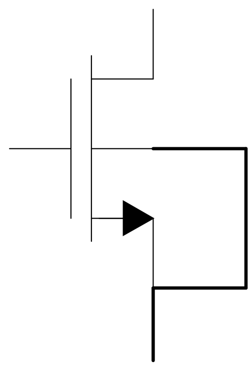
EE 434
Lecture 16

Small signal model

Small signal applications in amplifier analysis and design

Quiz 13

The gm of an n-channel MOS transistor that has a quiescent current of 5mA was measured to be 10mA/V. If the length of the transistor is .6u, determine the width. Assume the process parameters uC_{OX}=100uA/V^2, V_{TO}=700mV



And the number is ….

1 8 7 5 3

6 9 4 2

4

Quiz 13

The g_m of an n-channel MOS transistor that has a quiescent current of 5mA was measured to be 10mA/V. If the length of the transistor is .6u, determine the width. Assume the process parameters uC_{OX}=100uA/V^2, V_{TO}=700mV

Solution

$$g_m = \sqrt{\frac{2\mu C_{OX} W}{L}}\sqrt{I_{DQ}}$$

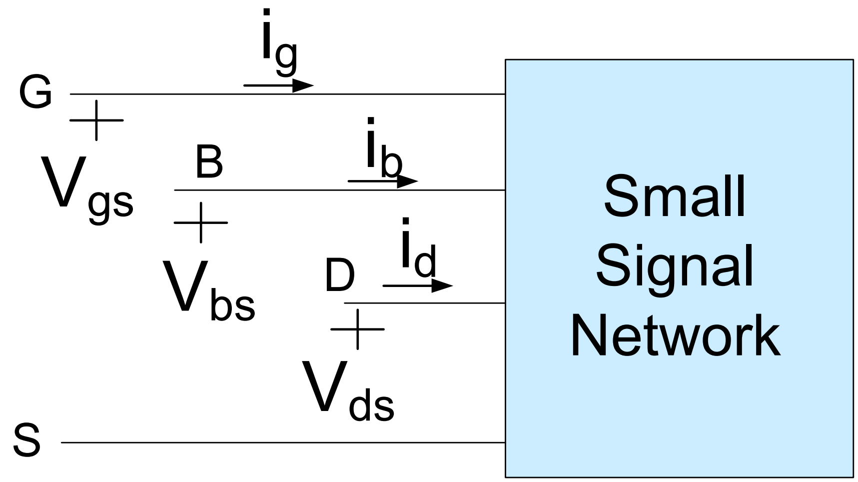
$$W = \frac{g_m^2}{I_{DQ}} \frac{L}{2\mu C_{Ox}}$$

$$W = \frac{(10E-3)^2}{5E-3} \frac{0.6\mu}{2 \bullet 100E-6} = 60\mu$$

Review from Last Time

- Model parameters available from MOSIS WEB site
  - Square-law model parameters
    - μ obtained from K' and C_{OX}
    - λ quite size and operating point dependent so obtain from simulation
  - BSIM Model Contains 97 parameters
    - Models best when close to size and operating point of device
    - When corner models added and binning models added, parameter space becomes much larger
    - Actual device parameters will vary due to nominal process variation and random die-level variation
    - Model still has some errors
- Considerable ongoing activity on device modeling
  - Several hundred papers per year appearing on MOSFET modeling alone
  - Level of activity in modeling probably increasing

Small Signal Model Summary



$$i_g = 0$$

$$i_b = 0$$

$$i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds}$$

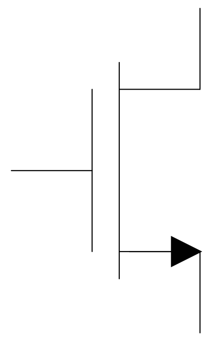
$$g_m = \frac{\mu C_{OX} W}{L}\left(V_{GSQ} - V_T\right)$$

$$g_o = \lambda I_{DQ}$$

$$g_{mb} = g_m\left(\frac{\gamma}{2\sqrt{\phi - V_{BSQ}}}\right)$$

Small Signal Model Summary

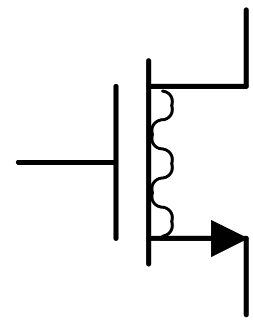
Large Signal Model



$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX} \frac{W}{L}\left(V_{GS} - V_T - \frac{V_{DS}}{2}\right) V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \frac{W}{2L}(V_{GS} - V_T)^2 \bullet (1 + \lambda V_{DS}) & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$

$$V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right)$$

Small Signal Model



$$i_g = 0$$

$$i_b = 0$$

$$i_d = g_m v_{gs} + g_{mb} v_{bs} + g_o v_{ds}$$

where

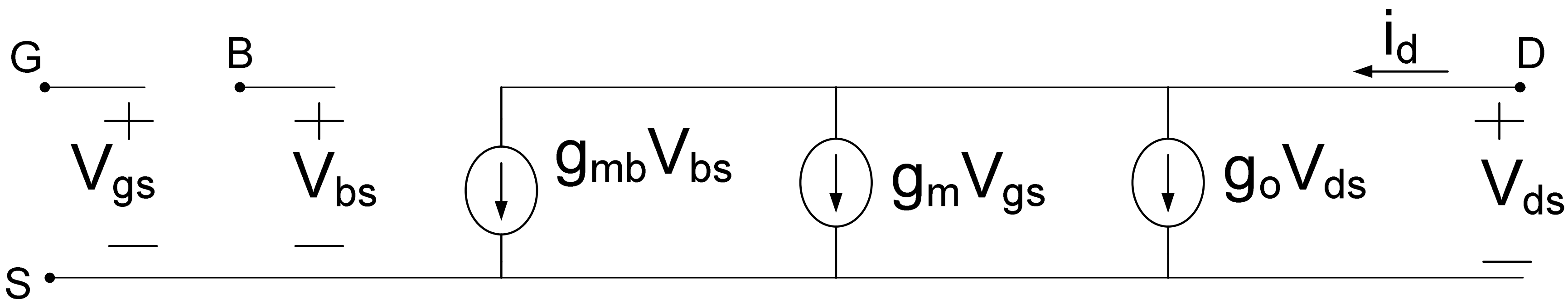
$$g_m = \frac{\mu C_{OX} W}{L}(V_{GSQ} - V_T)$$

$$g_{mb} = g_m \left(\frac{\gamma}{2\sqrt{\phi - V_{BSQ}}} \right)$$

$$g_o = \lambda I_{DQ}$$

Small Signal Model Summary

An equivalent circuit



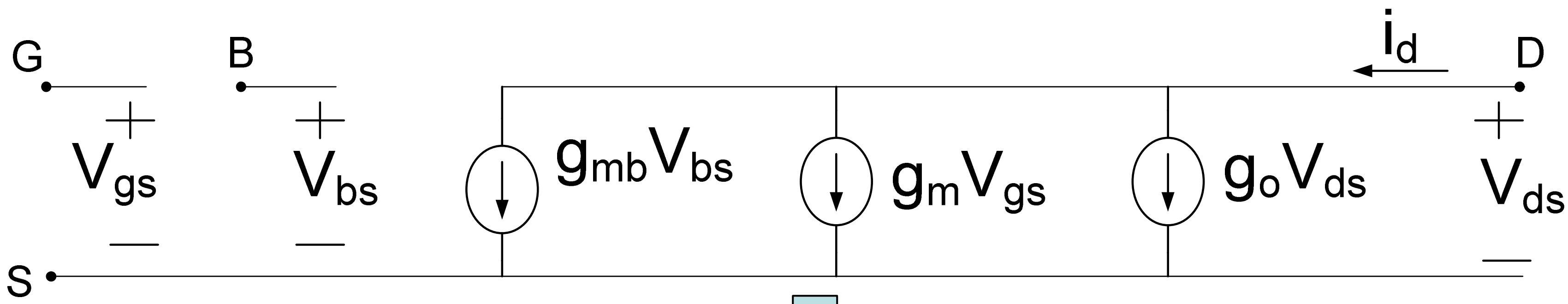
$$g_m = \frac{\mu C_{OX} W}{L}\left(V_{GSQ} - V_T\right)$$

$$g_o = \lambda I_{DQ}$$

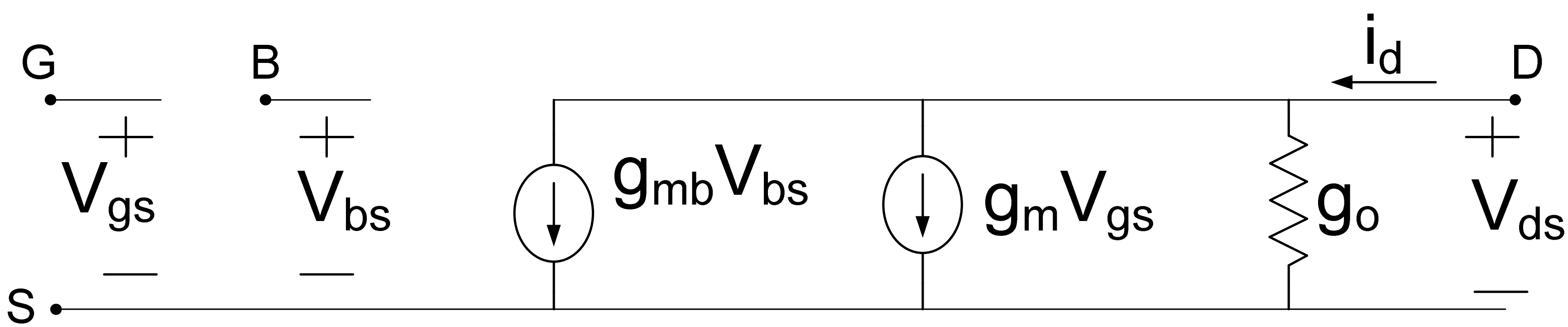
$$g_{mb} = g_m\left(\frac{\gamma}{2\sqrt{\phi - V_{BSQ}}}\right)$$

This contains absolutely no more information than the previous model

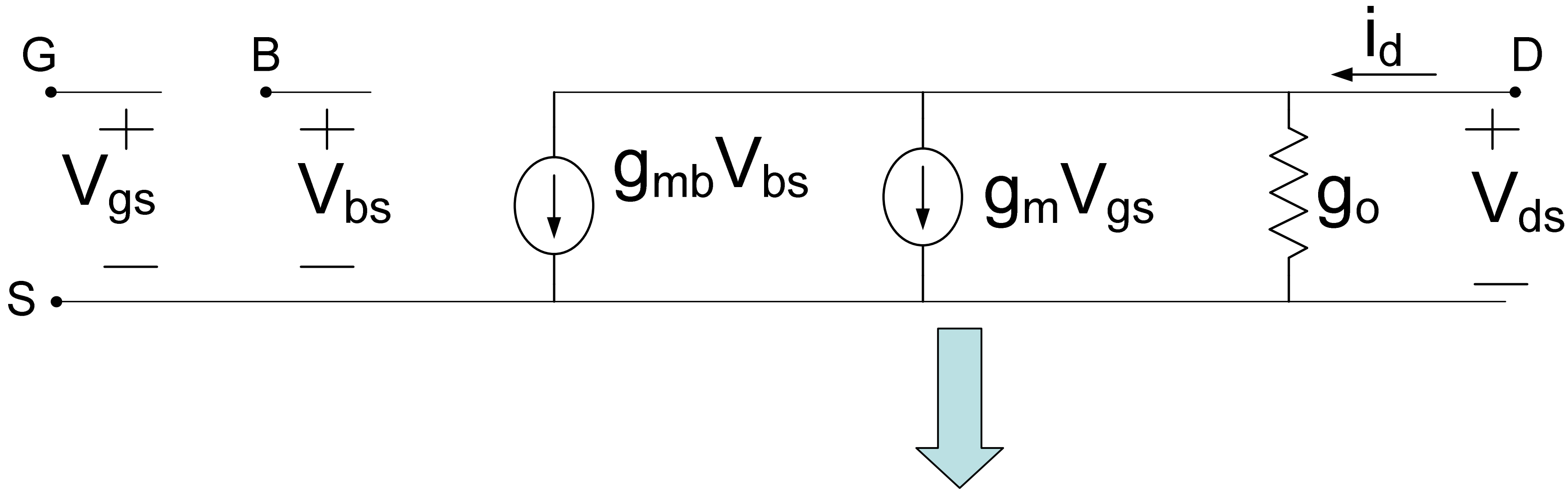
Small Signal Model Summary



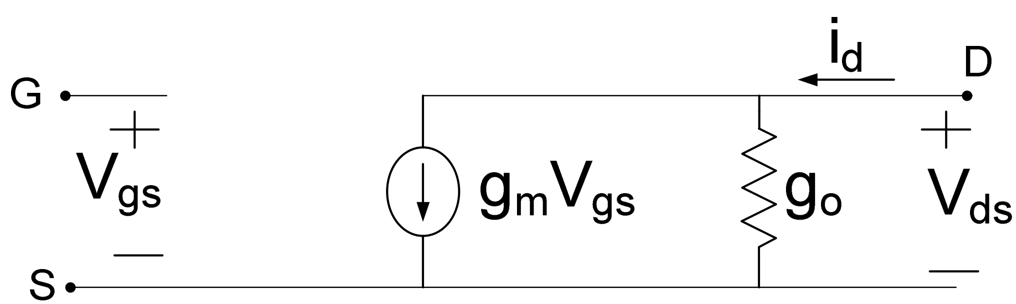
More convenient representation



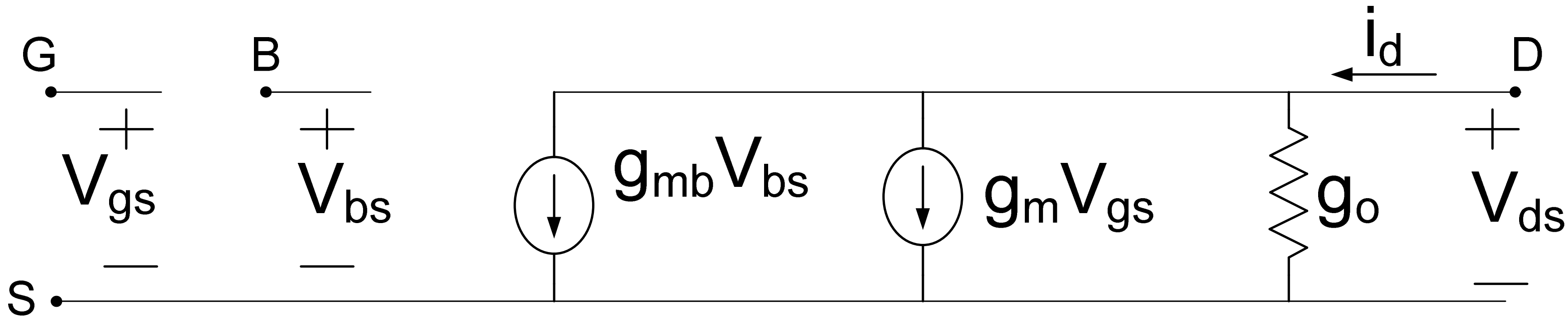
Small Signal Model Summary

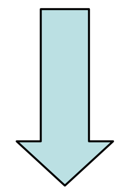


Simplification that is often adequate

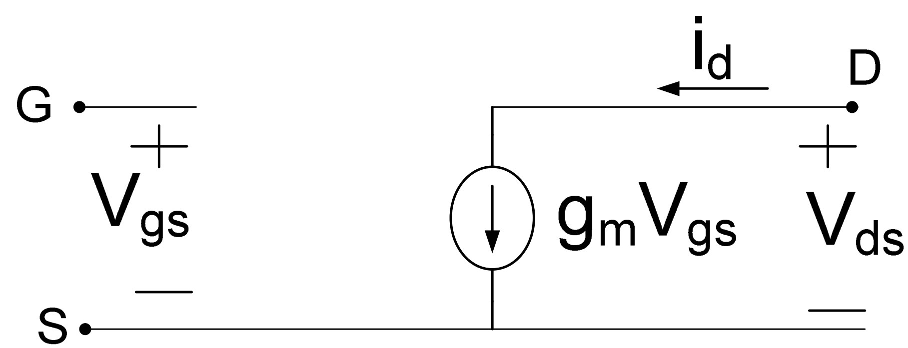


Small Signal Model Summary

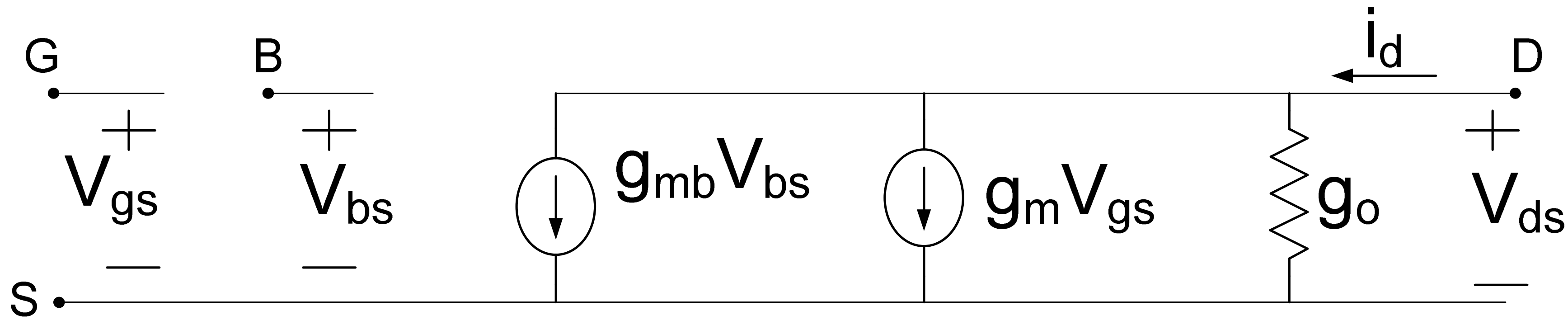




Even further simplification that is often adequate



Small Signal Model Summary



Alternate equivalent representations for g_m

$$g_m = \frac{\mu C_{OX} W}{L}\left(V_{GSQ} - V_T\right)$$ from $$I_D \cong \mu C_{OX}\frac{W}{2L}\left(V_{GS} - V_T\right)^2$$

$$g_m = \sqrt{\frac{2\mu C_{OX} W}{L}}\sqrt{I_{DQ}}$$

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} = \frac{2I_{DQ}}{V_{EBQ}}$$

How does g_m vary with I_{DQ}?

$$g_m = \sqrt{\frac{2\mu C_{OX} W}{L}} \sqrt{I_{DQ}}$$

Varies with the square root of I_{DQ}

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T} = \frac{2I_{DQ}}{V_{EBQ}}$$

Varies linearly with I_{DQ}

$$g_m = \frac{\mu C_{OX} W}{L} \left(V_{GSQ} - V_T \right)$$

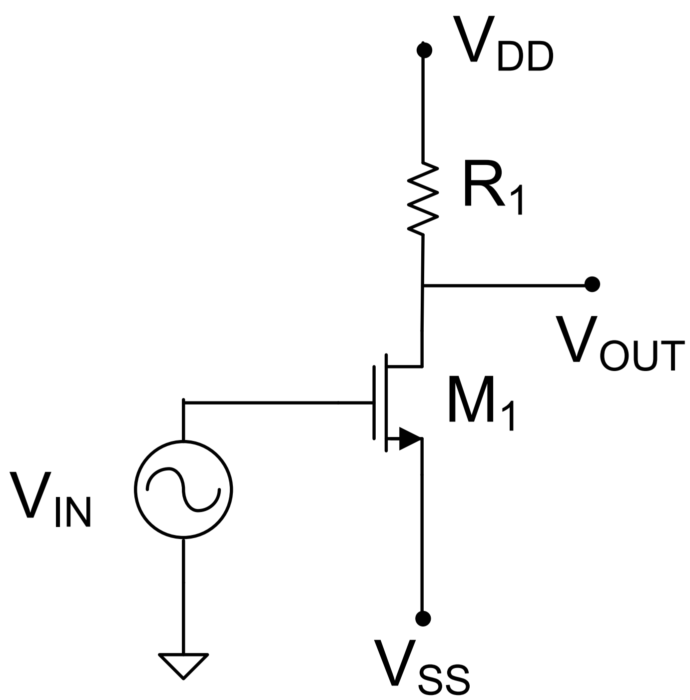
Doesn't vary with I_{DQ}

How does g_m vary with I_{DQ}?

All of the above are true – but with qualification

g_m is a function of more than one variable (I_{DQ}) and how it varies depends upon how the remaining variables are constrained

Small signal analysis example

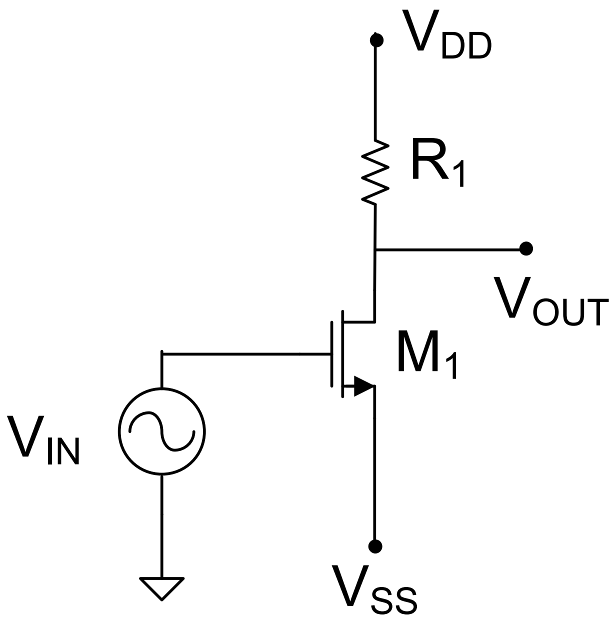


uC_{OX}=100uA/V^2
V_T=.75V
λ=.01V^{-1}

V_{DD}=8V
V_{SS} = -1.25V
W=16u
L=1u
R_1=15K

$$V_{IN} = V_m \sin \omega t$$

Small signal analysis example



$$V_{IN} = V_{GS} + V_{SS}$$

$$V_{DD} = I_D R_1 + V_{DS} + V_{SS}$$

$$I_D = \mu C_{OX} \frac{W}{2L} \left(V_{GS} - V_T\right)^2 \bullet \left(1 + \lambda V_{DS}\right)$$

$$V_{IN} = V_m \sin \omega t$$

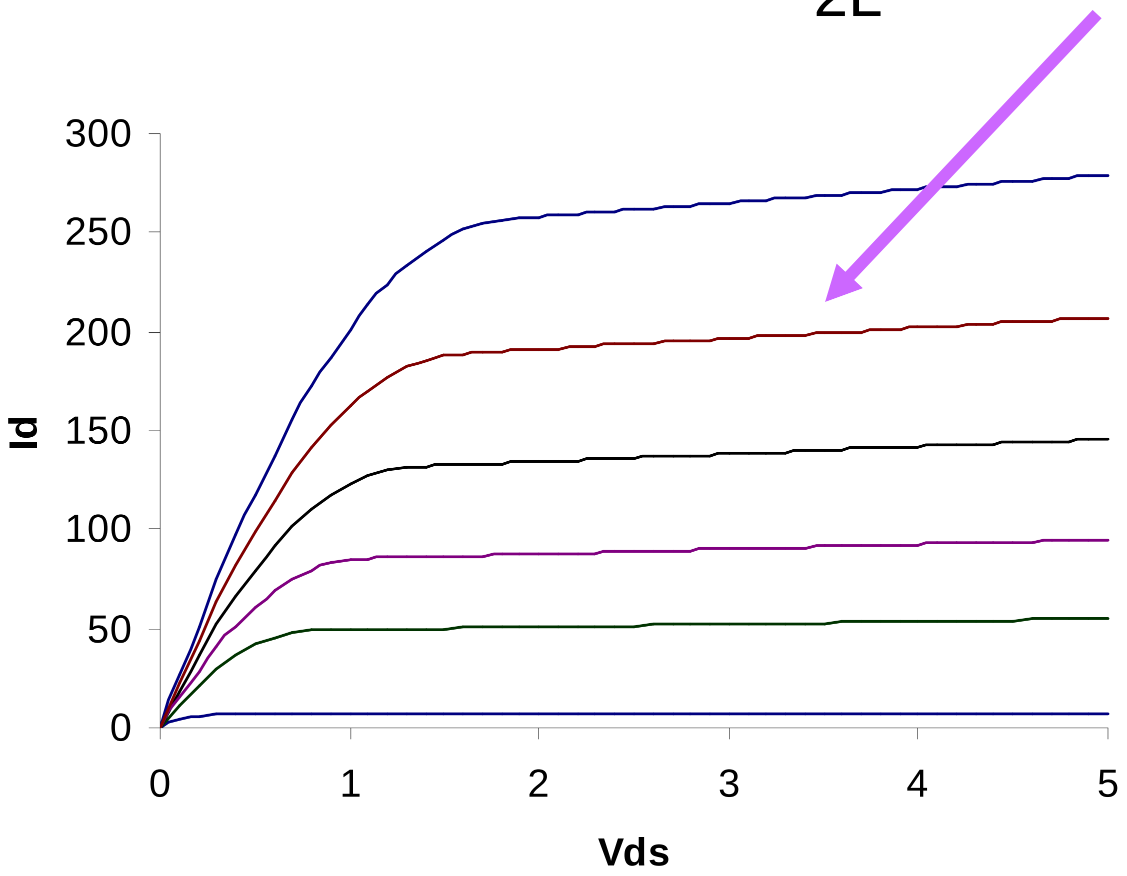
$$V_{OUT} = V_{DS} + V_{SS}$$

Must solve 5 simultaneous equations to obtain V_{OUT}

But one of these equations is nonlinear making the solution very tedious

Small signal analysis example

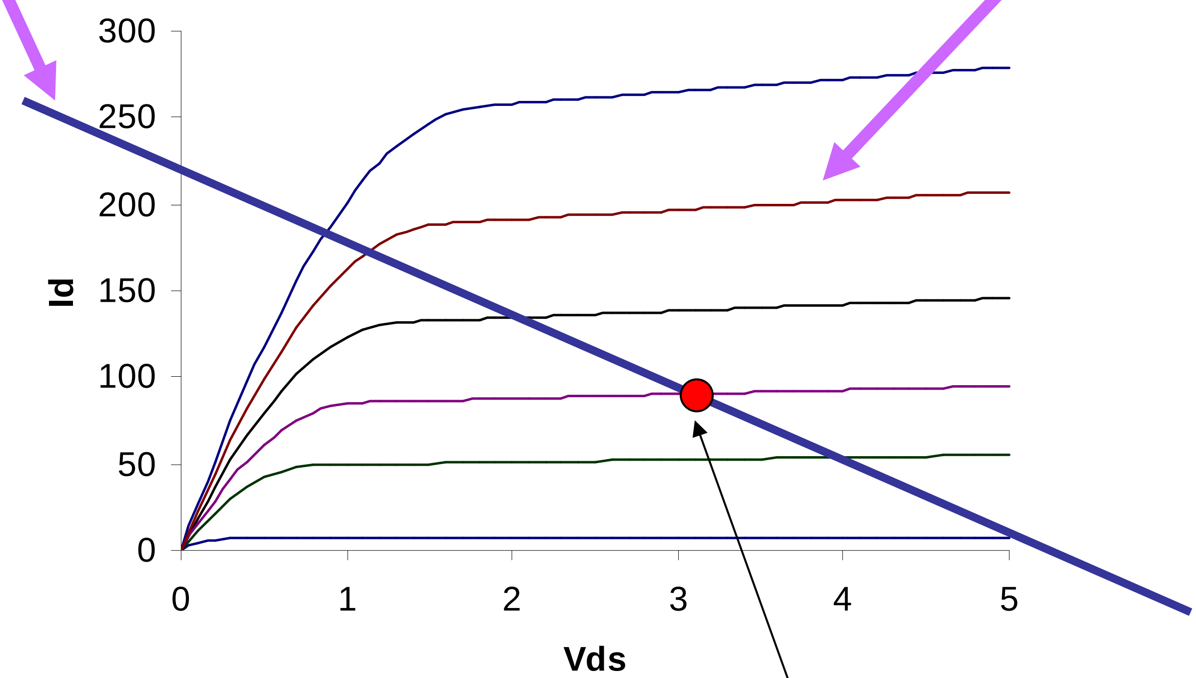
$$I_D = \mu C_{OX} \frac{W}{2L} (V_{GS} - V_T)^2 \bullet (1 + \lambda V_{DS})$$



Small signal analysis example

$$V_{DD} = I_D R_1 + V_{DS} + V_{SS}$$

$$I_D = \mu C_{OX} \frac{W}{2L} (V_{GS} - V_T)^2 \bullet (1 + \lambda V_{DS})$$

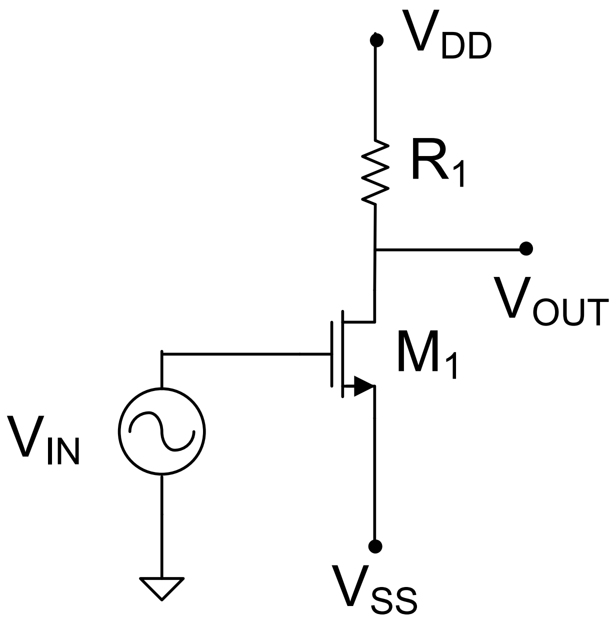


Q-point

$$V_{IN} = V_{GS} + V_{SS}$$

Consider V_{IN} very small

Small signal analysis example



$$V_{IN} = V_{GS} + V_{SS}$$

$$V_{DD} = I_D R_1 + V_{DS} + V_{SS}$$

$$I_D = \mu C_{OX} \frac{W}{2L} \left(V_{GS} - V_T\right)^2 \bullet \left(1 + \lambda V_{DS}\right)$$

$$V_{OUT} = V_{DS} + V_{SS}$$

Consider V_{IN}=0,V_m,-V_m

$$V_{IN} = V_m \sin \omega t$$

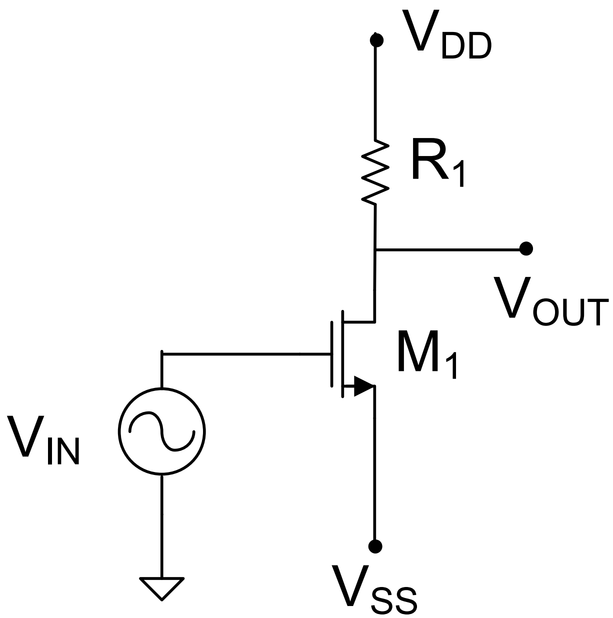
Define $V_{EB}=V_{GS}-V_T$ when $V_{IN}=0$

Thus $V_{EB}=-V_{SS}-V_T$

$$V_{OUT} = V_{DD} - I_D R_1$$

$$I_D \cong \mu C_{OX} \frac{W}{2L} \left(V_{IN} + V_{EB}\right)^2$$

Small signal analysis example



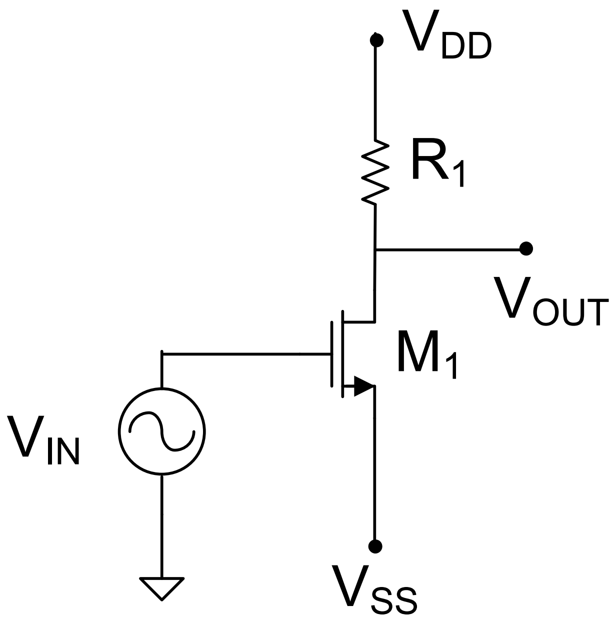
$$V_{OUT} = V_{DD} - I_D R_1$$

$$I_D \cong \mu C_{OX} \frac{W}{2L} (V_{IN} + V_{EB})^2$$

$$V_{OUT} = V_{DD} - R_1 \left[\mu C_{OX} \frac{W}{2L} \right] (V_{IN} + V_{EB})^2$$

$$V_{IN} = V_m \sin \omega t$$

Small signal analysis example



$$V_{IN} = V_m \sin \omega t$$

$$V_{OUT} = V_{DD} - R_1\left[\mu C_{OX}\frac{W}{2L}\right](V_{IN} + V_{EB})^2$$

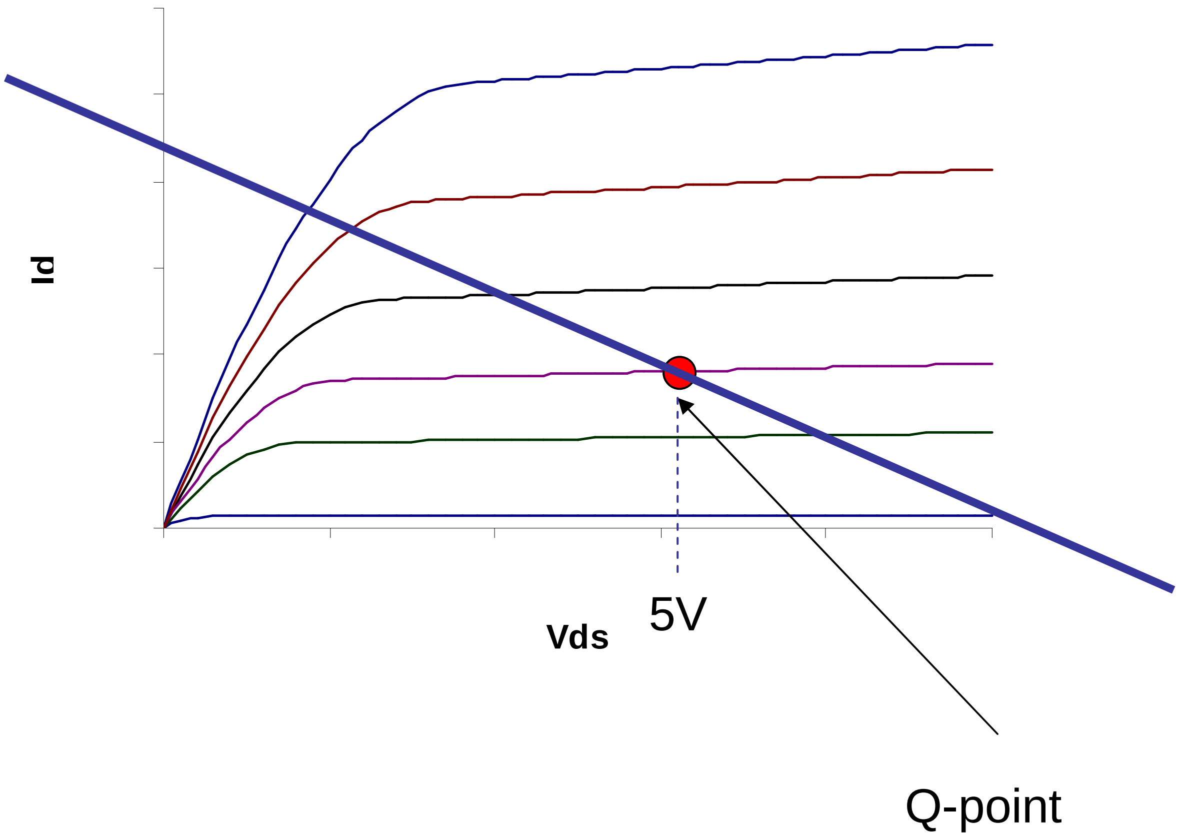
$$V_{EB} = -V_{SS} - V_T = 1.25V - .75V = 0.5V$$

$$V_{OUT}(V_{IN} = 0) = V_{DD} - R_1\left[\mu C_{OX}\frac{W}{2L}\right](V_{EB})^2$$

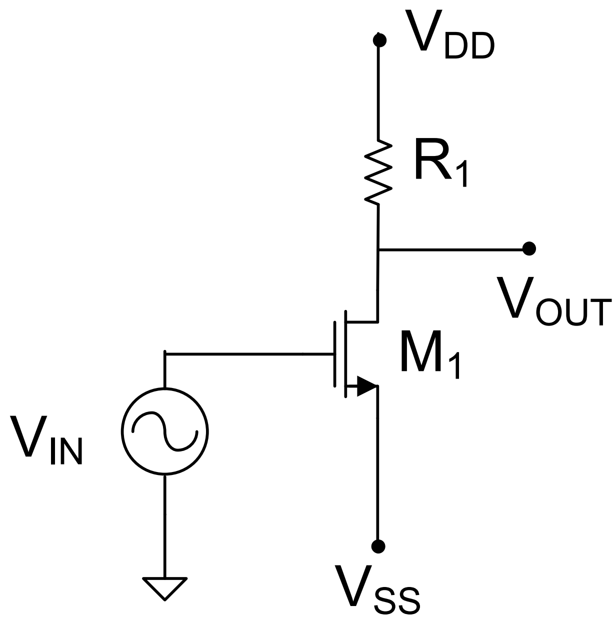
$$V_{OUT}(V_{IN} = 0) = 8 - 15K\left[10^{-4}\frac{16}{2 \bullet 1}\right]\left(\frac{1}{2}\right)^2 = 5V$$

This is termed the quiescent output voltage

Small signal analysis example



Small signal analysis example



$$V_{IN} = V_m \sin \omega t$$

$$V_{OUT} = V_{DD} - R_1 \left[\mu C_{OX} \frac{W}{2L} \right] (V_{IN} + V_{EB})^2$$

$$V_{EB} = 0.5V$$

$$V_{OUT}(V_{IN} = V_M) = V_{DD} - R_1 \left[\mu C_{OX} \frac{W}{2L} \right] (V_M + V_{EB})^2$$

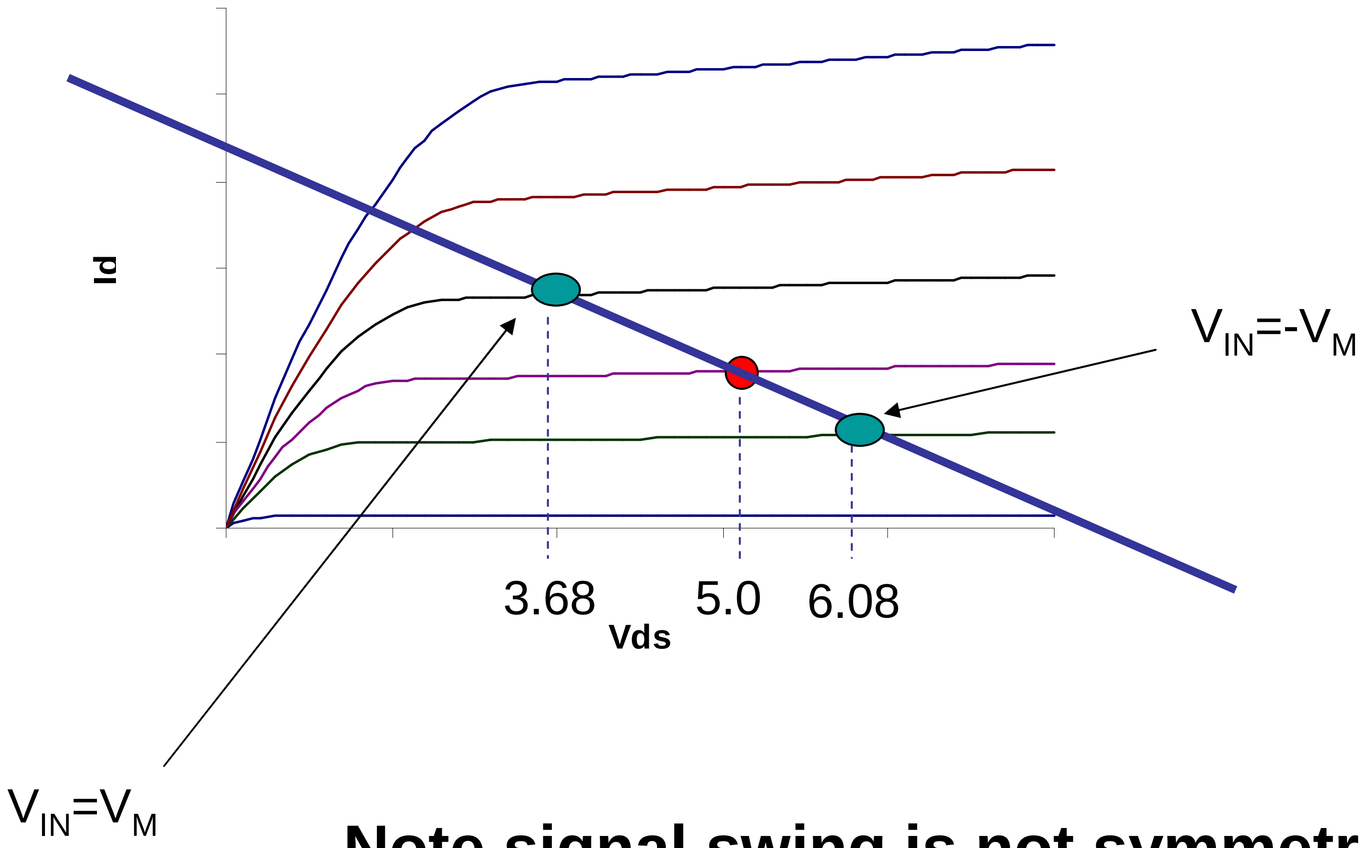
If V_M=0.1V

$$V_{OUT}(V_{IN} = V_M) = 8 - 15K \left[10^{-4} \frac{16}{2 \bullet 1} \right] (.1 + .5)^2 = 3.68$$

$$V_{OUT}(V_{IN} = -V_M) = V_{DD} - R_1 \left[\mu C_{OX} \frac{W}{2L} \right] (-V_M + V_{EB})^2$$

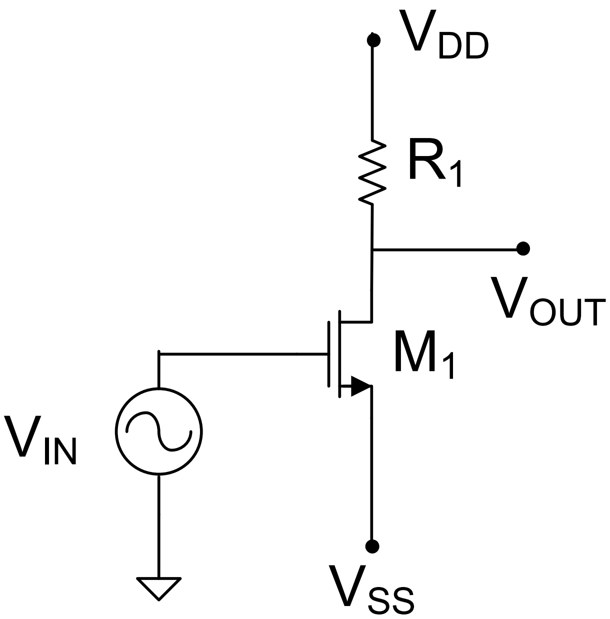
$$V_{OUT}(V_{IN} = -V_M) = 8 - 15K \left[10^{-4} \frac{16}{2 \bullet 1} \right] (-.1 + .5)^2 = 6.08$$

Small signal analysis example



Note signal swing is not symmetric

Small signal analysis example



$$V_{IN} = V_m \sin \omega t$$

$$\hat{A}_V = \frac{V_{OUT}\left(V_{IN} = V_M\right) - V_{OUT}\left(V_{IN} = -V_M\right)}{2V_M}$$

Parametric expression for apparent gain

$$V_{OUT}\left(V_{IN} = V_M\right) = V_{DD} - R_1\left[\mu C_{OX}\frac{W}{2L}\right]\left(V_M + V_{EB}\right)^2$$

$$V_{OUT}\left(V_{IN} = -V_M\right) = V_{DD} - R_1\left[\mu C_{OX}\frac{W}{2L}\right]\left(-V_M + V_{EB}\right)^2$$

$$\hat{A}_V = -R_1\frac{\mu C_{OX}W}{2L}\frac{\left(V_M + V_{EB}\right)^2 - \left(-V_M + V_{EB}\right)^2}{2V_M}$$

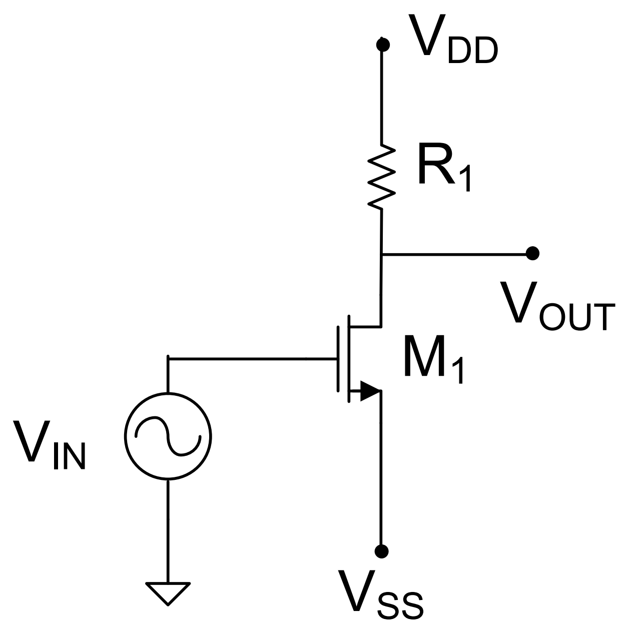
$$\hat{A}_V = -R_1\frac{\mu C_{OX}W}{2L}\frac{\left(V_M + V_{EB}\right)^2 - \left(-V_M + V_{EB}\right)^2}{2V_M}$$

$$\hat{A}_V = -R_1\frac{\mu C_{OX}W}{2L}\frac{\left(V_M^2 + 2V_MV_{EB} + V_{EB}^2\right) - \left(V_M^2 + 2V_{EB}V_M + V_{EB}^2\right)}{2V_M}$$

$$\hat{A}_V = -R_1\frac{\mu C_{OX}W}{2L}\frac{4V_MV_{EB}}{2V_M} = -R_1\frac{\mu C_{OX}W}{L}V_{EB}$$

Note: Apparent gain is independent of V_M

Small signal analysis example



$$V_{IN} = V_m \sin \omega t$$

$$\hat{A}_V = -R_1 \frac{\mu C_{OX} W}{L} V_{EB}$$

Very simple expression for apparent gain

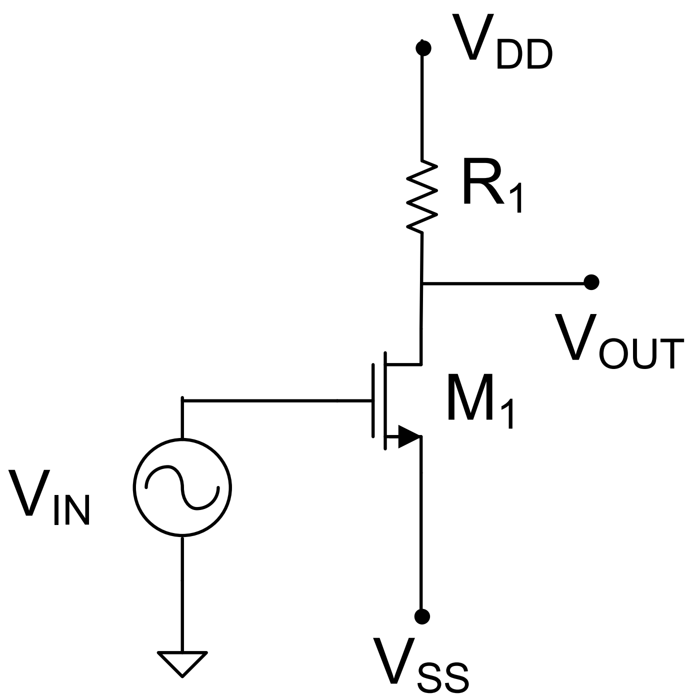
Derivation of apparent gain very tedious

Apparent gain gives minimal insight into design strategies

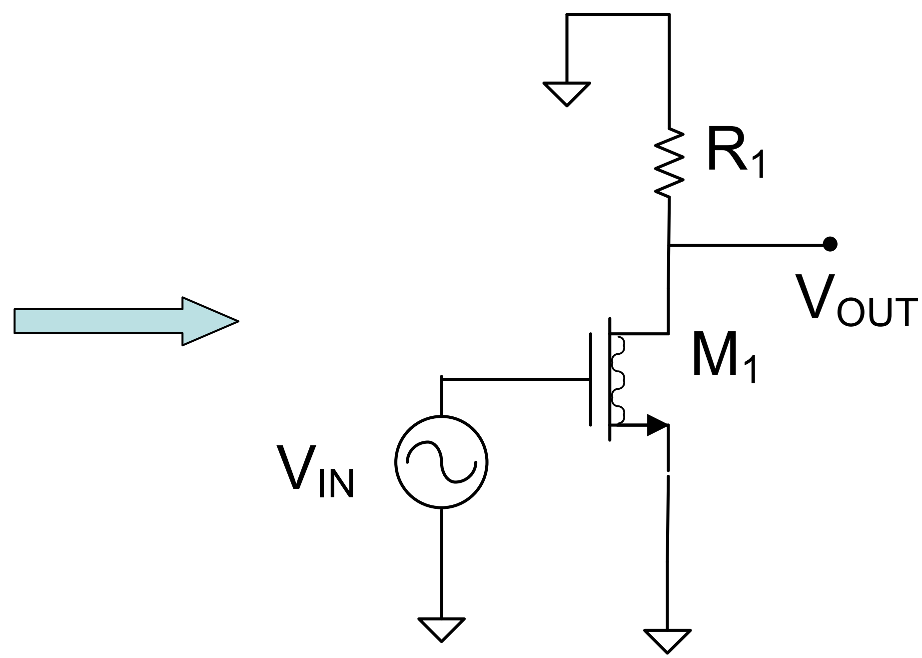
Near the Q-point, all well-behaved circuits operate linearly

Can this linear operation be exploited to simplify the analysis?

Small signal analysis example



Circuit Schematic

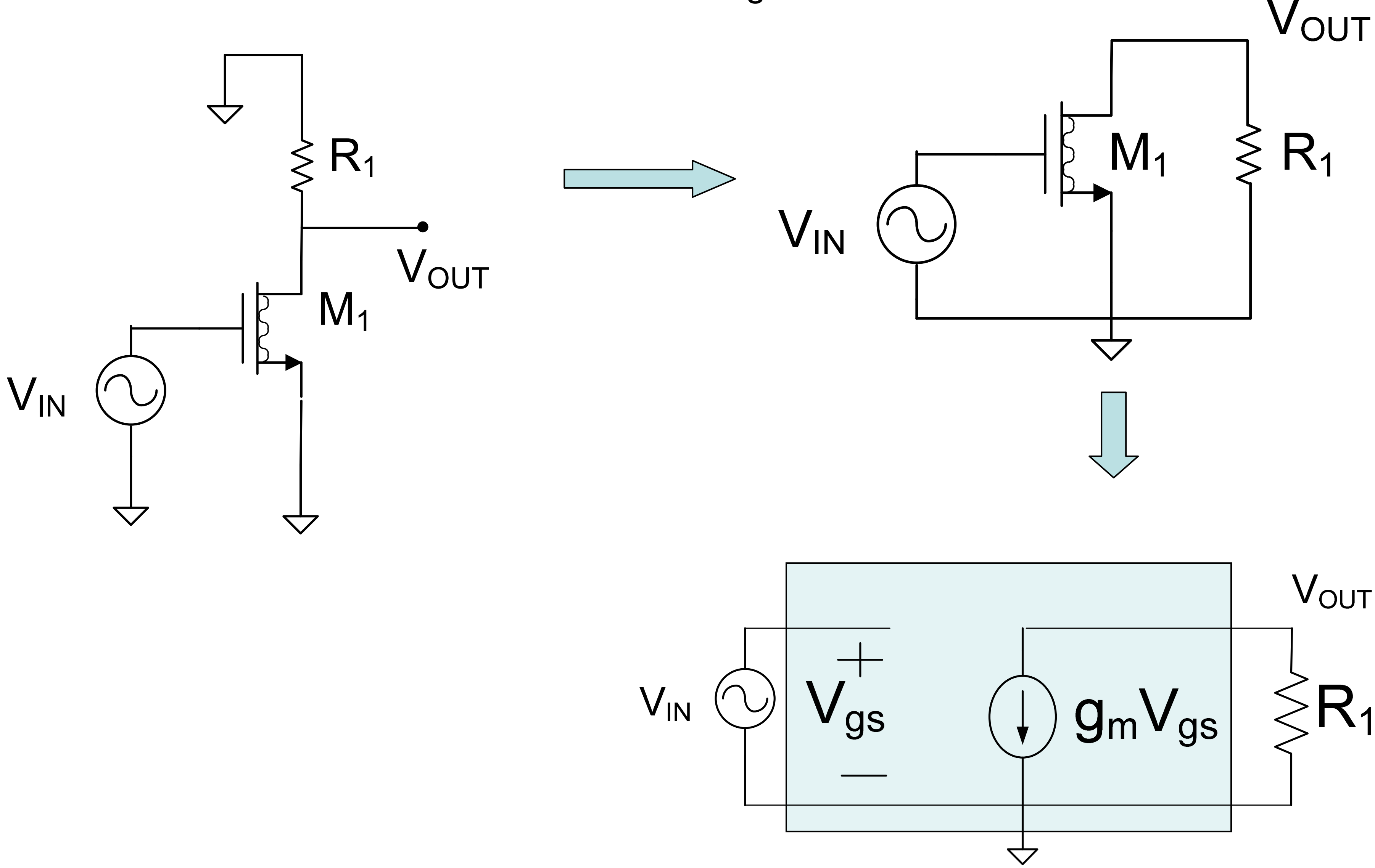


Linearized Small Signal Circuit

$$V_{IN} = V_m \sin \omega t$$

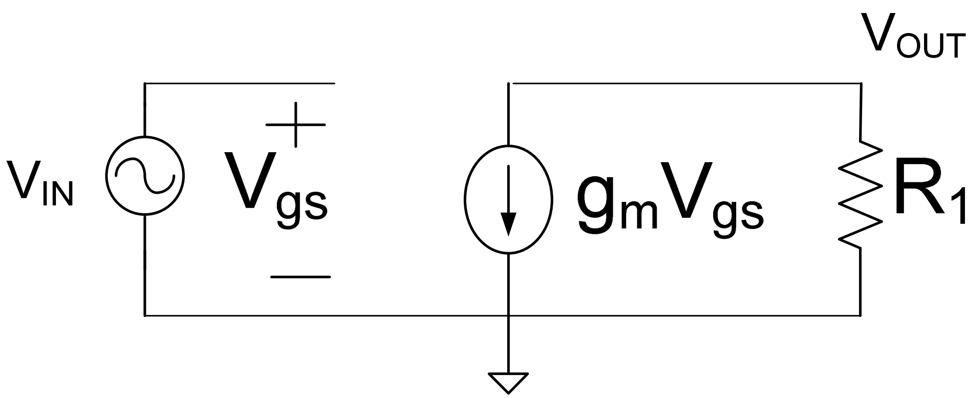
Small signal analysis example

Linearized Small Signal Circuits



Small signal analysis example

Linearized Small Signal Circuits



$$V_{OUT} = -g_m V_{gs} R_1$$
$$V_{IN} = V_{gs}$$

$$A_V = \frac{V_{OUT}}{V_{IN}} = -g_m R_1$$

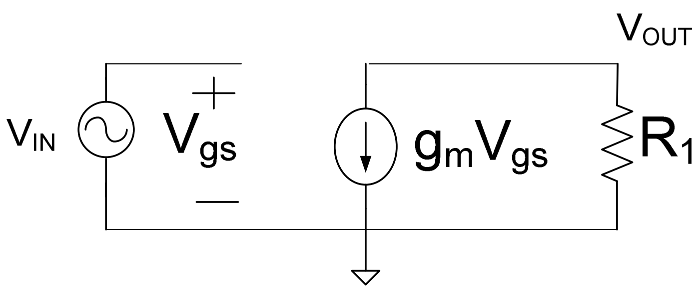
$$A_V = -\frac{2I_{DQ}}{V_{EB}} R_1$$

Still need I_{DQ} and V_{EB}

Small signal analysis much simpler (because linear)

Small signal analysis example

Linearized Small Signal Circuits



$$A_V = -\frac{2I_{DQ}}{V_{EB}}R_1$$

$$I_{DQ} \cong \mu C_{OX}\frac{W}{2L}(V_{EB})^2$$

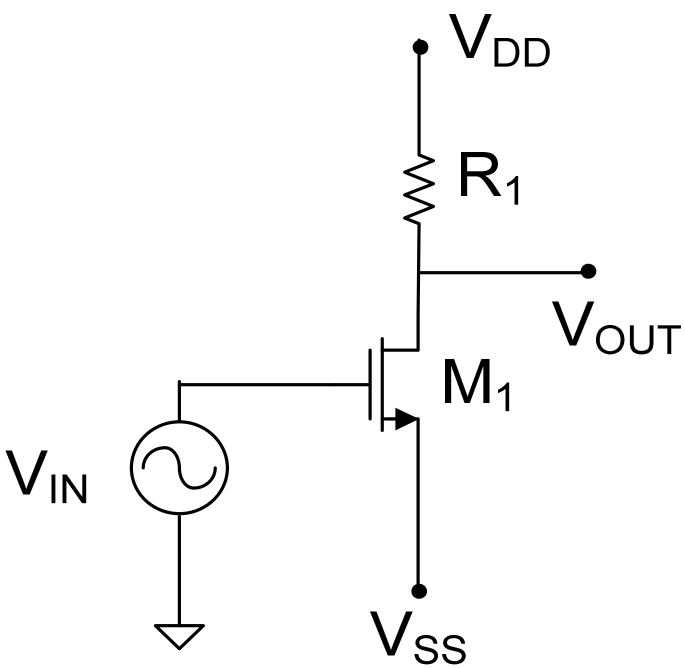
$$A_V = -\frac{2\mu C_{OX}\frac{W}{2L}(V_{EB})^2}{V_{EB}}R_1 = -\frac{\mu C_{OX}W\,V_{EB}R_1}{L}$$

$$V_{EB} = -V_{SS} - V_T = 1.25V - .75V = 0.5V$$

$$A_V = -\frac{\mu C_{OX}W\,V_{EB}R_1}{L} = -\frac{10^{-4}\bullet 16\bullet 0.5\bullet 15K}{1} = -12$$

This is identical to the numerical value obtained for the apparent gain !

Small signal analysis example



How does small signal gain compare to apparent gain for this circuit?

$$\hat{A}_V = -R_1 \frac{\mu C_{OX} W}{L} V_{EB}$$

$$A_V = -\frac{2I_{DQ}}{V_{EB}} R_1$$

$$A_V = -\frac{\mu C_{OX} W \, V_{EB} R_1}{L}$$

For this circuit the apparent gain and the actual gain are identical

This is not true in general but they will be close provided V_M is reasonably small and they become equal in the limit as V_M approaches 0