EE 434
Lecture 14

MOSFET Device Modeling

- Large signal model
- Small signal model

Quiz 11

Assume a device is characterized by the two equations

$$V_1 = 2V_2^{\,2} I_1 \qquad \& \qquad I_2 = 3V_1^3 + 2\ln\left(\frac{V_2}{2}\right)$$

Determine the

1) DC model for the device
2) Small signal Y parameter model for the device at the operating point $V_1 = 1V$, $V_2 = 1V$

And the number is ....

1 8 7 5 3

6 9 4 2

6

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Solution

1) DC model: $I_1 = V_1/2V_2^{\,2}$ & $I_2 = 3V_1^3 + 2\ln\left(\frac{V_2}{2}\right)$

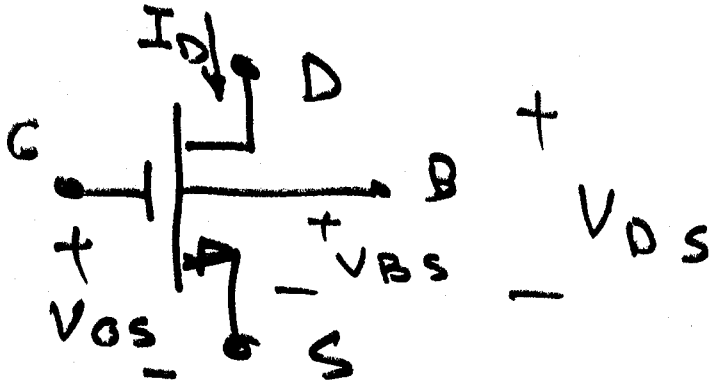
2) Small signal Y model:

$$y_{11} = \left.\frac{\partial I_1}{\partial V_1}\right|_Q = \left.\frac{1}{2V_2^2}\right|_Q = \frac{1}{2\times 1^2}\Omega^{-1} = 0.5\Omega^{-1}$$

$$y_{12} = \left.\frac{\partial I_1}{\partial V_2}\right|_Q = \left.\frac{V_1}{2}\left(-2V_2^{-3}\right)\right|_Q = \frac{1}{2}\left(-\frac{2}{1^3}\right)\Omega^{-1} = -1\Omega^{-1}$$

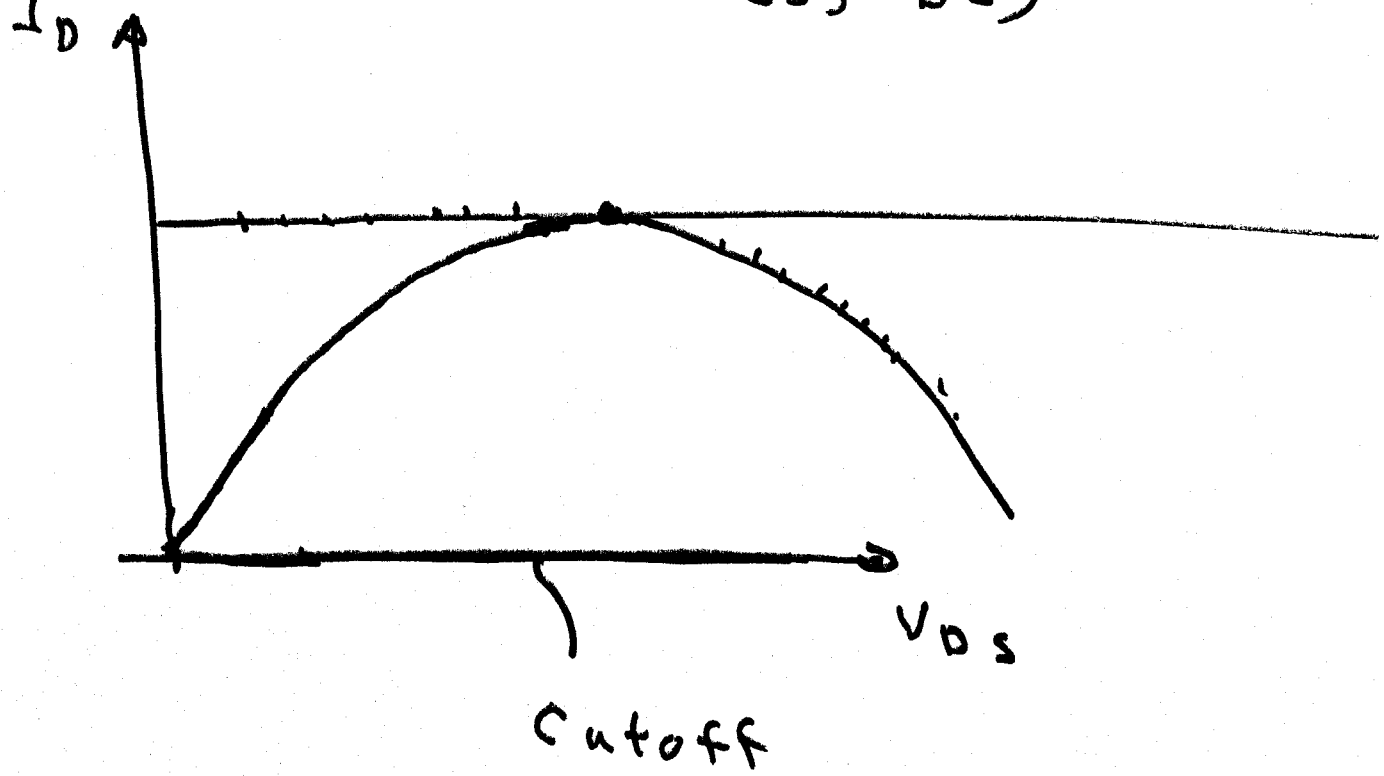
$$y_{21} = \left.\frac{\partial I_2}{\partial V_1}\right|_Q = \left.9V_1^2\right|_Q = 9\times 1^2\Omega^{-1} = 9\Omega^{-1}$$

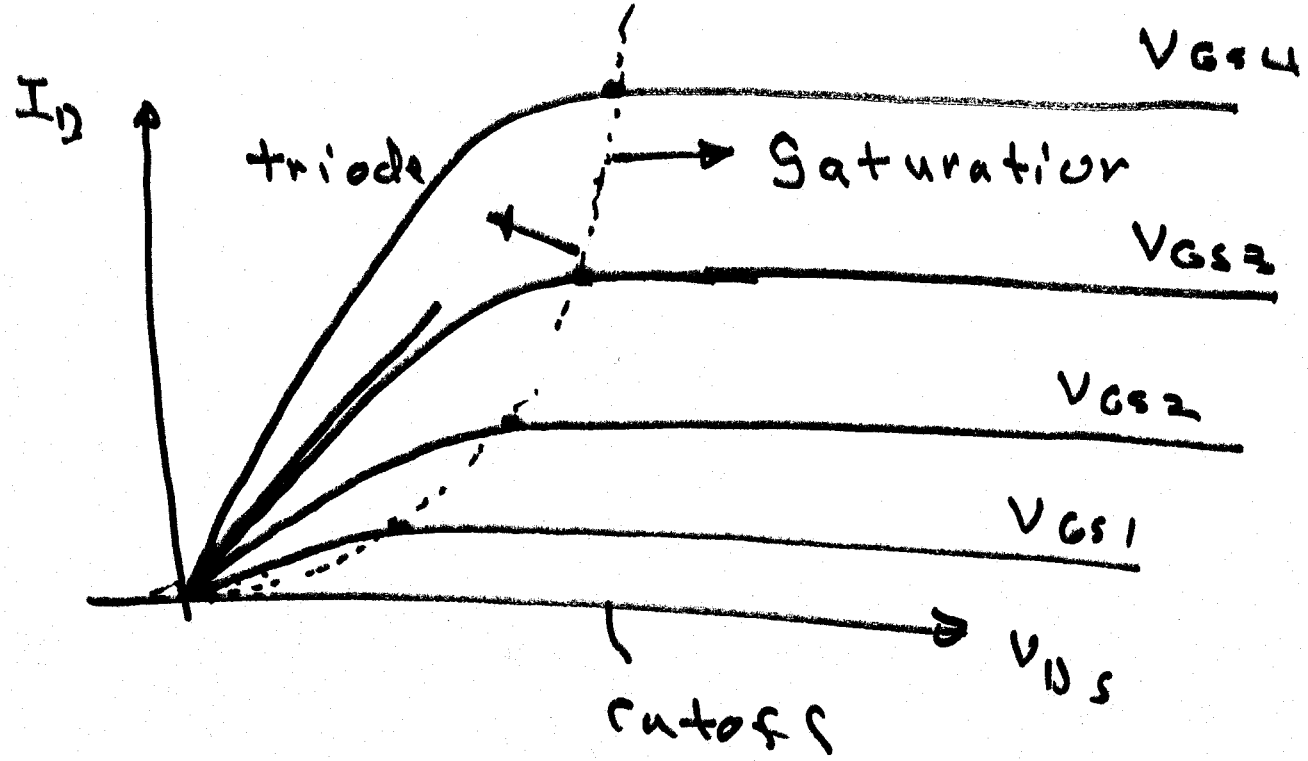
$$y_{22} = \left.\frac{\partial I_2}{\partial V_2}\right|_Q = \left.\frac{2}{V_2}\right|_Q = \frac{2}{1}\Omega^{-1} = 2\Omega^{-1}$$



$$I_D = \begin{cases} 0 & \text{"cutoff"} \quad V_{GS} < V_T \\ \dfrac{\mu C_{ox} W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right) V_{DS} & \text{"triode"} \quad V_{GS} > V_T, \quad V_{DS} < V_{GS} - V_T \\ \dfrac{\mu C_{ox} W}{2L}\left(V_{GS} - V_T\right)^2 & \text{"saturation"} \quad V_{GS} > V_T, \quad V_{DS} > V_{GS} - V_T \end{cases}$$

$$I_D = f(V_{GS}, V_{DS})$$





$$V_{DS} = V_{GS} - V_T$$

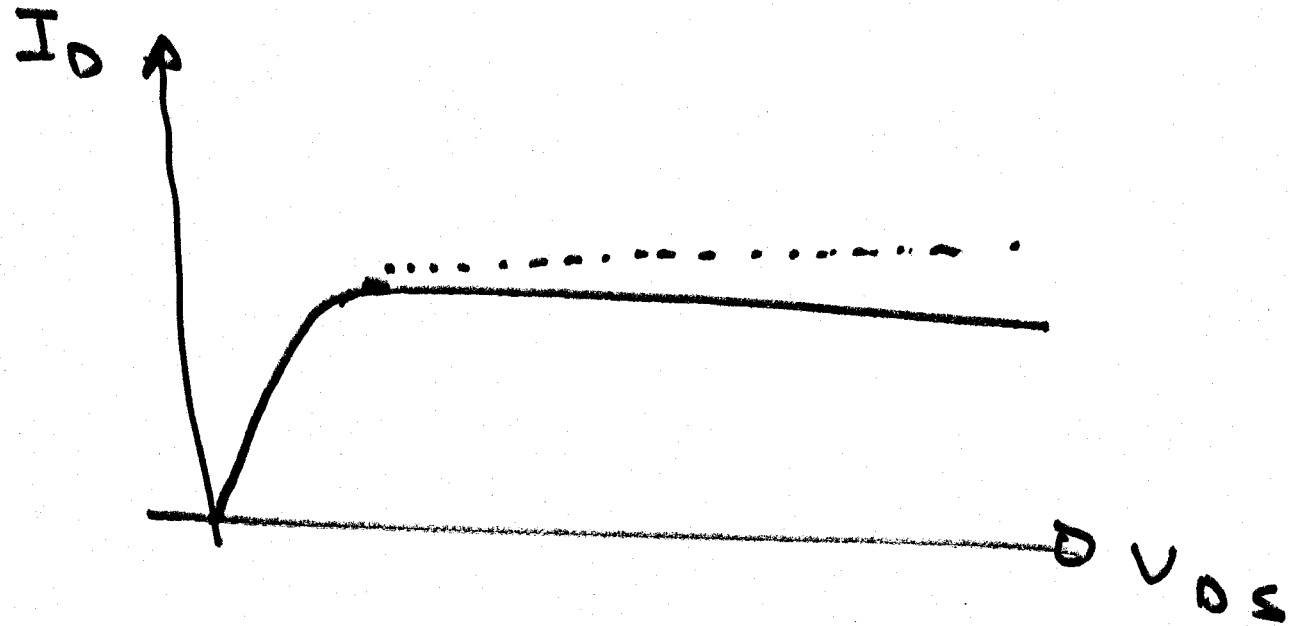
$$I_D = \frac{u C_{ox} W}{L} \left(V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS}$$

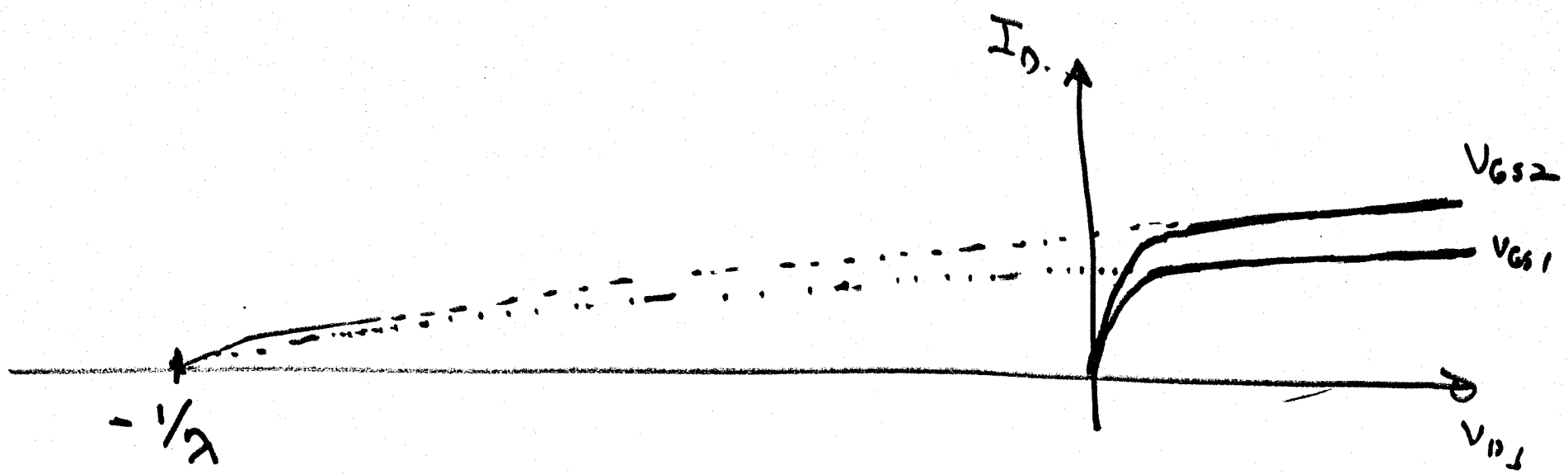
$$I_D = \frac{u C_{ox} W}{L} \left(\frac{V_{DS}^2}{2} \right)$$

$$I_D = \frac{u\, C_{ox} W}{L} \left(V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS}$$

$$\left. \frac{\partial I_D}{\partial V_{DS}} \right|_{V_{DS} = V_{GS} - V_T} = \left. \frac{u\, C_{ox} W}{L} \left(V_{GS} - V_T - V_{DS} \right) \right|_{V_{DS} = V_{GS} - V_T}$$

$$= 0$$

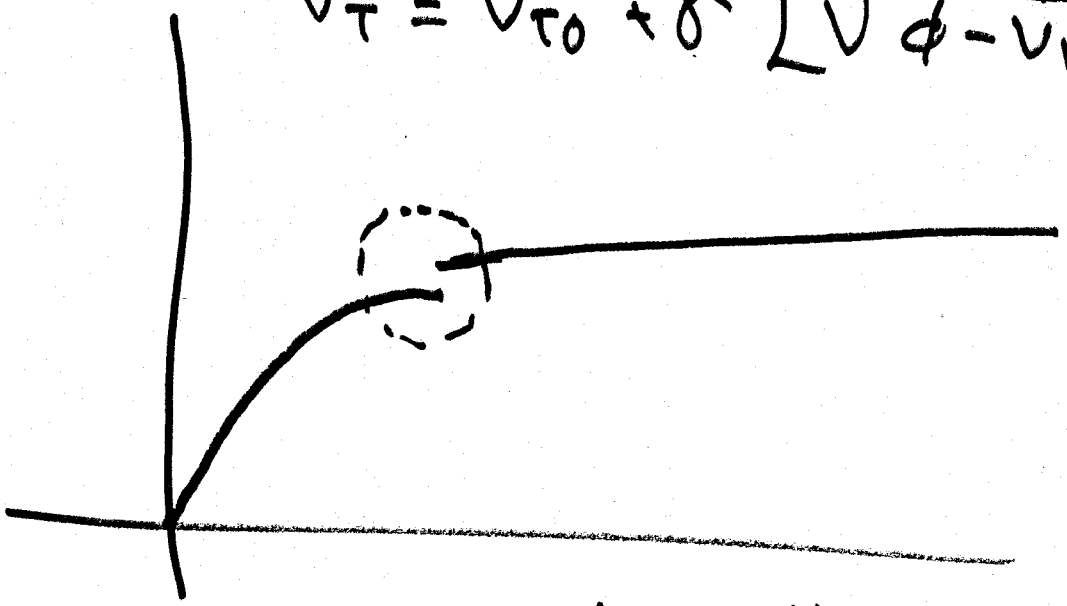




$$I_D = \frac{\mu C_{ox} W}{2L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

$$I_D = \begin{cases} 0 & V_{GS} < V_T \\ \dfrac{\mu C_{ox} W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right) V_{DS} & V_{GS} > V_T,\ V_{DS} < V_{GS} - V_T \\ \dfrac{\mu C_{ox} W}{2L}\left(V_{GS} - V_T\right)^2 (1 + \lambda V_{DS}) & V_{GS} > V_T,\ V_{DS} > V_{GS} - V_T \end{cases}$$

$$V_T = V_{T0} + \gamma\left[\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right]$$



$$\mu_p \simeq \frac{\mu_n}{3}$$

$\mu_n C_{ox} \simeq 100\,\mu A/V^2$

$V_{T_n} \simeq .75$

$V_{Tp} \simeq -.75\,V$

$\lambda \simeq \begin{cases} .01\,V^{-1} & \text{for } L > 1\mu \\ \text{bigger} & L < 1\mu \end{cases}$

$\gamma = .5\,V^{1/2}$

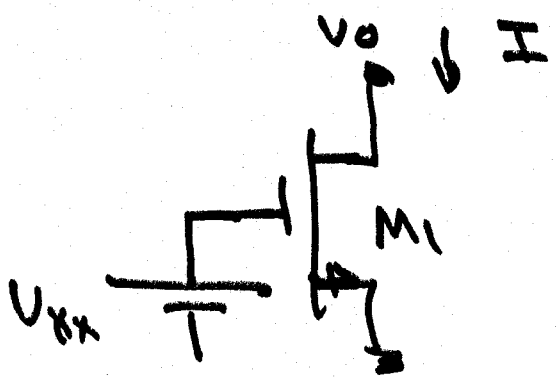
$\phi = .6\,V$

Process Parameters $\{u, C_{ox}, V_{TO}, \gamma, \phi, \lambda\}$

Design Parameters $\{W, L\}$

Note: only 1 degree of freedom: $\frac{W}{L}$

Example:



Assume V_o is small so that M_1 is deep in triode region

$$I_D = \frac{u C_{ox} W}{L} \left(V_{GS} - V_T - \cancel{\frac{V_{DS}}{2}}^{\,0} \right) V_{DS}$$

$$\therefore R_{FET} = \frac{L}{u C_{ox} W (V_{GS} - V_T)}$$

Consider $W = L = 1\mu$

$V_{GS} = 2V$

$V_T = 1V$

$$R_{FET} = \frac{1}{(100E-6)(1V)} = 10K\Omega$$

Small Signal Model of MOSFET

Region of Operation: Saturation

$$I_D = \frac{\mu C_{ox} W}{2L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

$$\left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_Q} = g_m = \frac{\mu C_{ox} W}{2L} (2)(V_{GS} - V_T)(1 + \lambda V_{DS})\Big|_Q$$

$$\simeq \frac{\mu C_{ox} W}{L} (V_{GSQ} - V_T)$$

$$\left.\frac{\partial I_D}{\partial V_{DS}}\right|_{V_Q} = g_o$$

$$\left.\frac{\partial I_D}{\partial V_{BS}}\right|_{V_Q} = g_{mb}$$

$$g_m = \frac{u C_{ox} W}{L}(V_{GSQ} - V_T)$$

$$I_D \simeq \frac{u C_{ox} W}{2L}(V_{GS} - V_T)^2 \Rightarrow V_{GSQ} - V_T = \sqrt{\frac{2L}{u C_{ox} W}}\sqrt{I_{DQ}}$$

$$g_m = \sqrt{I_{DQ}}\sqrt{\frac{2 u C_{ox} W}{L}}$$

$$g_m = \frac{I_D\, 2}{(V_{GS} - V_T)^{\cancel{2}}}\cancel{(V_{GS} - V_T)} \simeq \frac{2 I_{DQ}}{(V_{GSQ} - V_T)}$$

$$g_m = \frac{u C_{ox} W}{L}(V_{GSQ} - V_T) = \sqrt{I_{DQ}}\sqrt{\frac{2 u C_{ox} W}{L}} = \frac{2 I_{DQ}}{V_{GSQ} - V_T}$$

$$g_o = \left.\frac{\partial I_D}{\partial v_{DS}}\right|_Q = \lambda \left[\left.\frac{\mu C_{ox} W}{2L}(V_{GS}-V_T)^2\right|_Q\right]$$

$$g_o \simeq \lambda I_{DQ}$$

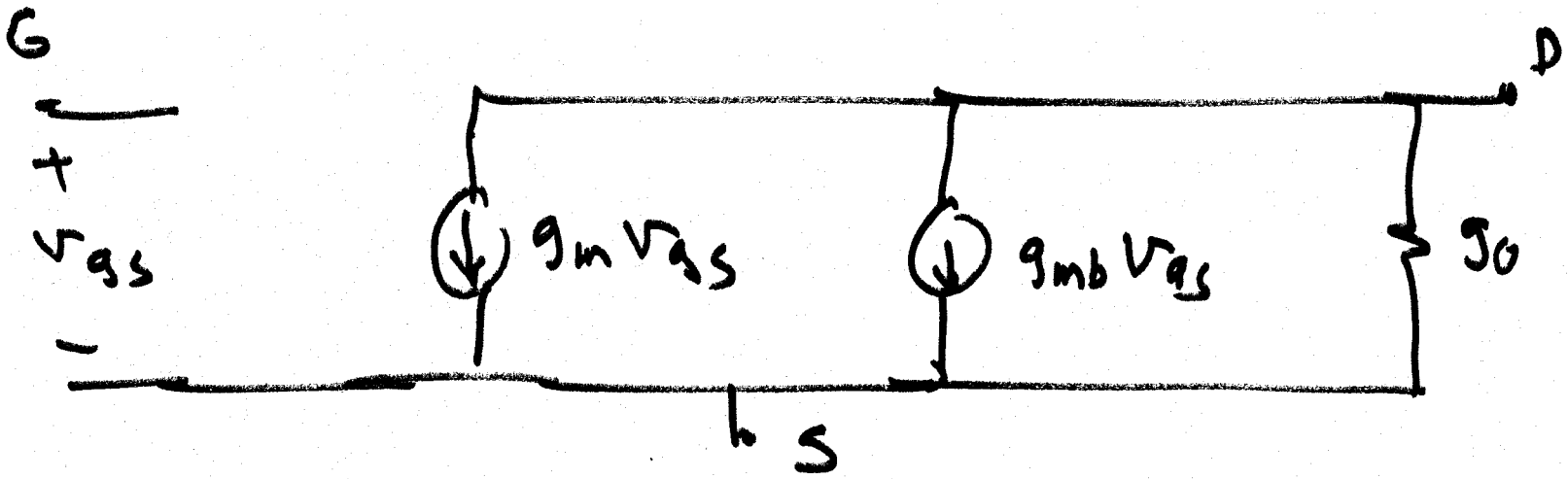
$$g_{mb} = \frac{\partial I_D}{\partial v_{BS}} = \frac{\mu C_{ox} W}{\cancel{2}L}(\cancel{2})(V_{GS}-V_T)(-1)\frac{\partial V_T}{\partial v_{BS}}(1+\lambda V_{DS})$$

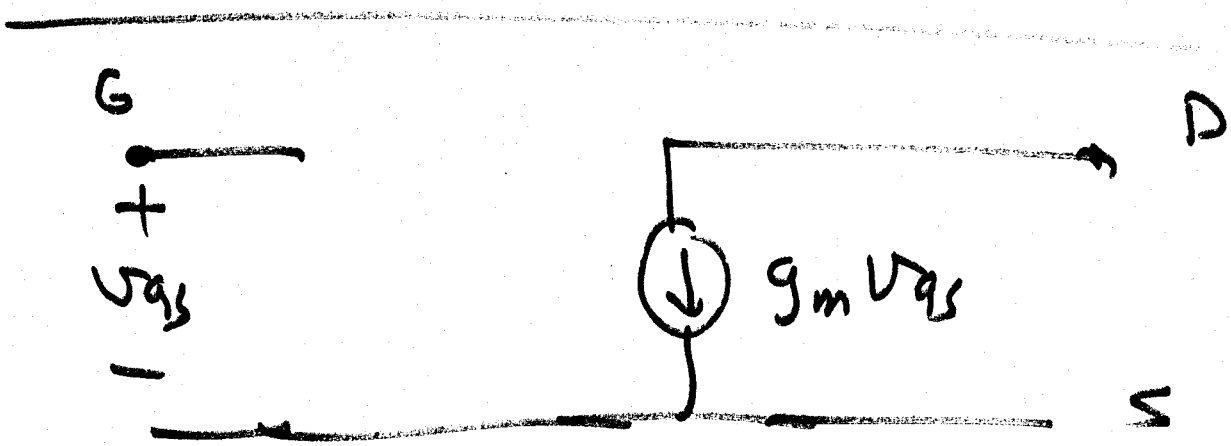
$$V_T = V_{T0} + \gamma\left[\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right]$$

$$\therefore \frac{\partial V_T}{\partial v_{BS}} = \gamma \frac{1}{2}(\phi - V_{BS})^{-1/2}(-1)$$

$$= \left.\frac{\mu C_{ox} W}{L}(V_{GS}-V_T)\frac{\gamma}{2\sqrt{\phi - V_{BS}}}\right|_Q$$

$$g_{mb} = \frac{g_m \gamma}{2\sqrt{\phi - v_{DSQ}}}$$





Simplified Small Signal Model