w.l.o.g., V_1, V_2, V_3 will be chosen as the independent variables

$\therefore$ Modeling problem reduces to finding f_1, f_2, f_3 such that

$$I_1 = f_1(V_1, V_2, V_3)$$

$$I_2 = f_2(V_1, V_2, V_3)$$

$$I_3 = f_3(V_1, V_2, V_3)$$

$$\binom{6}{3} = \frac{6!}{3!\,3!} = \frac{6 \cdot 5 \cdot 4}{6} = 20$$

Note: $20 \cdot 4 = 80$ different but equivalent models can be obtained for <u>any</u> 4-terminal device.

$$I_1 = f_1(V_1, V_2, V_3)$$

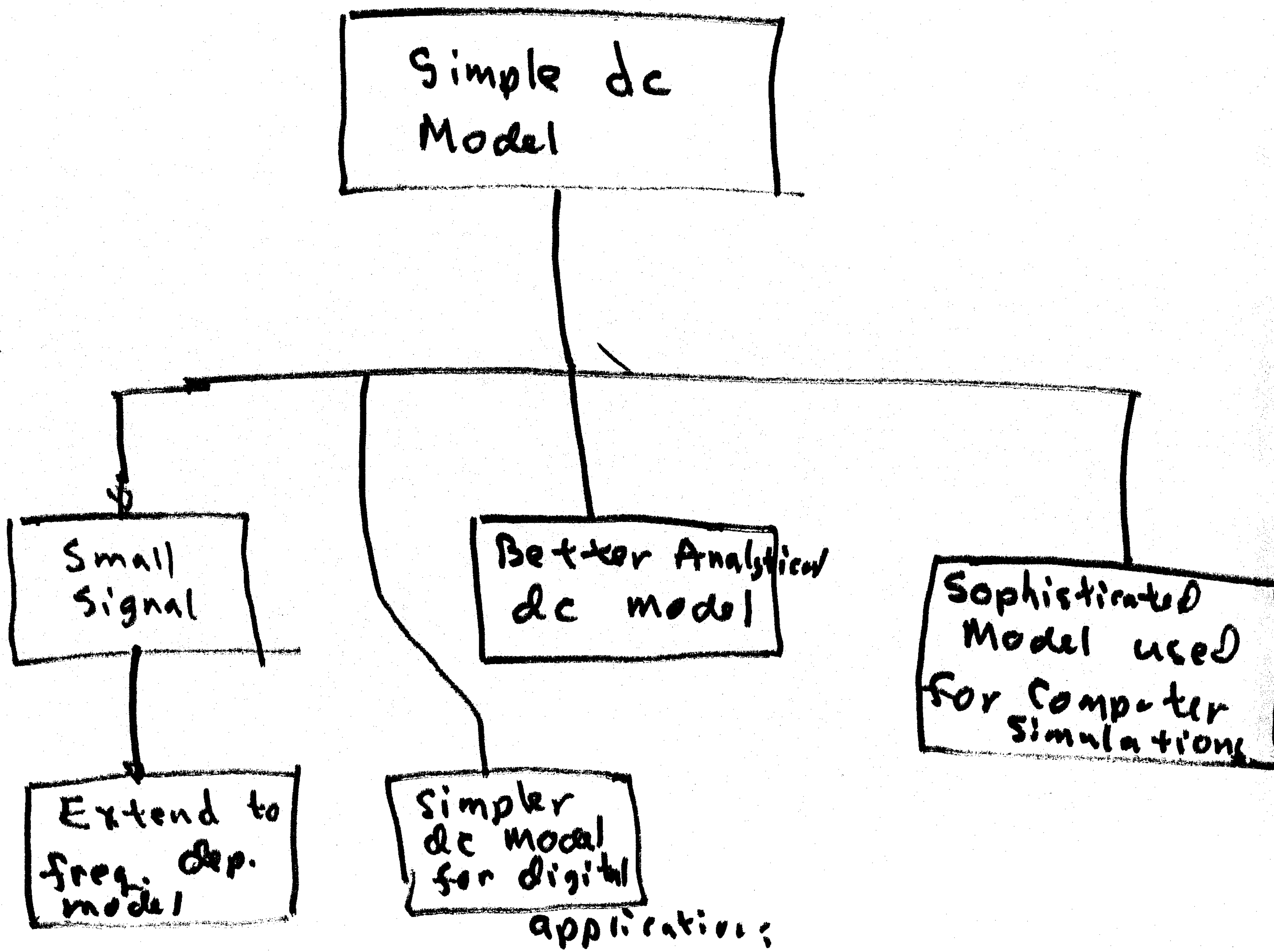
$$I_2 = f_2(V_1, V_2, V_3)$$

$$I_3 = f_3(V_1, V_2, V_3)$$

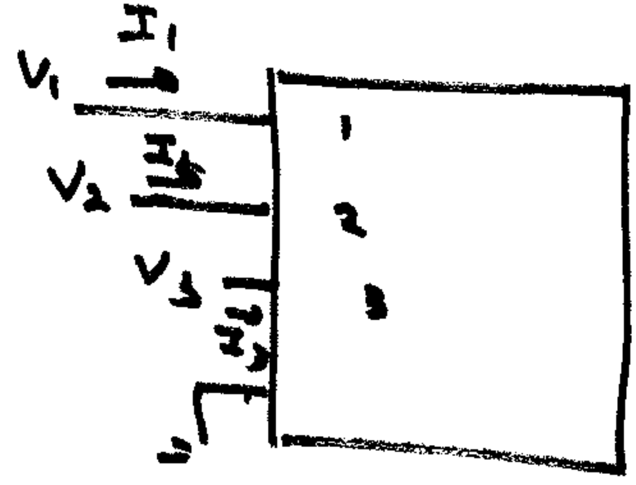
Strategy: - Develop multiple models that are useful for classes of applications

- Use as simple of a model as we can get away with

- Often must consider a modestly more complicated model to justify a given model.


Simple dc Model
Small Signal
Better Analytical dc model
Sophisticated Model used for Computer Simulations
Extend to freq. dep. model
Simpler dc model for digital applications

Small Signal Models



$$I_1 = f_1(V_1, V_2, V_3)$$

$$I_2 = f_2(V_1, V_2, V_3)$$

$$I_3 = f_3(V_1, V_2, V_3)$$

Develop the small signal model at the operating point (V_{1Q}, V_{2Q}, V_{3Q}) : $\vec{V}_Q = \begin{bmatrix} V_{1Q} \\ V_{2Q} \\ V_{3Q} \end{bmatrix}$

$$V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \quad I = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Similar eqn for $I_2(\vec{v})$ & $I_3(\vec{v})$

$$I_i(\vec{v}) = I_i(\vec{v}_Q) + \sum_{k=1}^{3} \left.\frac{\partial I_i}{\partial V_k}\right|_{\vec{v}_Q} (V_k - V_{kQ}) + \text{H.O.T.}$$

define the small signal quantities by

$$v_1 = V_1 - V_{1Q}$$

$$v_2 = V_2 - V_{2Q}$$

$$v_3 = V_3 - V_{3Q}$$

$$i_1 = I_1(\vec{v}) - I_1(\vec{v}_Q)$$

$$i_2 = I_2(\vec{v}) - I_2(\vec{v}_Q)$$

$$i_3 = I_3(\vec{v}) - I_3(\vec{v}_Q)$$

$$y_{ij} = \left.\frac{\partial I_i}{\partial V_k}\right|_{\vec{v}_Q}$$

$$i_1 = y_{11} v_1 + y_{12} v_2 + y_{13} v_3$$

$$i_2 = y_{21} v_1 + y_{22} v_2 + y_{23} v_3$$

$$i_3 = y_{31} v_1 + y_{32} v_2 + y_{33} v_3$$

Term the model characterized by

$$Y = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix}$$

the small signal model of the device.

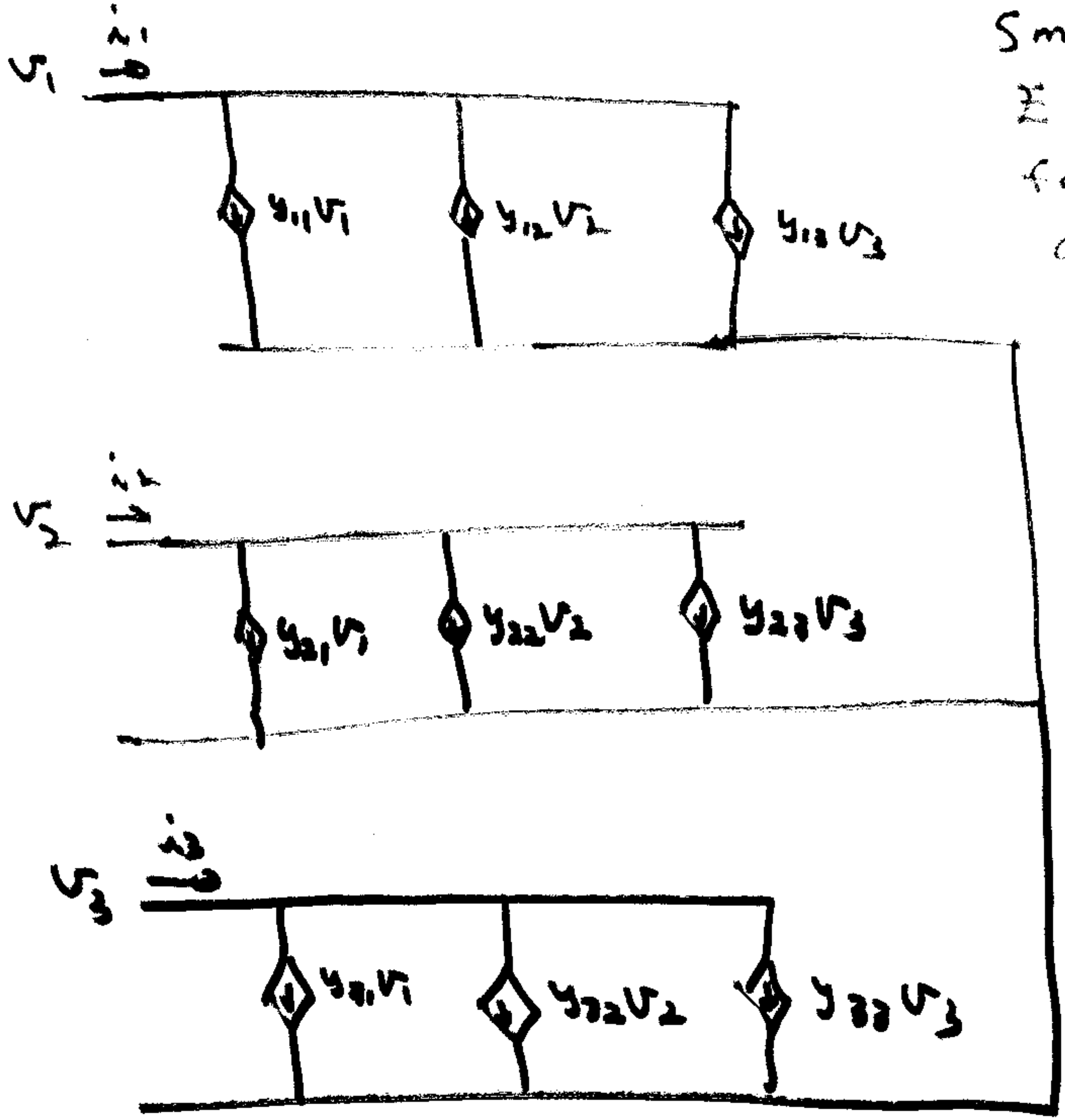
$$Y = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix}$$

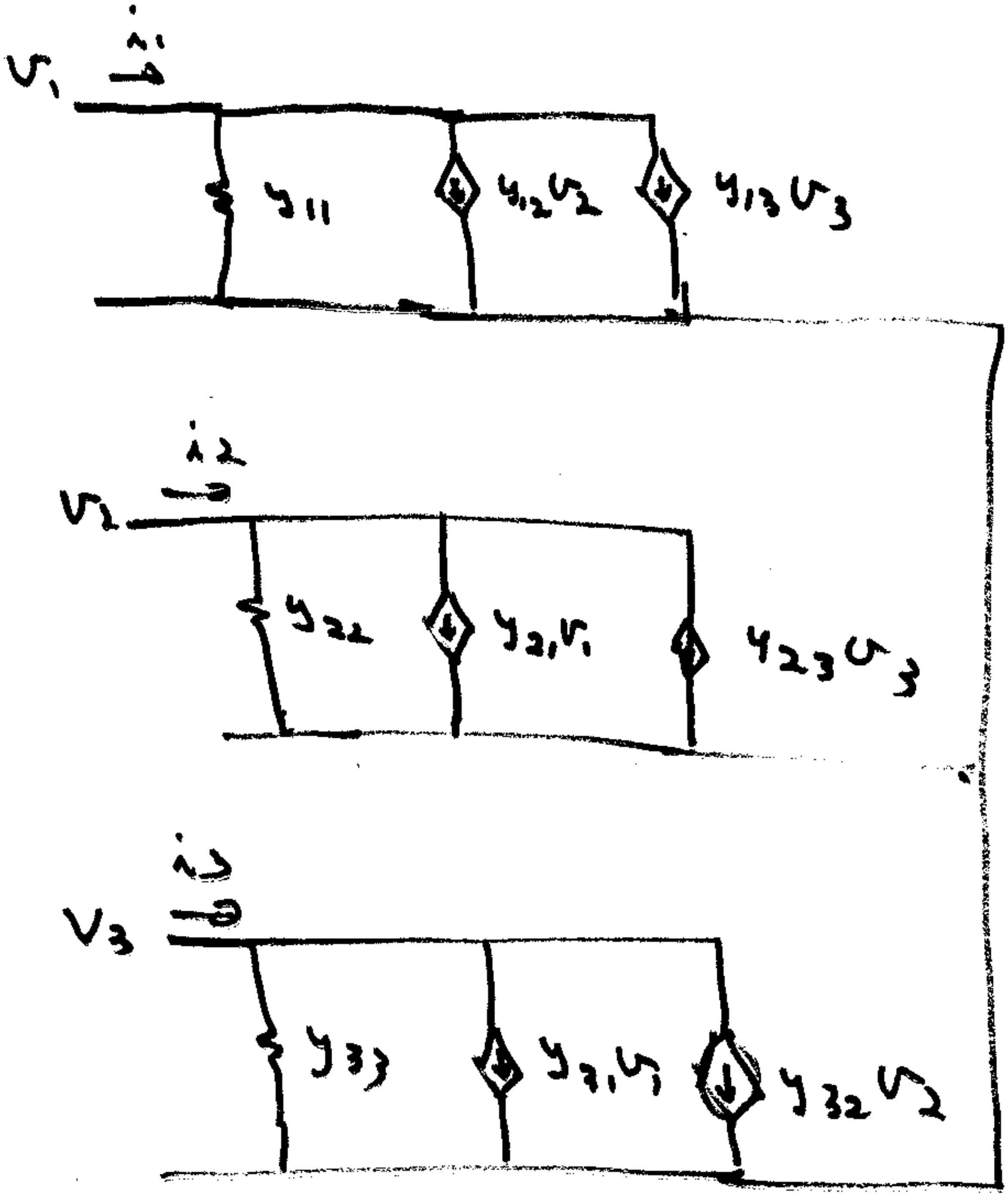
9 small signal parameters for a 4-term device

4 Small signal parameters for a 3-term device

1 Small signal parameter for a 2-term device

Small Signal
Equiv. Ckt
for 4-term
device



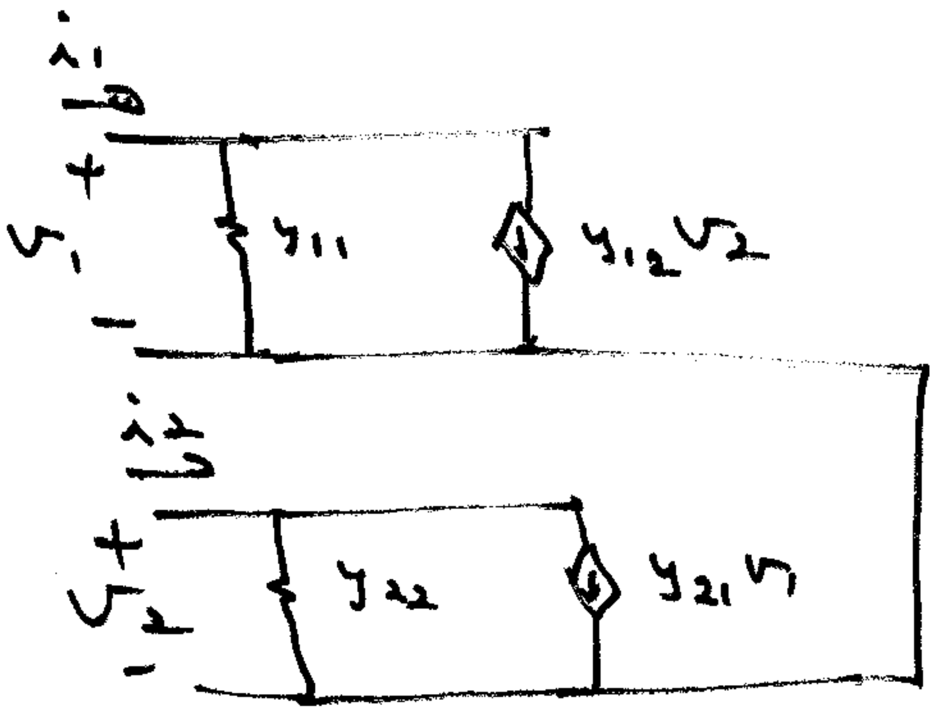


Alternate
Eq. Ckt
for
4-term
device

$$i_1 = y_{11} v_1 + y_{12} v_2$$

$$i_2 = y_{21} v_1 + y_{22} v_2$$

Equiv. Ckt for a 3-terminal device



Note: If the small signal quantities are time-dep, need to characterize them.

Assume all voltages & currents in device are periodic with period T

$$V_{1Q} = \frac{1}{T}\int_{t_1}^{t_1+T} V_1(t)\,dt$$

$$I_{1Q} = \frac{1}{T}\int_{t_1}^{t_1+T} I_1(t)\,dt$$

$\vdots$

$$v_1 = v_1 - V_{1Q}$$

$$i_1 = I_1 - I_{1Q}$$

$$\vdots$$

Example:

$$I_1 = 3V_1 + V_1 V_2^3$$
$$I_2 = V_1 e^{3V_2}$$

Obtain SS model & equiv. ckt at $V_{1Q} = 1V$, $V_{2Q} = 2V$

$$I_{1Q} = I_1(V_Q) = 3V_{1Q} + V_{1Q} V_{2Q}^3 = 3 + 8 = 11A$$
$$I_{2Q} = I_2(V_Q) = (1)e^{3\cdot 2} = e^6 A$$

$$i_1 = y_{11} v_1 + y_{12} v_2$$
$$i_2 = y_{21} v_1 + y_{22} v_2$$

$$y_{11} = \left.\frac{\partial I_1}{\partial v_1}\right|_Q = \left. 3 + V_2^3 \right|_Q = 3 + 2^3 = 11 \mho$$

$$y_{12} = \left.\frac{\partial I_1}{\partial v_2}\right|_Q = V_1 3V_2^2 = (1)(3)2^2 = 12 \mho$$

$$y_{21} = \frac{\partial I_2}{\partial v_1}\Big|_Q = e^{3V_2}\Big|_Q = e^{3\cdot 2} = e^{6}\ \mho$$

$$y_{22} = \frac{\partial I_2}{\partial v_2}\Big|_Q = 3V_1 e^{3V_2}\Big|_Q = 3e^{6}\ \mho$$