EE 434
Lecture 20

Bipolar
Device Models
Review from Last Time

- Bipolar device operation dependent upon how minority carriers in base contribute to collector current
- Bipolar transistor is inherently a current amplifier with exponential relationship between collector current and $V_{BE}$

\[ I_C = \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \]

This property makes BJT very useful
Bipolar Models

Simple dc Model

Small Signal Model

Better Analytical dc Models

Frequency-Dependent Small Signal Model

Sophisticated Model for Computer Simulations

Better Models for Predicting Device Operation
following convention, pick $I_C$ and $I_B$ as dependent variables and $V_{BE}$ and $V_{CE}$ as independent variables
Simple dc model

From last time:

\[ I_B = \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \]

\[ I_C = \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \]

\[ V_t = \frac{kT}{q} \]

This has the properties we are looking for but the variables we used in introducing these relationships are not standard.

It can be shown that \( \tilde{I}_S \) is proportional to the emitter area \( A_E \).

Define \( \tilde{I}_S = \beta J_S A_E \) and substitute this into the above equations.
Simple dc model

\[ I_B = \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \]
\[ I_C = \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}} \]
\[ V_t = \frac{kT}{q} \]

\[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]
\[ I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \]
\[ V_t = \frac{kT}{q} \]

\( J_S \) is termed the saturation current density

Process Parameters: \( J_S, \beta \)
Design Parameters: \( A_E \)

Environmental parameters and physical constants: \( k, T, q \)
At room temperature, \( V_t \) is around 26mV
\( J_S \) very small – around \( .25fA/u^2 \)
Transfer Characteristics

\[ J_S = 0.25 \text{fA/} \mu\text{m}^2 \]
\[ A_E = 400 \mu\text{m}^2 \]

\( V_{BE} \) close to 0.6V for a two decade change in \( I_C \) around 1mA
Transfer Characteristics

\[ J_S = 0.25 \text{fA}/u^2 \]

\[ A_E = 400u^2 \]

\[ V_{BE} \] close to 0.6V for a four decade change in \( I_C \) around 1mA
Simple dc model

Output Characteristics

\[ I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \]
Simple dc model

Better Model of Output Characteristics

\[ I_C \]

\[ V_{CE} \]

\[ V_{BE} \text{ or } I_B \]
Simple dc model

Typical Output Characteristics

Forward Active region of BJT is analogous to Saturation region of MOSFET
Saturation region of BJT is analogous to Triode region of MOSFET
Projections of these tangential lines all intercept the $-V_{\text{CE}}$ axis at the same place and this is termed the Early voltage, $V_{\text{AF}}$ (actually $-V_{\text{AF}}$ is intercept)

Typical values of $V_{\text{AF}}$ are in the 100V range
Simple dc model

Improved Model

\[ I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left( 1 + \frac{V_{CE}}{V_{AF}} \right) \]

\[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]

Valid only in Forward Active Region
Simple dc model

Improved Model

\[ V_t = \frac{kT}{q} \]

\[ I_E = -J_S A_E \left( e^{\frac{v_{BE}}{V_t}} - 1 \right) + J_S A_E \left( e^{\frac{v_{BC}}{V_t}} - 1 \right) \]

\[ I_C = J_S A_E \left( e^{\frac{v_{BE}}{V_t}} - 1 \right) - J_S A_E \left( e^{\frac{v_{BC}}{V_t}} - 1 \right) \]

Valid in All regions of operation
\( V_{AF} \) effects can be added
Not mathematically easy to work with
Note dependent variables changes
Termed Ebers-Moll model
Reduces to previous model in FA region
Simple dc model

Simplified Multi-Region Model

\[ I_C = J_S A_e \frac{V_{BE}}{V_t} \left(1 + \frac{V_{CE}}{V_{AF}}\right) \]

\[ I_B = \frac{J_S A_e}{\beta} \frac{V_{BE}}{V_t} \]

\[ V_t = \frac{kT}{q} \]

- **Forward Active**
  - \( V_{BE} = 0.7V \)
  - \( V_{CE} = 0.2V \)

- **Saturation**
  - \( I_C = I_B = 0 \)

- **Cutoff**
Simple dc model

**Simplified Multi-Region Model**

\[ I_C = J_s A E \frac{V_{BE}}{V_t} \left(1 + \frac{V_{CE}}{V_{AF}}\right) \]

\[ I_B = \frac{J_s A E}{\beta} e^{\frac{V_{BE}}{V_t}} \]

\[ V_t = \frac{kT}{q} \]

\[ V_{BE} = 0.7V \]

\[ V_{CE} = 0.2V \]

\[ I_C < \beta I_B \]

**Forward Active**

\[ V_{BE} > 0.4V \]

\[ V_{BC} < 0 \]

**Saturation**

\[ I_C = I_B = 0 \]

**Cutoff**

\[ V_{BE} < 0 \]

\[ V_{BC} < 0 \]

A small portion of the operating region is missed with this model but seldom operate in the missing region.
Simple dc model

Equivalent Simplified Multi-Region Model

\[ I_C = \beta I_B \left( 1 + \frac{V_{CE}}{V_{AF}} \right) \]

\[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]

\[ V_t = \frac{kT}{q} \]

\[ V_{BE} = 0.7V \]
\[ V_{CE} = 0.2V \]
\[ I_C = \beta I_B \]

\[ I_C = I_B = 0 \]
\[ V_{BE} < 0 \]
\[ V_{BC} < 0 \]

A small portion of the operating region is missed with this model but seldom operate in the missing region.
Bipolar Models

Simple dc Model

- Small Signal Model
  - Frequency-Dependent Small Signal Model
- Better Analytical dc Models
- Sophisticated Model for Computer Simulations

Better Models for Predicting Device Operation
Small Signal BJT Model

\[ y_{11} = \left. \frac{\partial I_B}{\partial V_{BE}} \right|_{Q-PT} = g_x \]

\[ y_{12} = \left. \frac{\partial I_B}{\partial V_{CE}} \right|_{Q-PT} = ? \]

\[ y_{21} = \left. \frac{\partial I_C}{\partial V_{BE}} \right|_{Q-PT} = g_m \]

\[ y_{22} = \left. \frac{\partial I_C}{\partial V_{CE}} \right|_{Q-PT} = g_o \]
Small Signal BJT Model

\[ y_{11} = \left. \frac{\partial I_B}{\partial V_{BE}} \right|_{Q-PT} = g_{\pi} \]

\[ y_{21} = \left. \frac{\partial I_C}{\partial V_{BE}} \right|_{Q-PT} = g_m \]

\[ y_{12} = \left. \frac{\partial I_B}{\partial V_{CE}} \right|_{Q-PT} = ? \]

\[ y_{22} = \left. \frac{\partial I_C}{\partial V_{CE}} \right|_{Q-PT} = g_o \]

Region of Operation for Small Signal Model:

Forward Active

\[ y_{11} = \left. \frac{\partial I_B}{\partial V_{BE}} \right|_{Q-PT} = \frac{1}{V_t} \left( \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \right) \]

\[ y_{12} = 0 \]

\[ y_{21} = \left. \frac{\partial I_C}{\partial V_{BE}} \right|_{Q-PT} = \frac{1}{V_t} \left( J_S A_E e^{\frac{V_{BE}}{V_t}} \right) \]

\[ y_{22} = \left. \frac{\partial I_C}{\partial V_{CE}} \right|_{Q-PT} = \frac{1}{V_{AF}} \left[ J_S A_E e^{\frac{V_{BE}}{V_t}} \right] \]

1. \[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]

2. \[ I_C = \beta I_B \left( 1 + \frac{V_{CE}}{V_{AF}} \right) \]
Small Signal BJT Model

\[ i_b = C Q V I g \]

\[ i_c = C Q V I g \]

\[ \pi = C Q V I g \]

\[ \alpha = C Q V I g \]

\[ g_m = \frac{I_{CQ}}{V_t} \]

\[ g_{\pi} = \frac{I_{CQ}}{\beta V_t} \]

\[ g_o = \frac{I_{CQ}}{V_{AF}} \]
Small Signal BJT Model

\[ g_m = \frac{I_{CQ}}{V_t} \]
\[ g_\pi = \frac{I_{CQ}}{\beta V_t} \]
\[ g_o \cong \frac{I_{CQ}}{V_{AF}} \]

Observe:

\[ g_{\pi} v_{be} = i_b \]
\[ g_m v_{be} = i_b \frac{g_m}{g_{\pi}} \]
\[ \frac{g_m}{g_{\pi}} = \frac{I_Q}{V_t} = \beta \]

\[ g_m v_{be} = \beta i_b \]
Small Signal BJT Model

\[ g_m = \frac{I_{CQ}}{V_t} \]
\[ g_\pi = \frac{I_{CQ}}{\beta V_t} \]
\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]

Alternate equivalent small signal model

\[ g_\pi = \frac{I_{CQ}}{\beta V_t} \]
\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]