \overline{A} \cdot C \]

in a basic CMOS process if static NAND and NOR gates are used? Assume A, B and C are available.
Example 1:

How many transistors are required to realize the function

\[ F = \overline{A \cdot B} + \overline{A} \cdot C \]

in a basic CMOS process if static NAND and NOR gates are used? Assume A, B and C are available.

Solution:

20 transistors and 5 levels of logic
Example 1:

How many transistors are required to realize the function

\[ F = \overline{A} \cdot \overline{B} + \overline{A} \cdot \overline{C} \]

in a basic CMOS process if static NAND and NOR gates are used? Assume A, B and C are available.

Solution (alternative): From basic Boolean Manipulations

\[ F = \overline{A} + \overline{B} + \overline{A} \cdot \overline{C} = \overline{A} + B + \overline{A} \cdot \overline{C} \]

\[ F = \overline{A} \cdot (1 + C) + B = \overline{A} + B \]

8 transistors and 3 levels of logic
Example 1:

How many transistors are required to realize the function

\[ F = \overline{A} \cdot \overline{B} + \overline{A} \cdot \overline{C} \]

in a basic CMOS process if static NAND and NOR gates are used? Assume A, B and C are available.

Solution (alternative): From basic Boolean Manipulations

\[ F = \overline{A} \cdot (1 + C) + B = \overline{A} + B \]

\[ F = \overline{A} + B = A \cdot \overline{B} \]

6 transistors and 2 levels of logic
Example 2: XOR Function

\[ Y = A \oplus B \]

A widely-used 2-input Gate

Static CMOS implementation

\[ Y = A\overline{B} + \overline{A}B \]

22 transistors  5 levels of logic

Delays unacceptable (will show later) and device count is too large!
Example 3: \[ Y = (A \cdot B) + (C \cdot D) \]

Standard Static CMOS Implementation

3 levels of Logic

16 Transistors if Basic CMOS Gates are Used

Can the same Boolean functionality be obtained with less transistors?
Observe:

Significant reduction in transistor count and levels of logic for realizing same Boolean function

Termed a “Complex Logic Gate” implementation

Some authors term this a “compound gate”
Complex Logic Gates

\[ Y = (A \cdot B) + (C \cdot D) \]
Complex Gates

Pull up and pull down network never both conducting

One of the two networks is always conducting
When the logic gate shown is not a multiple-input NAND or NOR gate but has Characteristics 1, 2, and 3 above, the gate will be referred to as a Complex Logic Gate.

Complex Logic Gates also implement static logic functions and some authors would refer to this as Static CMOS Logic as well but we will make the distinction and refer to this as “Complex Logic Gates”
Complex Gates

Complex Gate Design Strategy:

1. Implement $\overline{Y}$ in the PDN

2. Implement $Y$ in the PUN (must complement the input variables since p-channel devices are used)

($Y$ and $\overline{Y}$ often expressed in either SOP or POS form)
XOR in Complex Logic Gates

\[ Y = A \oplus B \]

Will express \( \bar{Y} \) and \( Y \) in standard SOP or POS form
XOR in Complex Logic Gates

\[ Y = A \oplus B \]

\[ Y = AB + \overline{AB} \]

\[ \overline{Y} = (AB + \overline{AB}) \]

\[ \overline{Y} = AB \cdot \overline{AB} \]

\[ \overline{Y} = (\overline{A} + B) \cdot (A + \overline{B}) \]
XOR in Complex Logic Gates

Y = \overline{A} \overline{B} + \overline{A}B
Y = (\overline{A} + B) \cdot (A + B)
XOR in Complex Logic Gates

\[ Y = \overline{A} \overline{B} + \overline{A}B \]

\[ \overline{Y} = (\overline{A} + B) \cdot (A + \overline{B}) \]

12 transistors and 2 levels of logic

Notice a significant reduction in the number of transistors required
XOR in Complex Logic Gates

\[ Y = AB + \overline{AB} \]

\[ \overline{Y} = (\overline{A} + B) \cdot (A + \overline{B}) \]

Multiple PU and PD networks can be used

\[ \overline{Y} = (\overline{A} + B) \cdot (A + \overline{B}) \]

\[ = (\overline{A} \cdot (A + \overline{B})) + (B \cdot (A + \overline{B})) \]

\[ = (\overline{A} \cdot \overline{B}) + (A \cdot B) \]
Complex Logic Gate Summary:

- **PU network** comprised of p-channel device
- **PD network** comprised of n-channel device
- One and only one of these networks is conducting at the same time

![Diagram](attachment:image.png)

If PUN and PDN satisfy the characteristics:

1. PU network comprised of p-channel device
2. PD network comprised of n-channel device
3. One and only one of these networks is conducting at the same time

**Properties of PU/PD logic of this type (with simple switch-level model):**

- Rail to rail logic swings
- Zero static power dissipation in both \( Y=1 \) and \( Y=0 \) states
- Arbitrarily fast (too good to be true? will consider again with better model)
Consider \[ Y = A \cdot B \] 

Standard CMOS Implementation

\[ \begin{align*} A & \quad \text{AND gate} \quad \text{OR gate} \quad Y \\ B & \quad \end{align*} \]

2 levels of Logic

6 Transistors if Basic CMOS Gates are Used

Basic noninverting functions generally require more complexity if basic CMOS gates are used for implementation
Pass Transistor Logic

\[ Y = A \cdot B \]

Requires only 2 transistors rather than 6 for a standard CMOS gate (and a resistor).
Pass Transistor Logic

\[ Y = A \cdot B \]

Even simpler pass transistor logic implementations are possible.

Requires only 1 transistor (and a resistor).

Will see later that the area of a single practical resistor for this circuit may be comparable to that needed for hundreds or even thousands of transistors.
May be able to replace resistor with transistor (one of several ways shown)

But high logic level can not be determined with existing device model (or even low logic level for circuit on right)

Power dissipation can not be determined with existing device model for circuit on right

Better device model is needed (Power? Signal Swing? Speed?)
Pass Transistor Logic

\[ Y = A \oplus B \]

6 transistors, 1 resistor, two levels of logic

(the 4 transistors in the two inverters are not shown)
Pass Transistor Logic

\[ Y = A \oplus B \]

2 transistors, 1 resistor, one level of logic
Pass Transistor Logic

Requires only 1 transistor (and a resistor)

- Pass transistor logic can offer significant reductions in complexity for some functions (particularly noninverting)
- Resistor may require more area than several hundred or even several thousand transistors
- Signal levels may not go to $V_{DD}$ or to 0V
- Static power dissipation may not be zero
- Signals may degrade unacceptably if multiple gates are cascaded
- “resistor” often implemented with a transistor to reduce area but signal swing and power dissipation problems still persist
- Pass transistor logic is widely used
Logic Design Styles

• Several different logic design styles are often used throughout a given design (3 considered thus far)
  – Static CMOS
  – Complex Logic Gates
  – Pass Transistor Logic

• The designer has complete control over what is placed on silicon and governed only by cost and performance

• New logic design strategies have been proposed recently and others will likely emerge in the future

• The digital designer needs to be familiar with the benefits and limitations of varying logic styles to come up with a good solution for given system requirements
Simple model of MOSFET was developed (termed switch-level model)

Simple gates designed in CMOS Process were introduced
  – Some have zero power dissipation
  – Some have or appeared to have rail to rail logic voltage swings
  – All appeared to be Infinitely fast
  – Logic levels of some can not be predicted with simple model
  – Simple model is not sufficiently accurate to provide insight relating to some of these properties

MOSFET modeling strategy
  – hierarchical model structure will be developed
  – generally use simplest model that can be justified
MOS Transistor Models

1. Switch-Level model

Advantages:
Simple, does not require understanding of semiconductor properties, does not depend upon process, adequate for understanding basic operation of many digital circuits

Limitations:
Does not provide timing information (surfaced when looking at static CMOS circuits, and several others that have not yet become apparent from the applications that have been considered) and cannot support design of "resistor" used in Pass Transistor Logic
Improved Device Models

With the simple switch-level model, it was observed that basic static CMOS logic gates have the following three properties:

- Rail to rail logic swings
- Zero static power dissipation in both \( Y=1 \) and \( Y=0 \) states
- Arbitrarily fast (too good to be true? will consider again with better model)

It can be shown that the first two properties are nearly satisfied in actual fabricated circuits but though the circuits are fast, they are observably not arbitrarily fast

Will now extend switch-level model to predict speed of basic gates
Recall

MOS Transistor
Qualitative Discussion of n-channel Operation

This was the first model introduced and was termed the basic switch-level model.
MOS Transistor
Qualitative Discussion of \textit{n}-channel Operation

Conceptual view of basic switch-level model
MOS Transistor
Qualitative Discussion of n-channel Operation

For $V_{GS}$ small

For $V_{GS}$ large
End of Lecture 5