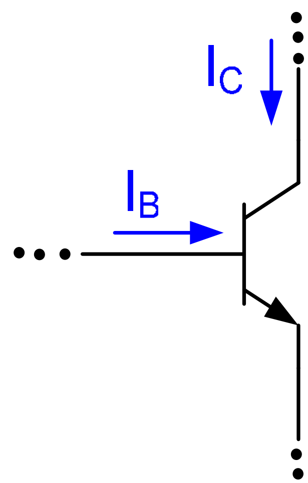
EE 434
Lecture 22

Bipolar

Device Models

Quiz 14

The collector current of a BJT was measured to be 20mA and the base current measured to be 0.1mA. What is the efficiency of injection of electrons coming from the emitter to the collector?



And the number is ….

1 8 7 5 3

6 9 4 2

And the number is ….

1 8 7 5 3

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Quiz 14

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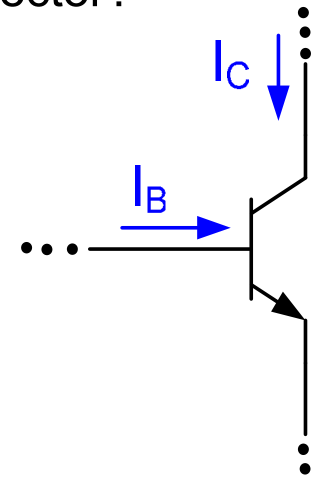


Solution:

Efficiency = α

$$\beta = \frac{I_C}{I_B} = \frac{20mA}{.1mA} = 200$$

$$\beta = \frac{\alpha}{1-\alpha} \Rightarrow \alpha = \frac{\beta}{1+\beta}$$

$$\alpha = \frac{200}{1+200} = .995$$

Review from Last Time

- Bipolar device operation dependent upon how minority carriers in base contribute to collector current
- Bipolar model (in high gain region)
  - Diode model for BE junction, injection efficiency for I_C α

$$I_B = \tilde{I}_S e^{\frac{V_{BE}}{V_t}}$$

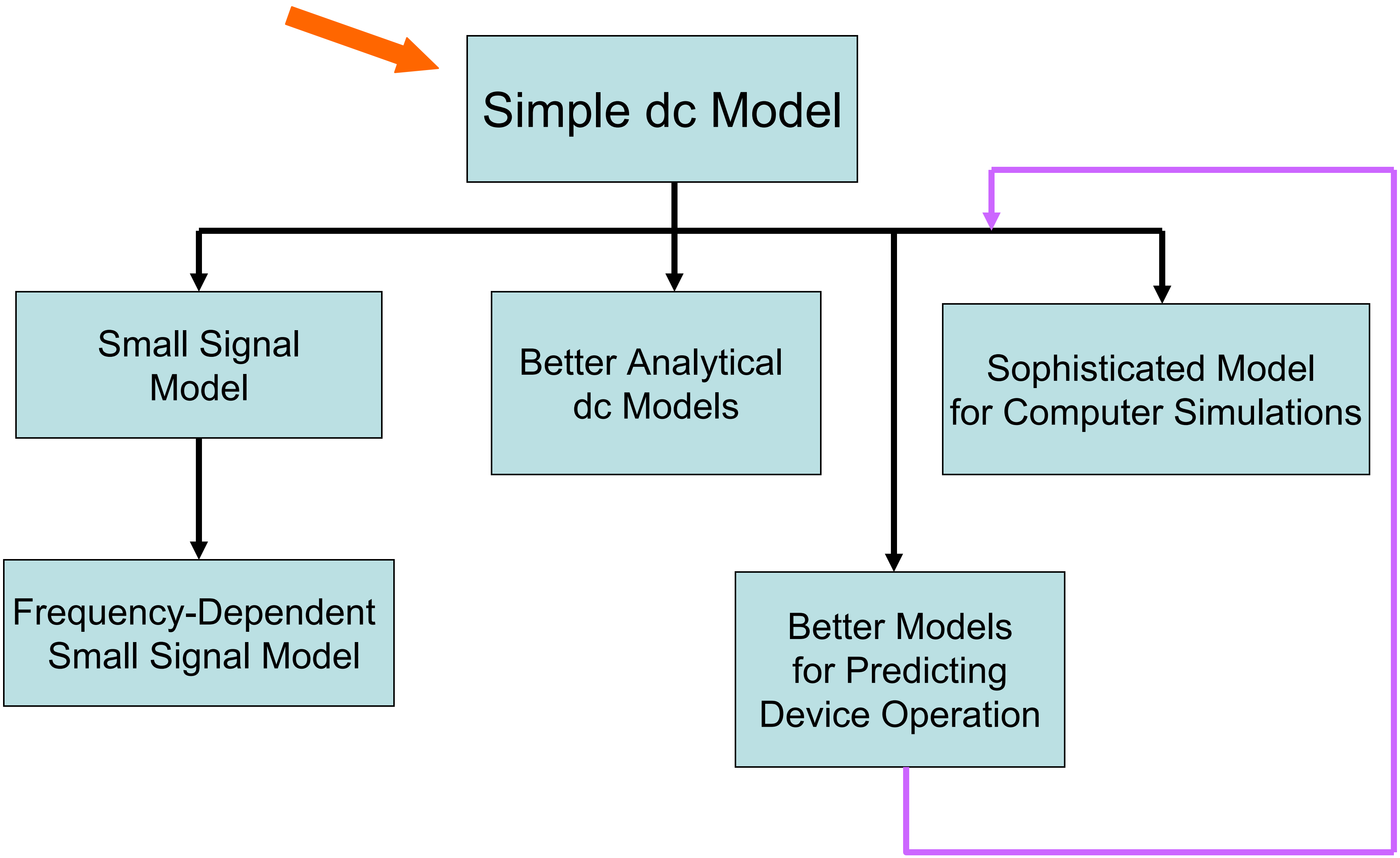
$$I_C = \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}}$$

$$\mathbf{I_B} = \frac{1}{\beta}\,\mathbf{I_C}$$

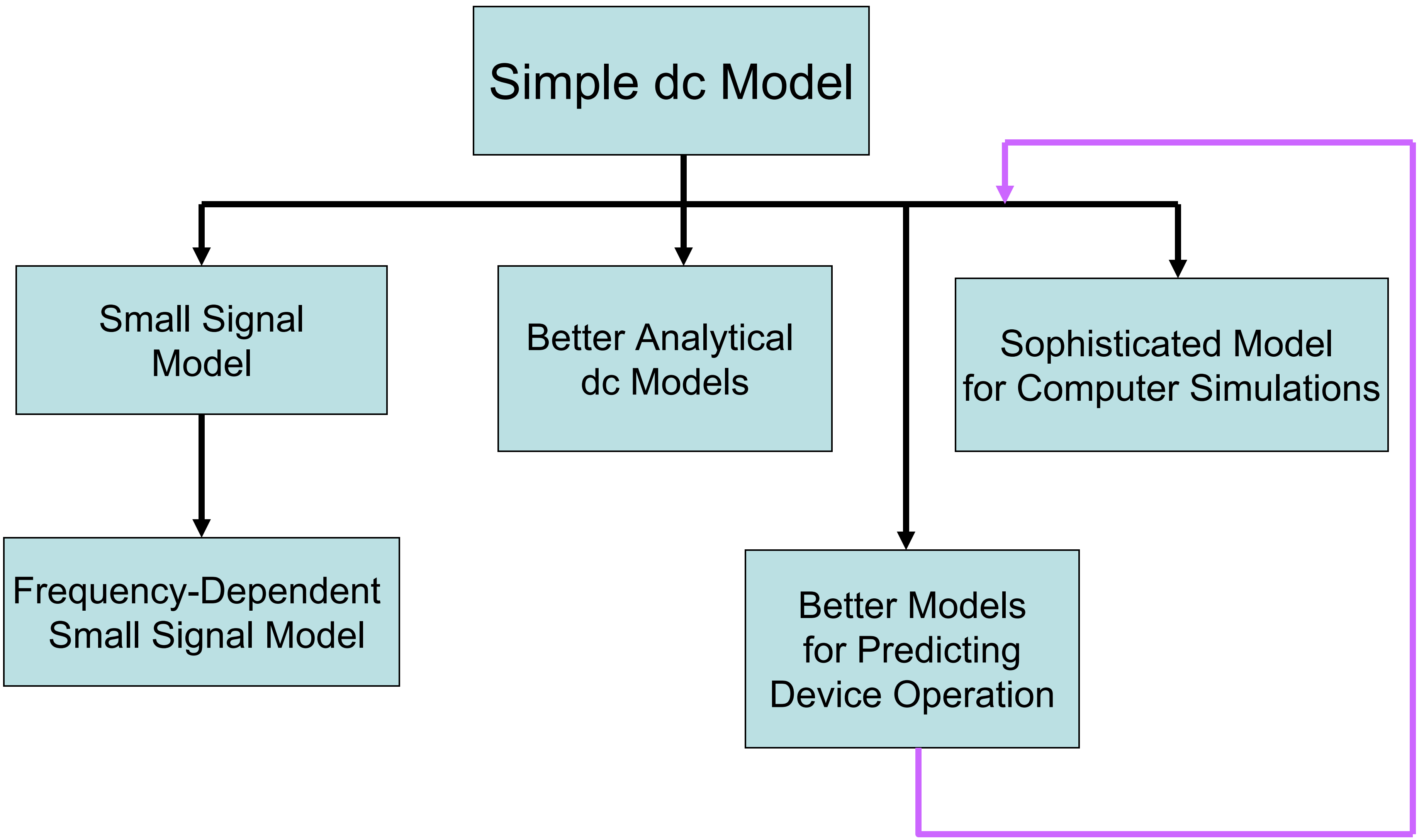
$$I_C = \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}}$$

- Bipolar transistor is inherently a current amplifier with exponential relationship between collector current and V_{BE}
  - This property makes BJT very useful

Bipolar Models

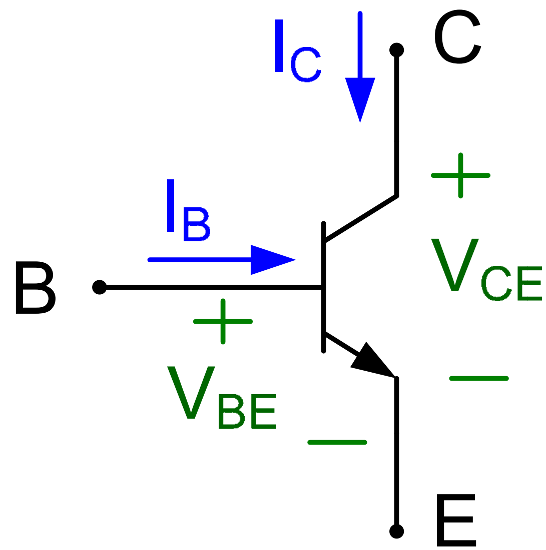


Bipolar Models



Bipolar Models

Simple dc Model



following convention, pick I_C and I_B as dependent variables and V_{BE} and V_{CE} as independent variables

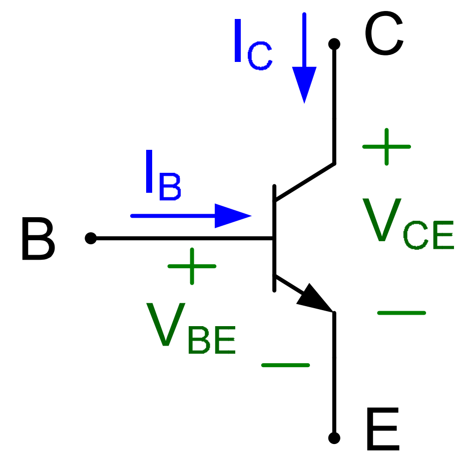
Simple dc model

From last time :

$$I_B = \tilde{I}_S e^{\frac{V_{BE}}{V_t}}$$

$$I_C = \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}}$$

$$V_t = \frac{kT}{q}$$



This has the properties we are looking for but the variables we used in introducing these relationships are not standard

It can be shown that $\tilde{I}_S$ is proportional to the emitter area A_E

Define $\tilde{I}_S = \beta^{-1} \mathbf{J_S A_E}$ and substitute this into the above equations

Simple dc model

$$I_B = \tilde{I}_S e^{\frac{V_{BE}}{V_t}}$$

$$I_C = \beta \tilde{I}_S e^{\frac{V_{BE}}{V_t}}$$

$$V_t = \frac{kT}{q}$$

$\Rightarrow$

$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}}$$

$$V_t = \frac{kT}{q}$$

J_S is termed the saturation current density

Process Parameters : J_S, β

Design Parameters: A_E

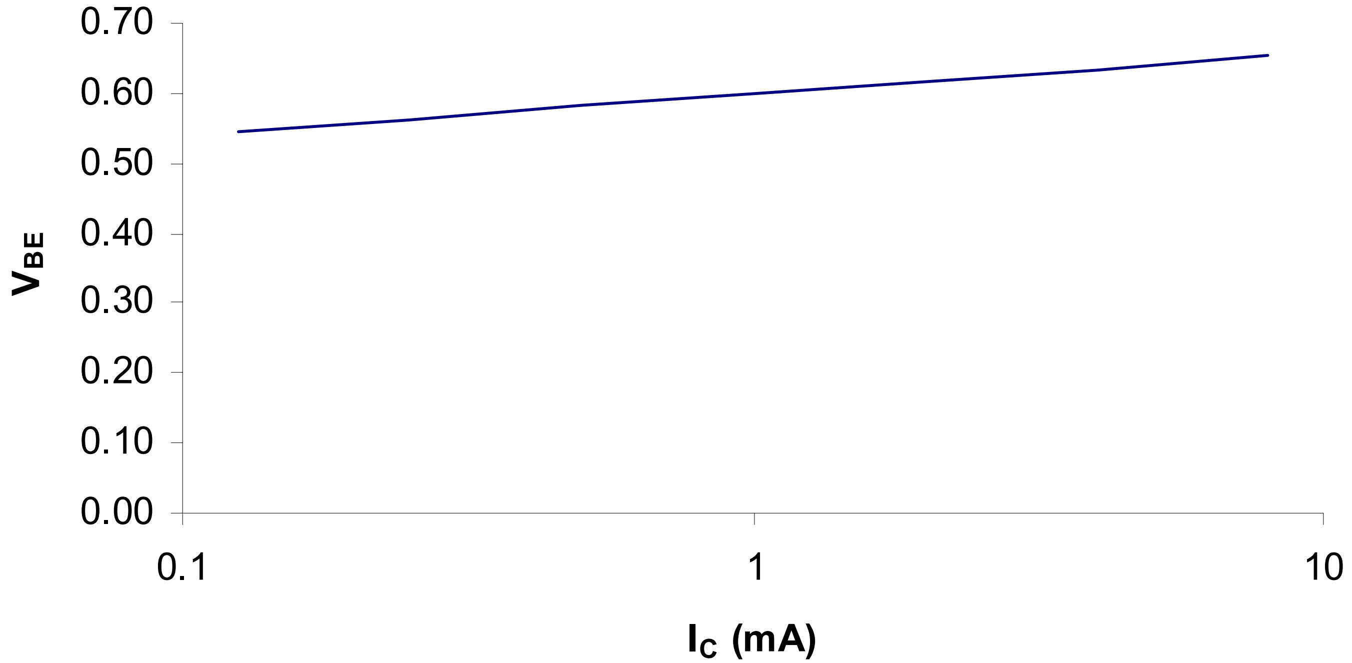
Environmental parameters and physical constants: k,T,q

At room temperature, V_t is around 26mV

J_S very small – around .25fA/u^2

Transfer Characteristics

J_S=.25fA/u^2
A_E=400u^2



V_{BE} close to 0.6V for a two decade change in I_C around 1mA

Transfer Characteristics

J_S=.25fA/u^2
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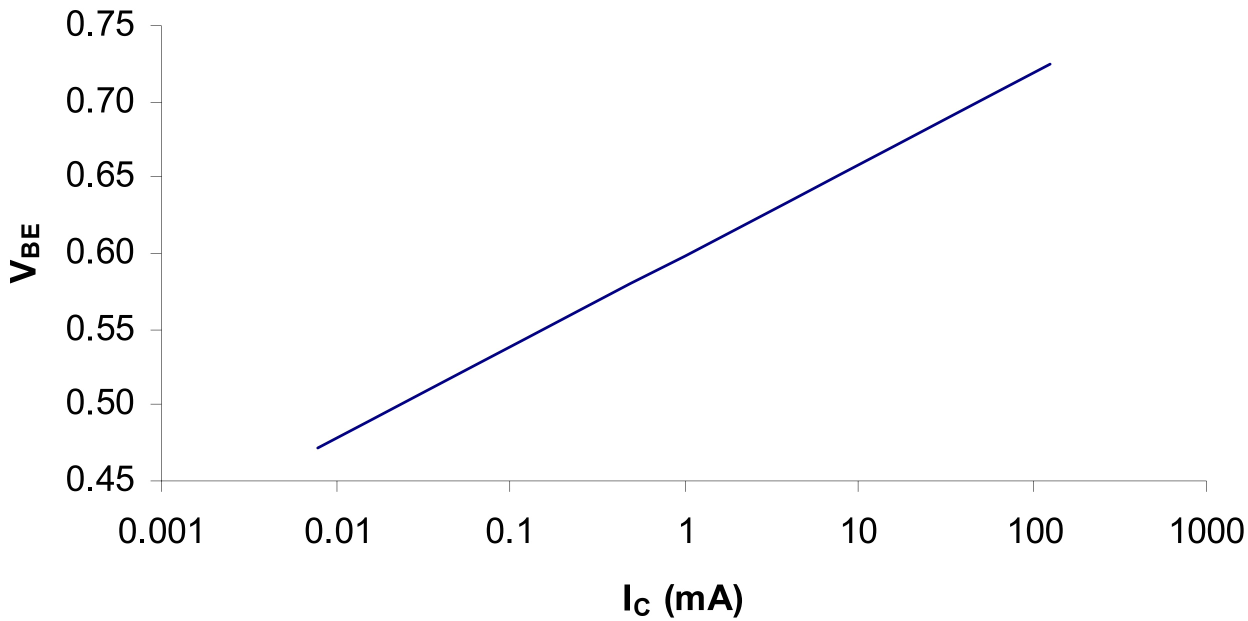
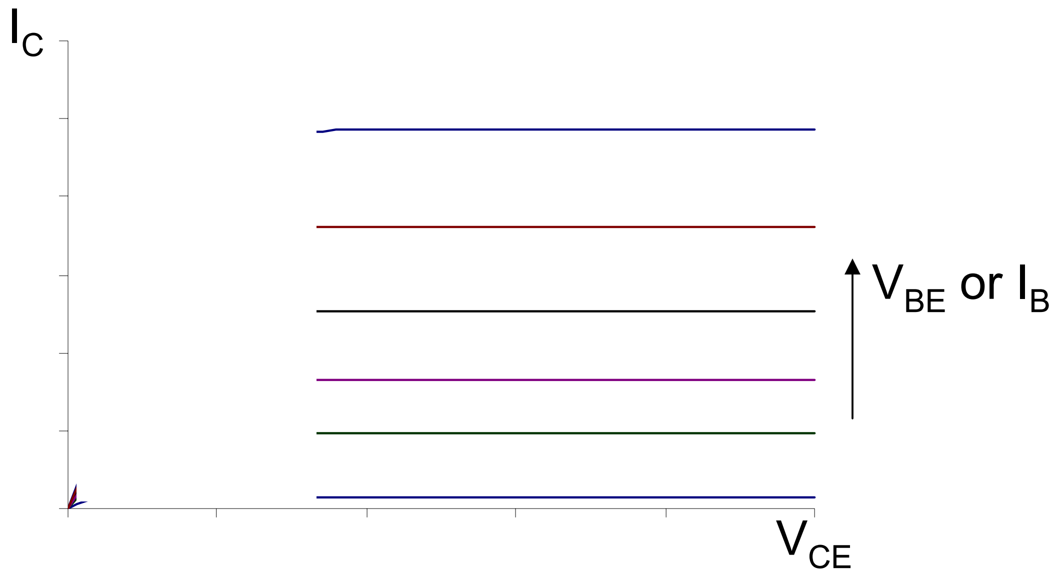


V_{BE} close to 0.6V for a four decade change in I_C around 1mA

Simple dc model

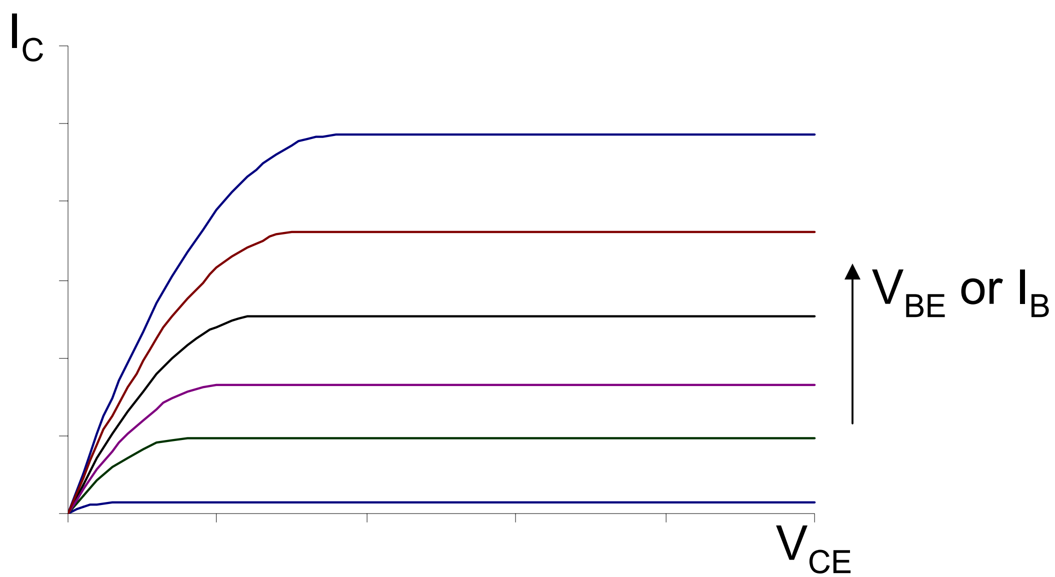
Output Characteristics



$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}}$$

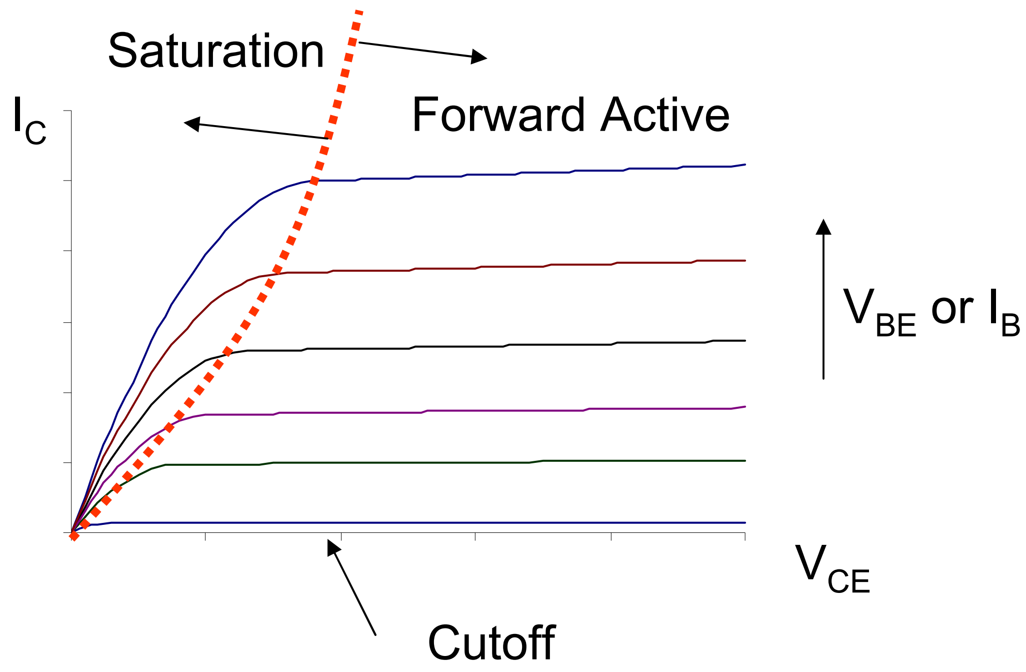
Simple dc model

Better Model of Output Characteristics



Simple dc model

Typical Output Characteristics

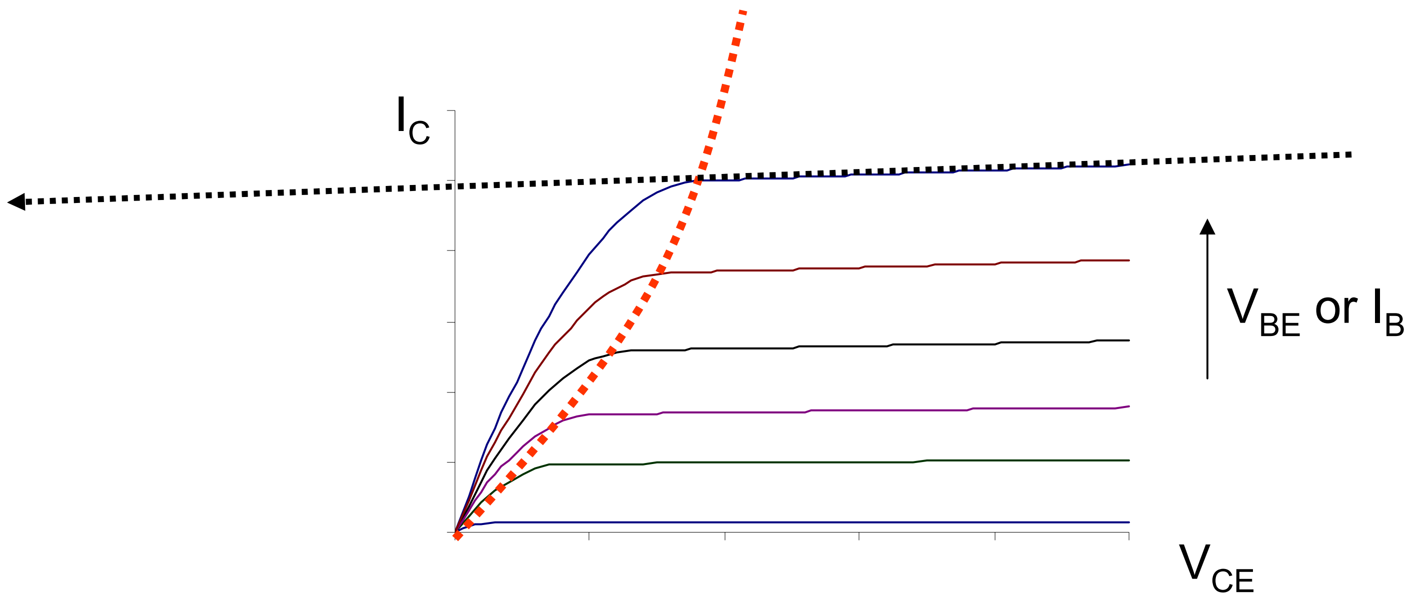


Forward Active region of BJT is analogous to Saturation region of MOSFET

Saturation region of BJT is analogous to Triode region of MOSFET

Simple dc model

Typical Output Characteristics

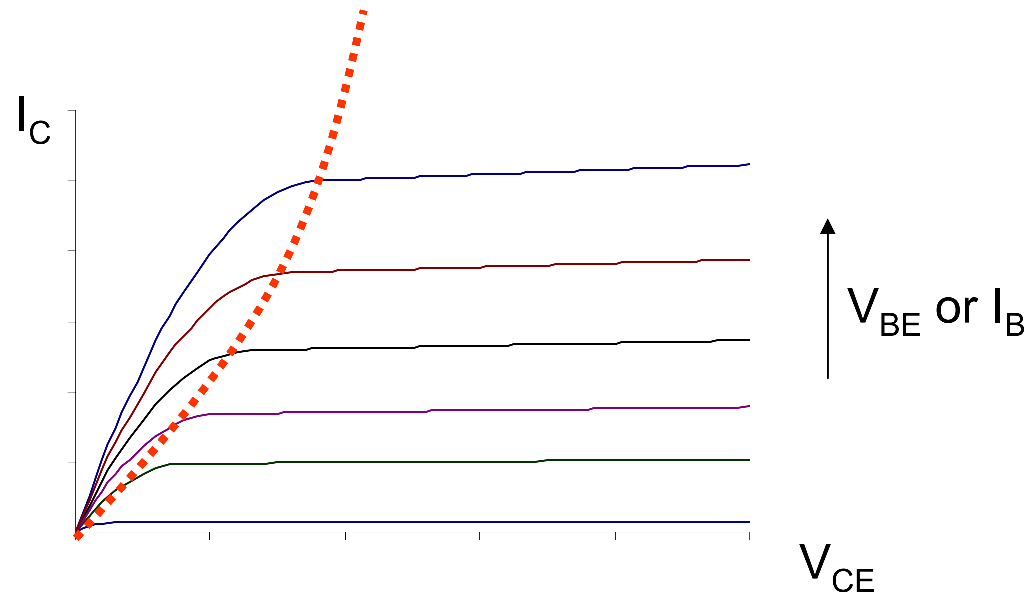


Projections of these tangential lines all intercept the $-V_{CE}$ axis at the same place and this is termed the Early voltage, V_{AF} (actually $-V_{AF}$ is intercept)

Typical values of V_{AF} are in the 100V range

Simple dc model

Improved Model



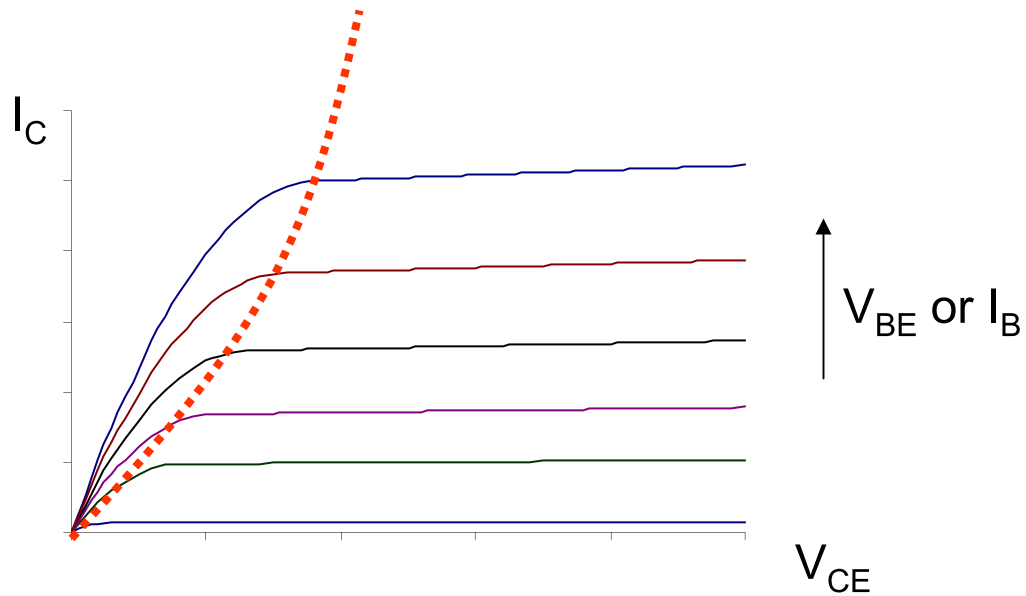
$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$$I_C = J_S e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

Valid only in Forward Active Region

Simple dc model

Improved Model



$$V_t = \frac{kT}{q}$$

$$I_E = -\frac{J_S A_E}{\alpha_F}\left(e^{\frac{V_{BE}}{V_t}} - 1\right) + J_S A_E\left(e^{\frac{V_{BC}}{V_t}} - 1\right)$$

$$I_C = J_S A_E\left(e^{\frac{V_{BE}}{V_t}} - 1\right) - \frac{J_S A_E}{\alpha_R}\left(e^{\frac{V_{BC}}{V_t}} - 1\right)$$

Valid in All regions of operation

V_{AF} effects can be added

Not mathematically easy to work with

Note dependent variables changes

Termed Ebers-Moll model

Reduces to previous model in FA region

Simple dc model

Ebers-Moll model

$$\left.\begin{aligned} I_E &= -\frac{J_S A_E}{\alpha_F}\left(e^{\frac{V_{BE}}{V_t}} - 1\right) + J_S A_E\left(e^{\frac{V_{BC}}{V_t}} - 1\right) \\ I_C &= J_S A_E\left(e^{\frac{V_{BE}}{V_t}} - 1\right) - \frac{J_S A_E}{\alpha_R}\left(e^{\frac{V_{BC}}{V_t}} - 1\right) \end{aligned}\right\}$$

$$V_t = \frac{kT}{q}$$

Process Parameters: $\{J_S, \alpha_F, \alpha_R\}$

Design Parameters: $\{A_E\}$

α_F is the parameter α discussed earlier
α_R is termed the "reverse α"

$$\beta_F = \frac{\alpha_F}{1-\alpha_F} \qquad \beta_R = \frac{\alpha_R}{1-\alpha_R}$$

Typical values for process parameters:

$J_S \sim 10^{-16} A/\mu^2$ $\beta_F \sim 100$, $\beta_R \sim 0.4$

Simple dc model

Ebers-Moll model

$$I_E = -\frac{J_S A_E}{\alpha_F}\left(e^{\frac{V_{BE}}{V_t}} - 1\right) + J_S A_E\left(e^{\frac{V_{BC}}{V_t}} - 1\right)$$

$$V_t = \frac{kT}{q} \qquad I_C = J_S A_E\left(e^{\frac{V_{BE}}{V_t}} - 1\right) - \frac{J_S A_E}{\alpha_R}\left(e^{\frac{V_{BC}}{V_t}} - 1\right)$$

With typical values for process parameters in forward active region (V_{BE}~0.6V, V_{BC}~-3V), with V_t=26mV and if A_E=100μ^2:

$J_S \sim 10^{-16} A/\mu^2$ $\qquad$ $\beta_F \sim 100$, $\beta_R \sim 0.4$

$$I_C = J_S A_E\left(e^{\frac{V_{BE}}{V_t}} - 1\right) - \frac{J_S A_E}{\alpha_R}\left(e^{\frac{V_{BC}}{V_t}} - 1\right)$$

$$I_C = 10^{-14}\left(1.05\text{x}10^{10} - 1\right) - \frac{10^{-14}}{.28}\left(7.7\text{x}10^{-51} - 1\right)$$

Completely dominant!

Makes no sense to keep anything other than $I_C = J_S A_E\left(e^{\frac{V_{BE}}{V_t}}\right)$ in forward active

Simple dc model

Ebers-Moll model

$$I_E = -\frac{J_S A_E}{\alpha_F}\left(e^{\frac{V_{BE}}{V_t}} - 1\right) + J_S A_E\left(e^{\frac{V_{BC}}{V_t}} - 1\right)$$

$$V_t = \frac{kT}{q} \qquad I_C = J_S A_E\left(e^{\frac{V_{BE}}{V_t}} - 1\right) - \frac{J_S A_E}{\alpha_R}\left(e^{\frac{V_{BC}}{V_t}} - 1\right)$$

Alternate equivalent expressions for dependent variables $\{I_C, I_B\}$ defined earlier for Ebers-Moll equations in terms of independent variables $\{V_{BE}, V_{CE}\}$

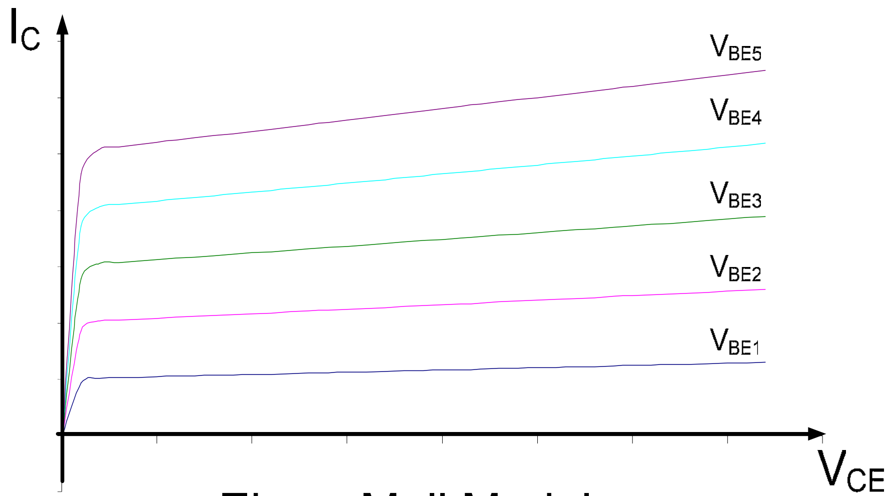
$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}}\left(1 - \left[\frac{1+\beta_R}{\beta_R}\right] e^{\frac{-V_{CE}}{V_t}}\right)$$

$$I_B = J_S A_E e^{\frac{V_{BE}}{V_t}}\left(\frac{1}{\beta_F} - \frac{1}{\beta_R} e^{\frac{-V_{CE}}{V_t}}\right)$$

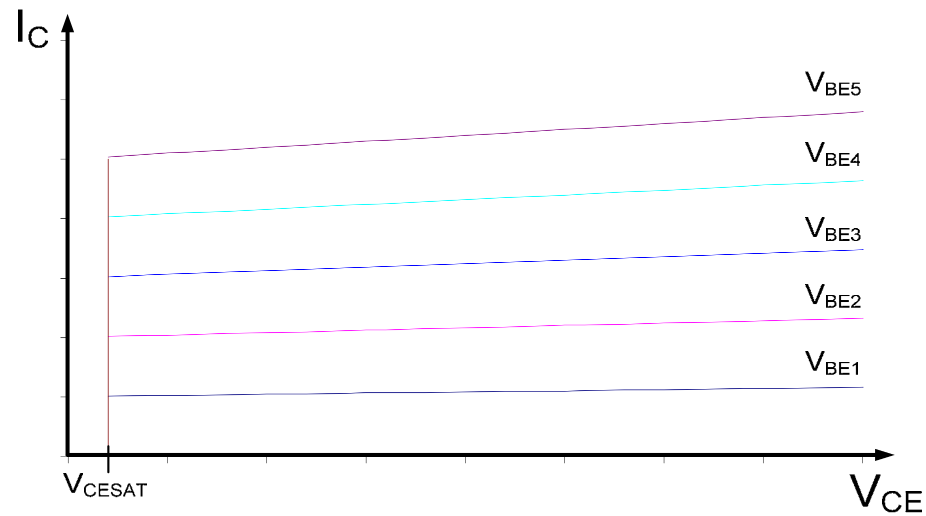
No more useful than previous equation but in form consistent with notation
Introduced earlier

Simple dc model

Simplified Multi-Region Model



Ebers-Moll Model



Simplified Multi-Region Model

$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 - \left[\frac{1+\beta_R}{\beta_R}\right] e^{\frac{-V_{CE}}{V_t}}\right)\left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

$$I_B = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(\frac{1}{\beta_F} - \frac{1}{\beta_R} e^{\frac{-V_{CE}}{V_t}}\right)$$

Forward Active:

$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

Saturation:

$V_{BE}=0.7V$

$V_{CE}=0.2V$

Cutoff:

$I_C=I_B=0$

Simple dc model

Simplified Multi-Region Model

$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

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$$V_t = \frac{kT}{q}$$

Forward Active

V_{BE}=0.7V
V_{CE}=0.2V

Saturation

I_C=I_B=0

Cutoff

Simple dc model

Simplified Multi-Region Model

$$I_C = J_S A_E e^{\frac{V_{BE}}{V_t}} \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$

$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$$V_t = \frac{kT}{q}$$

$V_{BE}>0.4V$

$V_{BC}<0$

Forward Active

$V_{BE}=0.7V$
$V_{CE}=0.2V$

$I_C<\beta I_B$

Saturation

$I_C=I_B=0$

$V_{BE}<0$

$V_{BC}<0$

Cutoff

A small portion of the operating region is missed with this model but seldom operate in the missing region

Simple dc model

Equivalent Simplified Multi-Region Model

Model	Conditions	Region
$I_C = \beta I_B\left(1+\frac{V_{CE}}{V_{AF}}\right)$ $I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$ $V_t = \frac{kT}{q}$	$V_{BE}>0.4V$ $V_{BC}<0$	Forward Active
$V_{BE}=0.7V$ $V_{CE}=0.2V$	$I_C<\beta I_B$	Saturation
$I_C=I_B=0$	$V_{BE}<0$ $V_{BC}<0$	Cutoff

A small portion of the operating region is missed with this model but seldom operate in the missing region