EE 434
Lecture 20

Bipolar
Device Models

Review from Last Time

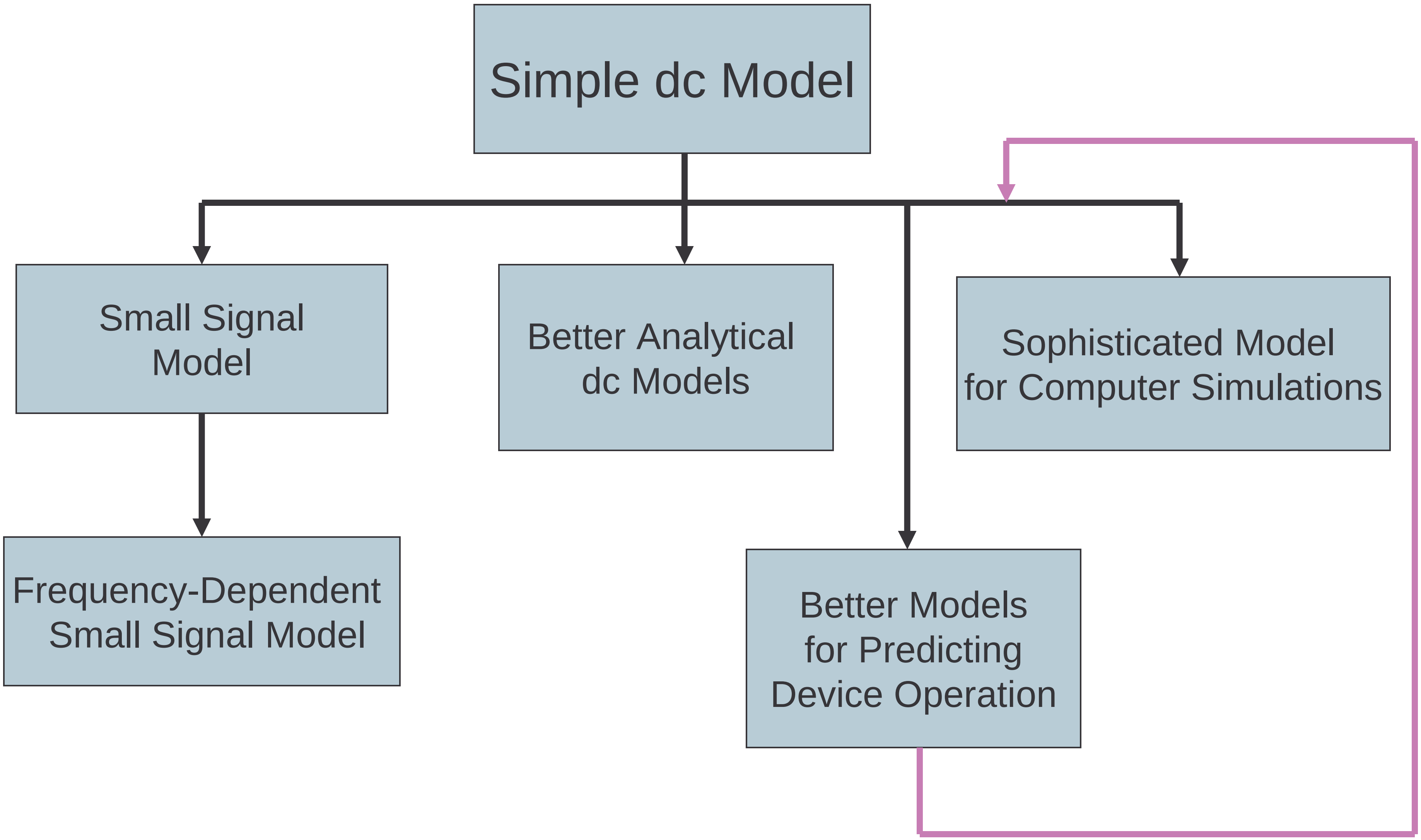
– Bipolar device operation dependent upon how minority carriers in base contribute to collector current

– Bipolar transistor is inherently a current amplifier with exponential relationship between collector current and V_{BE}

$$I_C = \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}}$$

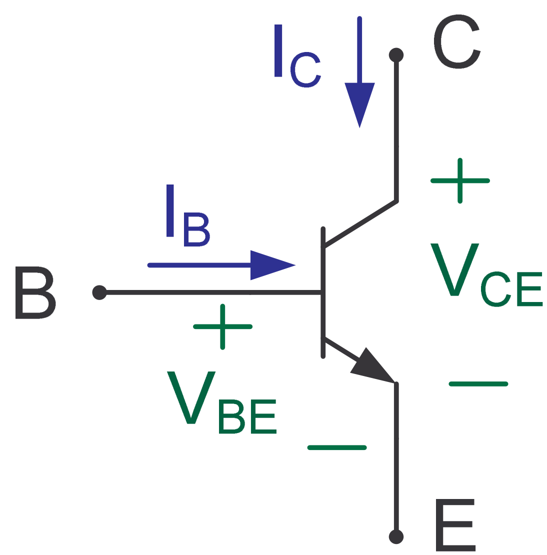
This property makes BJT very useful

Bipolar Models



Bipolar Models

Simple dc Model



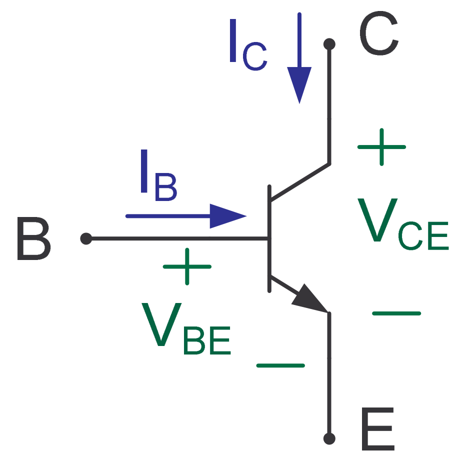
following convention, pick I_C and I_B as dependent variables and V_{BE} and V_{CE} as independent variables

Simple dc model

From last time :

$$I_B = \tilde{I}_S e^{\frac{V_{BE}}{V_t}}$$

$$I_C = \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}}$$



$$V_t = \frac{kT}{q}$$

This has the properties we are looking for but the variables we used in introducing these relationships are not standard

It can be shown that $\tilde{I}_S$ is proportional to the emitter area A_E

Define $\tilde{I}_S = \beta J_S A_E$ and substitute this into the above equations

Simple dc model

$$I_B = \tilde{I}_S e^{\frac{V_{BE}}{V_t}}$$

$$I_C = \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}}$$

$$V_t = \frac{kT}{q}$$

$\Rightarrow$

$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}}$$

$$V_t = \frac{kT}{q}$$

J_S is termed the saturation current density

Process Parameters : J_S,β

Design Parameters: A_E

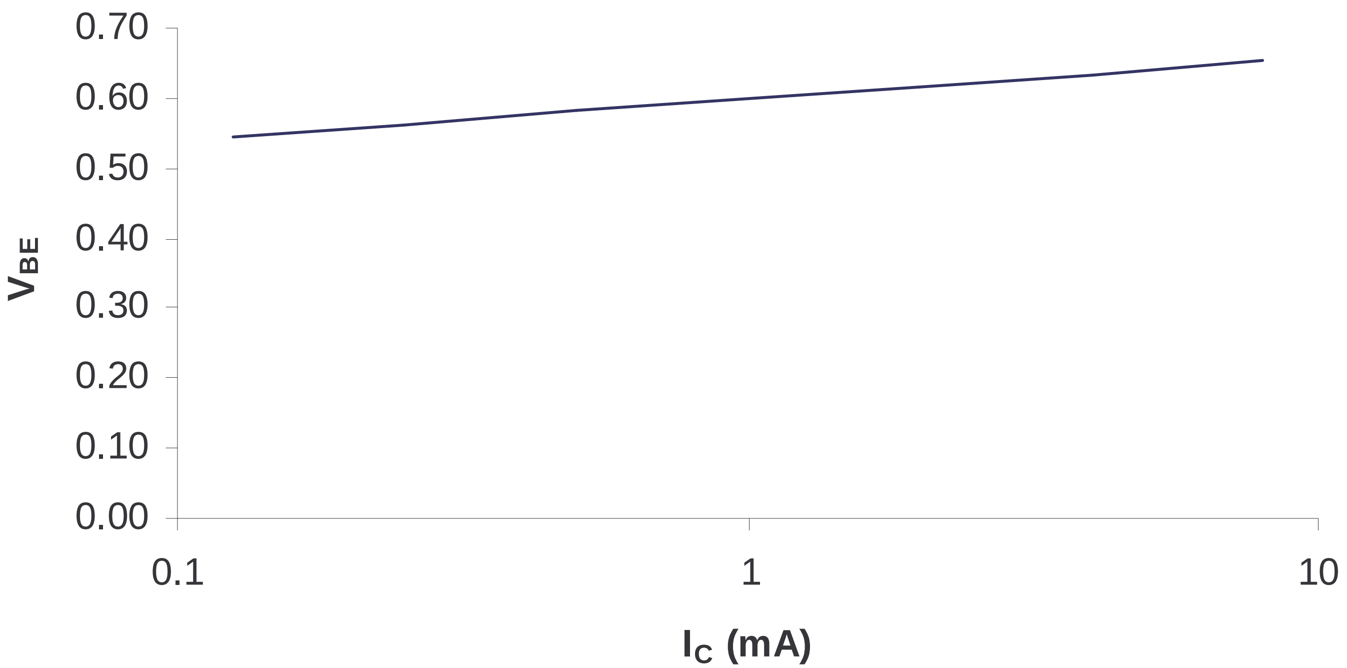
Environmental parameters and physical constants: k,T,q

At room temperature, V_t is around 26mV

J_S very small – around .25fA/u^2

Transfer Characteristics

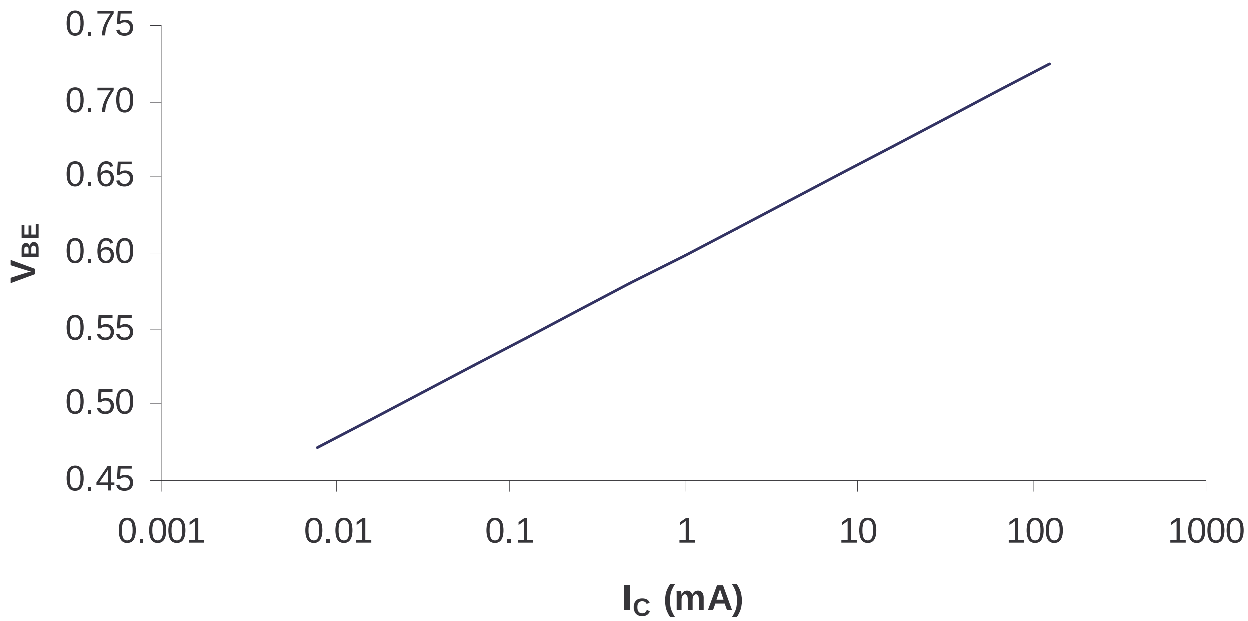
J_S=.25fA/u^2
A_E=400u^2



V_{BE} close to 0.6V for a two decade change in I_C around 1mA

Transfer Characteristics

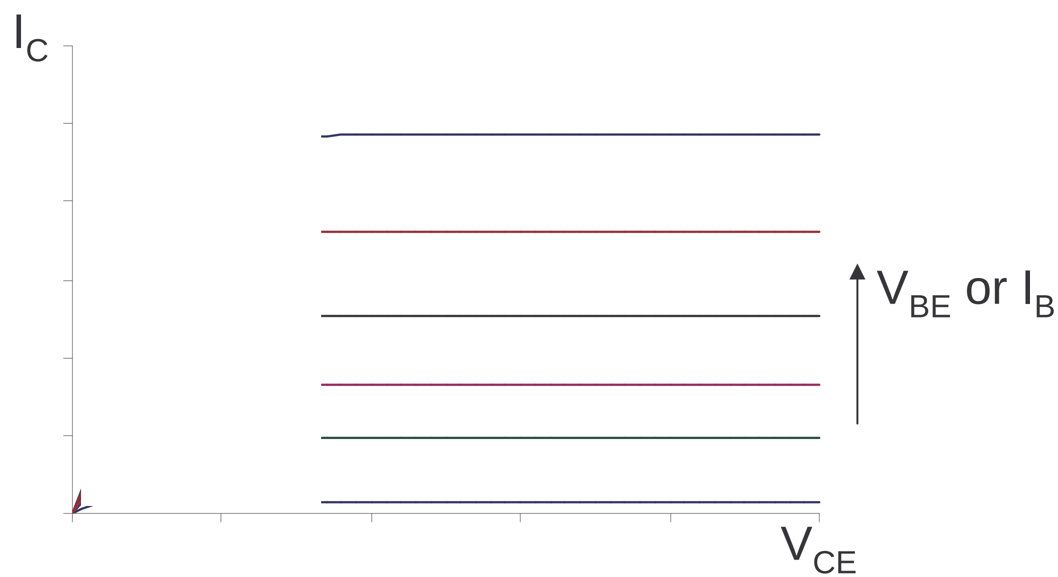
J_S=.25fA/u^2
A_E=400u^2



V_{BE} close to 0.6V for a four decade change in I_C around 1mA

Simple dc model

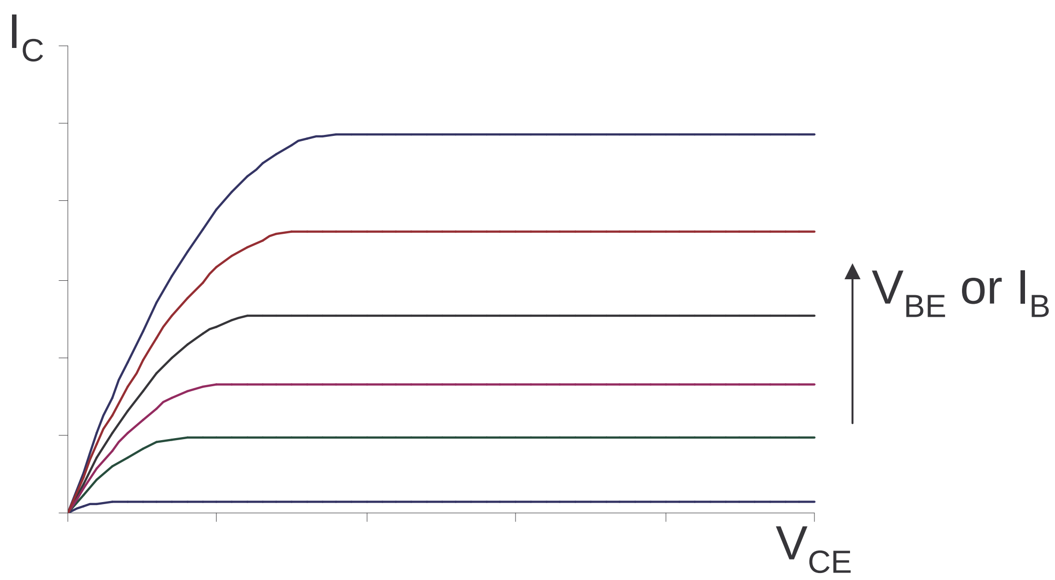
Output Characteristics



$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}}$$

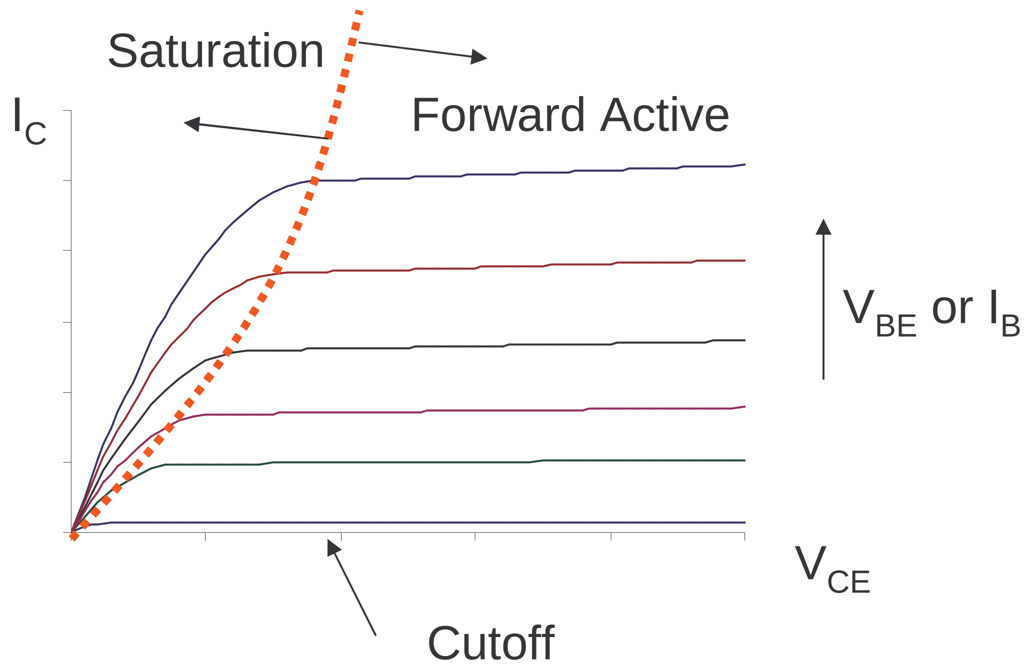
Simple dc model

Better Model of Output Characteristics



Simple dc model

Typical Output Characteristics

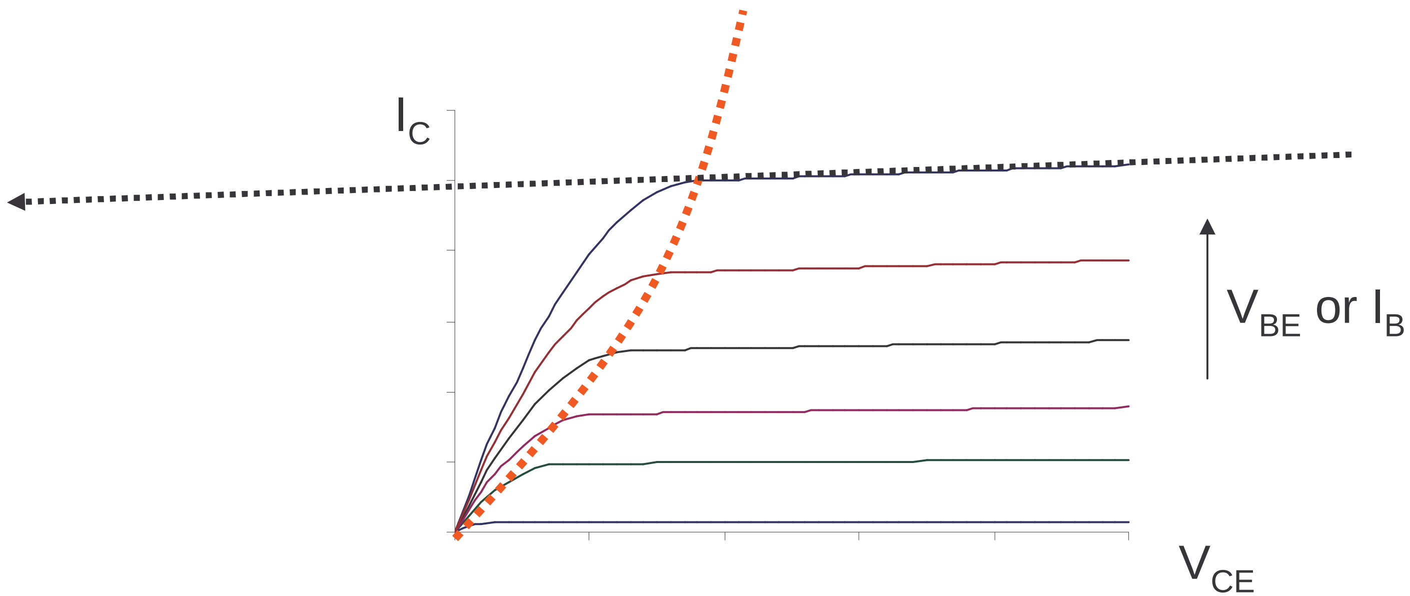


Forward Active region of BJT is analogous to Saturation region of MOSFET

Saturation region of BJT is analogous to Triode region of MOSFET

Simple dc model

Typical Output Characteristics

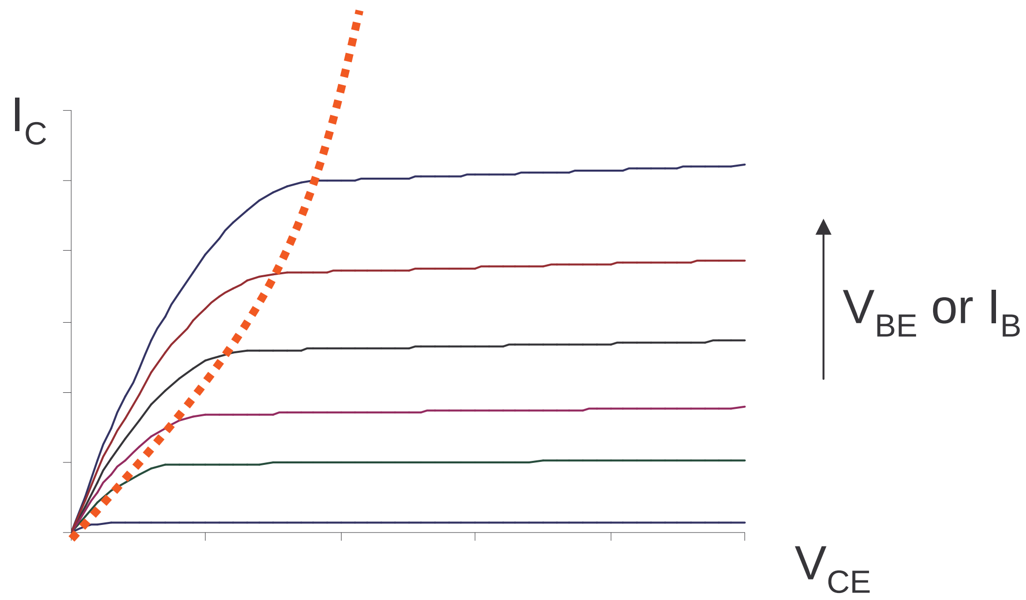


Projections of these tangential lines all intercept the $-V_{CE}$ axis at the same place and this is termed the Early voltage, V_{AF} (actually $-V_{AF}$ is intercept)

Typical values of V_{AF} are in the 100V range

Simple dc model

Improved Model



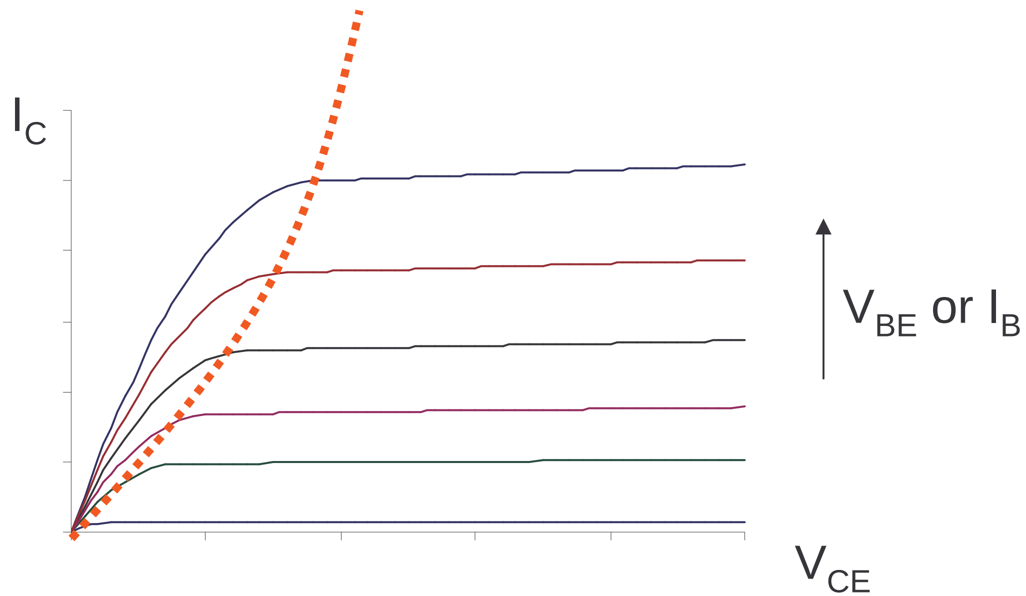
$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$$I_C = J_S e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

Valid only in Forward Active Region

Simple dc model

Improved Model



$$V_t = \frac{kT}{q}$$

$$I_E = -\frac{J_S A_E}{\alpha_F}\left(e^{\frac{V_{BE}}{V_t}} - 1\right) + J_S A_E\left(e^{\frac{V_{BC}}{V_t}} - 1\right)$$

$$I_C = J_S A_E\left(e^{\frac{V_{BE}}{V_t}} - 1\right) - \frac{J_S A_E}{\alpha_R}\left(e^{\frac{V_{BC}}{V_t}} - 1\right)$$

Valid in All regions of operation

V_{AF} effects can be added

Not mathematically easy to work with

Note dependent variables changes

Termed Ebers-Moll model

Reduces to previous model in FA region

Simple dc model

Simplified Multi-Region Model

$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$$V_t = \frac{kT}{q}$$

Forward Active

V_{BE}=0.7V
V_{CE}=0.2V

Saturation

I_C=I_B=0

Cutoff

Simple dc model

Simplified Multi-Region Model

$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}}\left(1+\frac{V_{CE}}{V_{AF}}\right)$ $I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$ $V_t = \frac{kT}{q}$	$V_{BE}>0.4V$ $V_{BC}<0$	Forward Active
$V_{BE}=0.7V$ $V_{CE}=0.2V$	$I_C<\beta I_B$	Saturation
$I_C=I_B=0$	$V_{BE}<0$ $V_{BC}<0$	Cutoff

A small portion of the operating region is missed with this model but seldom operate in the missing region

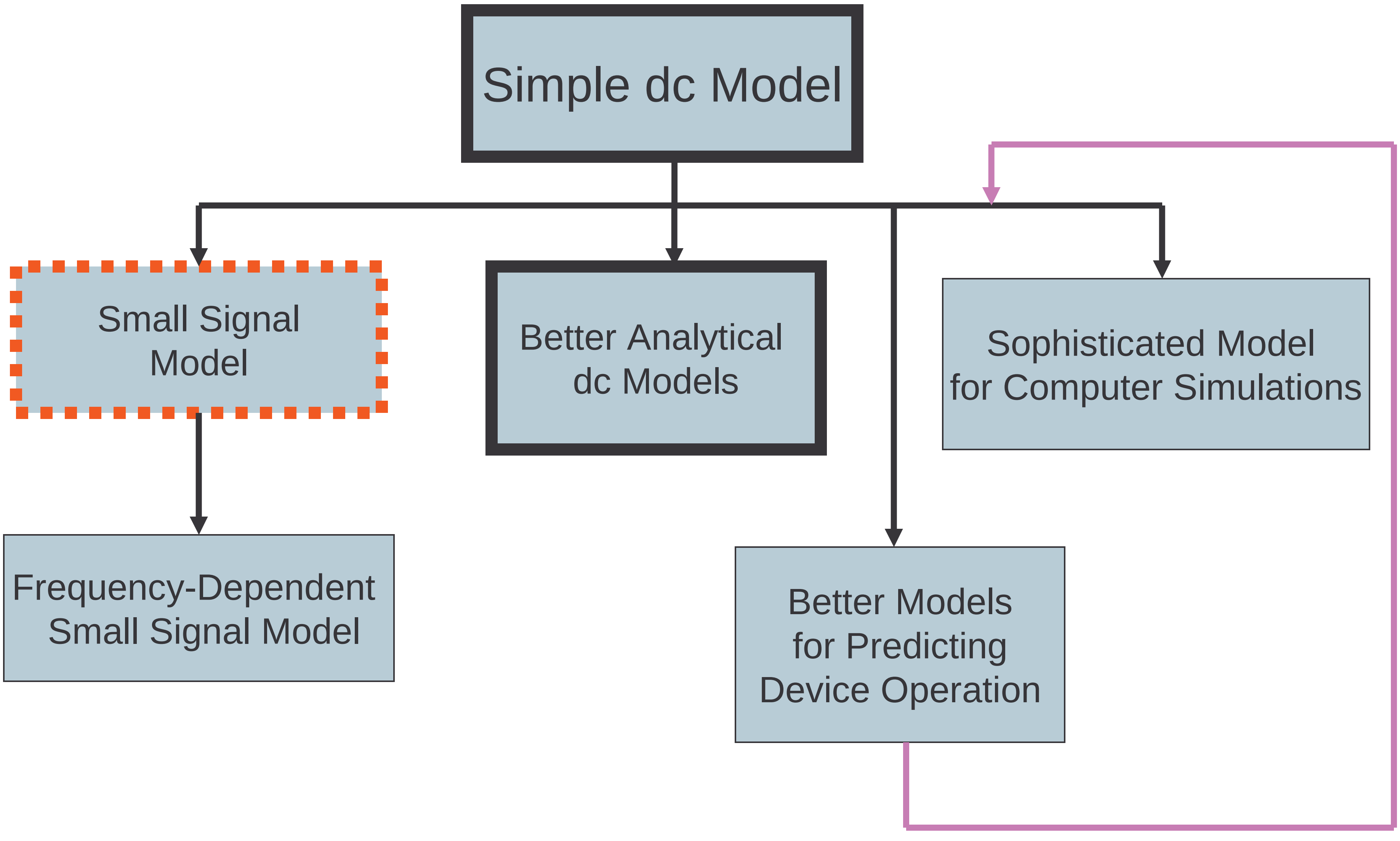
Simple dc model

Equivalent Simplified Multi-Region Model

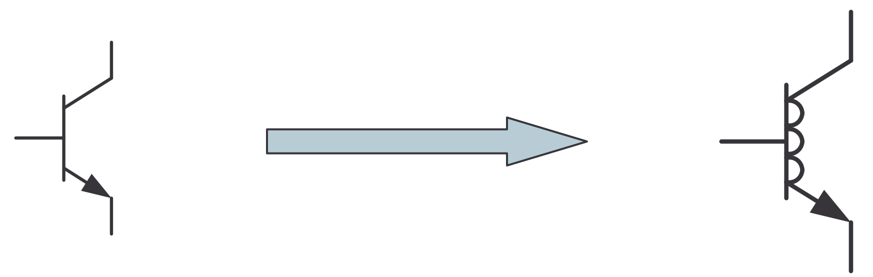
$I_C = \beta I_B \left(1 + \frac{V_{CE}}{V_{AF}}\right)$ $I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$ $V_t = \frac{kT}{q}$	$V_{BE}>0.4V$ $V_{BC}<0$	Forward Active
$V_{BE}=0.7V$ $V_{CE}=0.2V$	$I_C<\beta I_B$	Saturation
$I_C=I_B=0$	$V_{BE}<0$ $V_{BC}<0$	Cutoff

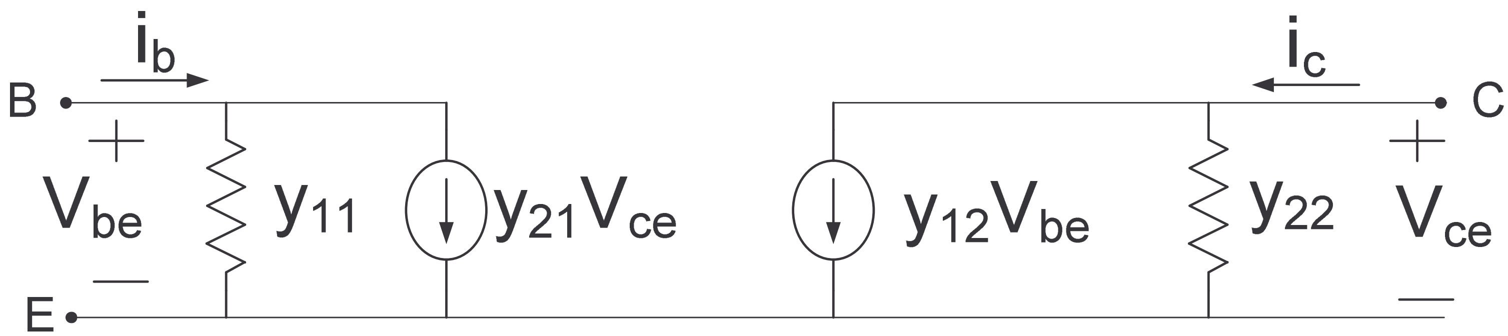
A small portion of the operating region is missed with this model but seldom operate in the missing region

Bipolar Models



Small Signal BJT Model





$$y_{11} = \left.\frac{\partial I_B}{\partial V_{BE}}\right|_{Q-PT} \overset{defn}{=} g_\pi$$

$$y_{21} = \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{Q-PT} \overset{defn}{=} g_m$$

$$y_{12} = \left.\frac{\partial I_B}{\partial V_{CE}}\right|_{Q-PT} \overset{defn}{=} ?$$

$$y_{22} = \left.\frac{\partial I_C}{\partial V_{CE}}\right|_{Q-PT} \overset{defn}{=} g_o$$

Small Signal BJT Model

$$y_{11} = \left.\frac{\partial I_B}{\partial V_{BE}}\right|_{Q-PT} \overset{defn}{=} g_\pi \qquad y_{21} = \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{Q-PT} \overset{defn}{=} g_m$$

$$y_{12} = \left.\frac{\partial I_B}{\partial V_{CE}}\right|_{Q-PT} \overset{defn}{=} ? \qquad y_{22} = \left.\frac{\partial I_C}{\partial V_{CE}}\right|_{Q-PT} \overset{defn}{=} g_o$$

Region of Operation for Small Signal Model : Forward Active

"1" $$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

"2" $$I_C = \beta I_B \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

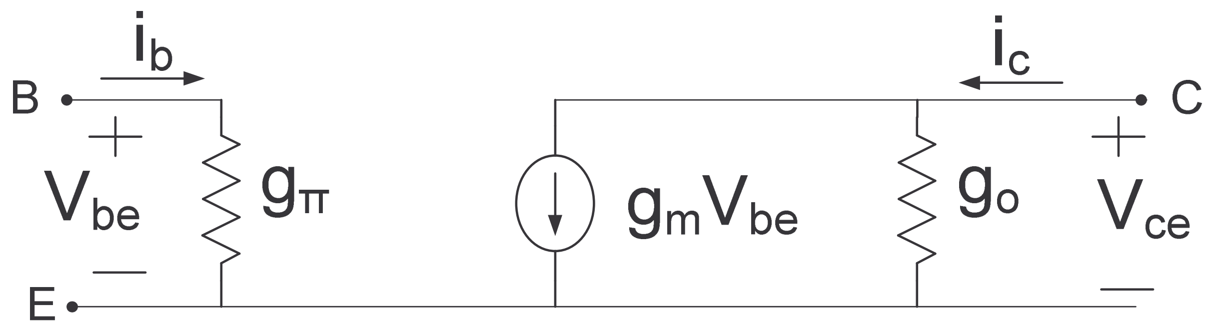
$$y_{11} = \left.\frac{\partial I_B}{\partial V_{BE}}\right|_{Q-PT} = \frac{1}{V_t}\left.\left(\frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}\right)\right|_{Q=PT} = \frac{I_{BQ}}{V_t} = \frac{I_{CQ}}{\beta V_t}$$

$$y_{12} = \left.\frac{\partial I_B}{\partial V_{CE}}\right|_{Q-PT} = 0$$

$$y_{21} = \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{Q-PT} = \frac{1}{V_t}\left.\left(J_S A_E e^{\frac{V_{BE}}{V_t}}\right)\right|_{Q-PT} = \frac{I_{CQ}}{V_t}$$

$$y_{22} = \left.\frac{\partial I_C}{\partial V_{CE}}\right|_{Q-PT} = \frac{1}{V_{AF}}\left[J_S A_E e^{\frac{V_{BE}}{V_t}}\Big|_{Q-PT}\right]_0 \cong \frac{I_{CQ}}{V_{AF}}$$

Small Signal BJT Model

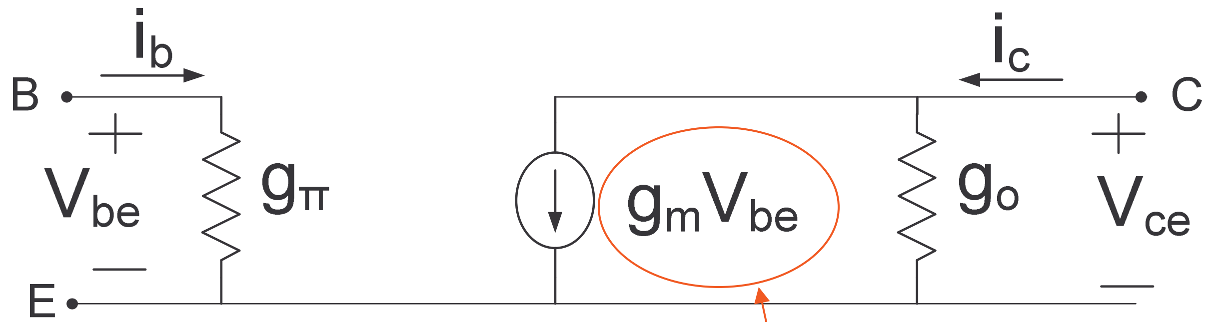


$$g_m = \frac{I_{CQ}}{V_t}$$

$$g_\pi = \frac{I_{CQ}}{\beta V_t}$$

$$g_o \cong \frac{I_{CQ}}{V_{AF}}$$

Small Signal BJT Model



$$g_m = \frac{I_{CQ}}{V_t} \qquad g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$

Observe :

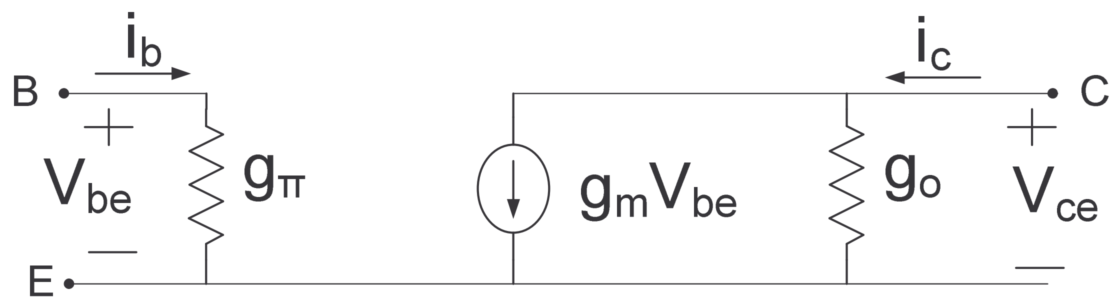
$$g_\pi v_{be} = i_b$$

$$g_m v_{be} = i_b \frac{g_m}{g_\pi}$$

$$\frac{g_m}{g_\pi} = \frac{\left[\dfrac{I_Q}{V_t}\right]}{\left[\dfrac{I_Q}{\beta V_t}\right]} = \beta$$

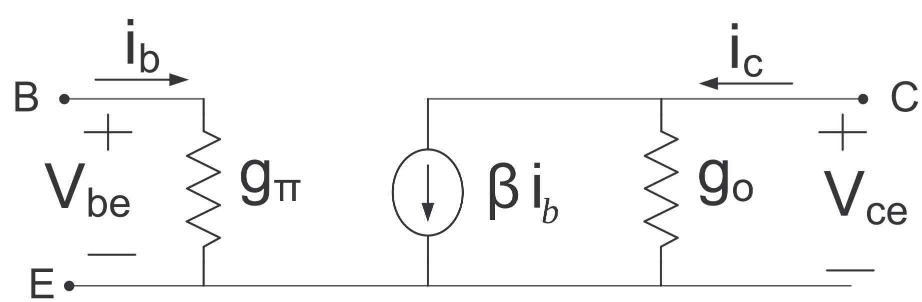
$$g_m v_{be} = \beta\, i_b$$

Small Signal BJT Model



$$g_m = \frac{I_{CQ}}{V_t} \qquad g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$

Alternate equivalent small signal model



$$g_\pi = \frac{I_{CQ}}{\beta V_t} \qquad g_o \cong \frac{I_{CQ}}{V_{AF}}$$