EE 434
Lecture 24

Bipolar Small Signal Device Models
Quiz 16

What is a “binning model” and what is the purpose of using “binning models”?
And the number is ....

1       8       7       5       3
6       9       4       2
And the number is ....

1 8 7 5 3

6 9 4 2
Quiz 16

What is a “binning model” and what is the purpose of using “binning models”?

Solution:

A binning model is actually a set of models whereby the model derived for dimensions close to those of a specific device is used rather than using the same model for each device (the functional form of most binning models does not change, simply the parameters in the model)

A good binning model will more closely predict the actual characteristics of a device than a model that does not change with device dimensions.
Models for Computer Simulation

- Simple dc Model
  - Small Signal Model
    - Frequency-Dependent Small Signal Model
  - Better Analytical dc Models
    - Better Models for Predicting Device Operation
  - Sophisticated Model for Computer Simulations
Concept in modeling is to partition model into two parts, one that characterizes the technology and the other that characterizes the geometric aspects of a device:

- Technology part of the model common to all devices in a process (Level 1, BSIM4, PSP models – over 100 parameters in BSIM 4 model)

- Geometric information unique to each device: \{W, L, N_{RD}, N_{RS}, A_D, A_S, P_D, P_S\}, (default values used in not specified)

- Models based upon physical principles but empirically modified to either simplify model or improve validity

Geometric description may not be unique.

Anticipated parasitics often included at schematic level for design prior to layout.

Hierarchy used in models.
Bipolar Models

Simple dc Model

- Small Signal Model
  - Frequency-Dependent Small Signal Model
- Better Analytical dc Models
  - Better Models for Predicting Device Operation
- Sophisticated Model for Computer Simulations
Recall:

Small-Signal Model

\[
\begin{align*}
    i_1 &= g_1(v_1, v_2, v_3) \\
    i_2 &= g_2(v_1, v_2, v_3) \\
    i_3 &= g_3(v_1, v_2, v_3)
\end{align*}
\]

\[
\begin{align*}
    i_1 &= y_{11}v_1 + y_{12}v_2 + y_{13}v_3 \\
    i_2 &= y_{21}v_1 + y_{22}v_2 + y_{23}v_3 \\
    i_3 &= y_{31}v_1 + y_{32}v_2 + y_{33}v_3
\end{align*}
\]

- Small signal circuit model is linear (and unique at a Q-point)
- Small signal equivalent circuits are not unique
Recall:

**Small-Signal Model**

Small signal model is that which represents the relationship between the small signal voltages and the small signal currents.

For small signals, this relationship should be linear.
Recall:

**Small-Signal Model**

- Small signal circuit model is linear (and unique at a Q-point)
- Small signal equivalent circuits are not unique

\[
\begin{align*}
\mathbf{i}_1 &= g_1(v_1, v_2, v_3) \\
\mathbf{i}_2 &= g_2(v_1, v_2, v_3) \\
\mathbf{i}_3 &= g_3(v_1, v_2, v_3)
\end{align*}
\]

\[
\begin{align*}
\mathbf{i}_1 &= y_{11}v_1 + y_{12}v_2 + y_{13}v_3 \\
\mathbf{i}_2 &= y_{21}v_1 + y_{22}v_2 + y_{23}v_3 \\
\mathbf{i}_3 &= y_{31}v_1 + y_{32}v_2 + y_{33}v_3
\end{align*}
\]

\[
y_{ij} = \left. \frac{\partial f_i(v_1, v_2, v_3)}{\partial v_j} \right|_{\bar{v}=v_q}
\]
Recall:

**Small-Signal Model**

Small signal model is that which represents the relationship between the small signal voltages and the small signal currents.

Define

\[ i_1 = I_1 - I_{1Q} \]
\[ i_2 = I_2 - I_{2Q} \]
\[ u_1 = V_1 - V_{1Q} \]
\[ u_2 = V_2 - V_{2Q} \]

\[
\begin{align*}
I_1 &= f_1(V_1, V_2) \\
I_2 &= f_2(V_1, V_2)
\end{align*}
\]
Recall:

**Small-Signal Model**

\[
\begin{align*}
    i_1 &= y_{11} V_1 + y_{12} V_2 \\
    i_2 &= y_{21} V_1 + y_{22} V_2
\end{align*}
\]

\[
y_{ij} = \frac{\partial f_i(V_1, V_2)}{\partial V_j} \bigg|_{V=\bar{V}_Q}
\]

\[
\bar{V} = \begin{pmatrix}
    V_{1Q} \\
    V_{2Q}
\end{pmatrix}
\]
Small Signal BJT Model

\[ y_{11} = \frac{\partial I_B}{\partial V_{BE}} \bigg|_{Q-PT} \quad \text{defn} \quad = g_n \]

\[ y_{12} = \frac{\partial I_B}{\partial V_{CE}} \bigg|_{Q-PT} \quad \text{defn} \quad = ? \]

\[ y_{21} = \frac{\partial I_C}{\partial V_{BE}} \bigg|_{Q-PT} \quad \text{defn} \quad = g_m \]

\[ y_{22} = \frac{\partial I_C}{\partial V_{CE}} \bigg|_{Q-PT} \quad \text{defn} \quad = g_o \]
Small Signal BJT Model

\[ y_{11} = \frac{\partial I_B}{\partial V_{BE}} \bigg|_{Q-PT} \quad \text{defn} \quad = g_\pi \]

\[ y_{21} = \frac{\partial I_C}{\partial V_{BE}} \bigg|_{Q-PT} \quad \text{defn} \quad = g_m \]

\[ y_{12} = \frac{\partial I_B}{\partial V_{CE}} \bigg|_{Q-PT} \quad \text{defn} \quad = ? \]

\[ y_{22} = \frac{\partial I_C}{\partial V_{CE}} \bigg|_{Q-PT} \quad \text{defn} \quad = g_o \]

Region of Operation for Small Signal Model:

Forward Active

\[ y_{11} = \frac{\partial I_B}{\partial V_{BE}} \bigg|_{Q-PT} = \frac{1}{V_t} \left( \frac{J_S A_E}{\beta} \right) \left( \frac{V_{BE}}{V_t} \right) \quad \approx \frac{I_{BQ}}{V_t} = \frac{I_{CQ}}{\beta V_t} \]

"1"

\[ I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}} \]

\[ y_{12} = \frac{\partial I_B}{\partial V_{CE}} \bigg|_{Q-PT} = 0 \]

"2"

\[ I_C = \beta I_B \left( 1 + \frac{V_{CE}}{V_{AF}} \right) \]

\[ y_{21} = \frac{\partial I_C}{\partial V_{BE}} \bigg|_{Q-PT} = \frac{1}{V_t} \left( J_S A_E e^{\frac{V_{BE}}{V_t}} \right) \bigg|_{Q-PT} = \frac{I_{CQ}}{V_t} \]

\[ y_{22} = \frac{\partial I_C}{\partial V_{CE}} \bigg|_{Q-PT} = \frac{1}{V_{AF}} \left( J_S A_E e^{\frac{V_{BE}}{V_t}} \right) \bigg|_{Q-PT} \approx \frac{I_{CQ}}{V_{AF}} \]
Small Signal BJT Model

\[ g_m = \frac{\frac{I_{CQ}}{V_t}}{V_t} \]
\[ g_\pi = \frac{\frac{I_{CQ}}{\beta V_t}}{\beta V_t} \]
\[ g_o \approx \frac{\frac{I_{CQ}}{V_{AF}}}{V_{AF}} \]
Small Signal BJT Model

Observe:

\[ g_{\pi} V_{be} = i_b \]

\[ g_m V_{be} = i_b \frac{g_m}{g_{\pi}} \]

\[ g_m = \frac{I_{CQ}}{V_t} \]

\[ g_{\pi} = \frac{I_{CQ}}{\beta V_t} \]

\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]

\[ g_m V_{be} = \beta i_b \]
Small Signal BJT Model

\[ g_m = \frac{I_{CQ}}{V_t} \]
\[ g_\pi = \frac{I_{CQ}}{\beta V_t} \]
\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]

Alternate equivalent small signal model

\[ g_\pi = \frac{I_{CQ}}{\beta V_t} \]
\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]
Properties of the BJT

Alternate Equivalent Small Signal Model

- Relative magnitude of small signal parameters
  - Simplified small signal model
Small Signal BJT Model

Observe:

\[ g_{\Pi} v_{be} = i_b \]

\[ g_m v_{be} = i_b \frac{g_m}{g_{\Pi}} \]

\[ g_m = \frac{I_Q}{V_t} \]
\[ g_{\Pi} = \begin{bmatrix} \frac{I_Q}{V_t} \\ \frac{I_Q}{\beta V_t} \end{bmatrix} = \beta \]

\[ g_m \approx \frac{I_Q}{V_{AF}} \]

\[ g_o \approx \frac{I_Q}{V_t} \]

\[ g_m v_{be} = \beta i_b \]
Small Signal BJT Model

\[ g_m = \frac{l_{CQ}}{V_t} \quad g_\pi = \frac{l_{CQ}}{\beta V_t} \quad g_o \approx \frac{l_{CQ}}{V_{AF}} \]

Alternate equivalent small signal model

\[ g_\pi = \frac{l_{CQ}}{\beta V_t} \quad g_o \approx \frac{l_{CQ}}{V_{AF}} \]
Properties of the BJT

• Alternate Equivalent Small Signal Model
  
  Magnitude of small signal parameters
Relative Magnitude of Small Signal Parameters

\[ g_m = \frac{I_{CQ}}{V_t} \quad g_{\pi} = \frac{I_{CQ}}{\beta V_t} \quad g_o \approx \frac{I_{CQ}}{V_{AF}} \]

\[
\begin{bmatrix}
\frac{I_Q}{V_t} \\
\frac{I_Q}{\beta V_t}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{I_Q}{\beta V_t} \\
\frac{I_Q}{V_{AF}}
\end{bmatrix}
\]

\[ g_m \gg g_{\pi} \gg g_o \]

Often the go term can be neglected in the small signal model because it is so small
Relative Magnitude of Small Signal Parameters

\[
\begin{align*}
g_m &= \frac{I_{CQ}}{V_t} \\
g_\pi &= \frac{I_{CQ}}{\beta V_t} \\
g_o &\approx \frac{I_{CQ}}{V_{AF}}
\end{align*}
\]

\[
\begin{align*}
g_m &= \left[ \begin{array}{c} I_Q \\ V_t \end{array} \right] = \beta \\
g_\pi &= \left[ \begin{array}{c} I_Q \\ \beta V_t \end{array} \right] \\
g_o &= \left[ \begin{array}{c} I_Q \\ V_{AF} \end{array} \right] = \frac{V_{AF}}{\beta V_t} \approx \frac{200V}{100 \cdot 26mV} = 77
\end{align*}
\]

\[
g_m >> g_\pi >> g_o
\]

Often the go term can be neglected in the small signal model because it is so small.
Simplified small signal model
Comparison of BJT and MOSFET
Comparison of MOSFET and BJT

BJT

\[ g_m = \frac{I_{CQ}}{V_t} \]

\[ g_{mBJT} = \frac{I_{CQ}}{V_t} \cdot \frac{V_t}{2I_{DQ}} = \frac{2I_{DQ}}{V_{EBQ}} \]

MOSFET

\[ g_m = \frac{\mu C_{OX} W}{L} V_{EB} \]

\[ g_m = \sqrt{\frac{2\mu C_{OX} W}{L}} \sqrt{I_{DQ}} \]

The transconductance of the BJT is typically much larger than that of the MOSFET (and larger is better!)
This is due to the exponential rather than quadratic output/input relationship.
The transconductance of the BJT is typically much larger than that of the MOSFET (and larger is better)
This is due to the exponential rather than quadratic output/input relationship
Comparison of MOSFET and BJT

BJT

\[ g_o \approx \frac{I_{CQ}}{V_{AF}} \]

MOSFET

\[ g_o = \lambda I_{DQ} \]

\[
\frac{g_{o_{BJT}}}{g_{o_{MOS}}} = \frac{I_{CQ}}{V_{AF}} \frac{1}{\lambda I_{DQ}} \approx \frac{1}{\lambda V_{AF}} \frac{1}{.01V^{-1} 200V} = 0.5
\]

The output conductances are comparable but that of the BJT is usually modestly smaller (and smaller is better!)
Comparison of MOSFET and BJT

BJT

\[ g_\pi = \frac{I_{cQ}}{\beta V_t} \]

MOSFET

\[ g_\pi = 0 \]

g_\pi is the reciprocal of the input impedance

g_\pi of a MOSFET is much smaller than that of a BJT (and smaller is better!)
Comparison of MOSFET and BJT

Assume BJT operating in FA region, MOSFET operating in Saturation
Assume same quiescent output voltage and same resistor $R_1$
One of the most widely used amplifier architectures
Comparison of MOSFET and BJT

BJT

MOSFET

\[ V_{IN(t)} \]

\[ V_{CC} \]

\[ R_1 \]

\[ V_{OUT} \]

\[ Q_1 \]

\[ V_{EE} \]

\[ V_{OUT} \]

\[ V_{IN} \]

\[ V_{DD} \]

\[ R_1 \]

\[ V_{OUT} \]

\[ M_1 \]

\[ V_{SS} \]
Comparison of MOSFET and BJT

**BJT**

**MOSFET**

\[ V_{\text{IN}} \quad V_{\text{OUT}} \]

\[ V_{\text{IN}} \quad V_{\text{OUT}} \]

assume \( g_o \) can be neglected

\[ V_{\text{IN}} \quad V_{\text{OUT}} \]

\[ V_{\text{IN}} \quad V_{\text{OUT}} \]

assume \( g_o \) can be neglected
Comparison of MOSFET and BJT

The functional form of the gain is the same for both circuits!
Comparison of MOSFET and BJT

For the same power level and the same quiescent voltage drop across $R_1$, the BJT will generally have a much larger gain since usually $V_t << V_{EB}$.
Comparison of MOSFET and BJT

BJT

Assume BJT operating in FA region, MOSFET operating in Saturation
Assume same bias current

MOSFET

One of the most widely used amplifier architectures in integrated applications
Special Case of Previous Architecture
Comparison of MOSFET and BJT

**BJT**

- $V_{CC}$
- $V_{EE}$
- $I_{CB}$
- $V_{OUT}$
- $V_{IN(t)}$

**MOSFET**

- $V_{DD}$
- $V_{SS}$
- $I_{DB}$
- $V_{OUT}$
- $V_{IN(t)}$
Comparison of MOSFET and BJT

- **BJT**

  - $v_{OUT}$
  - $v_{IN}$
  - $v_A = -\infty$
  - Assume $g_o$ can be neglected

- **MOSFET**

  - $v_{OUT}$
  - $v_{IN}$
  - $v_A = -\infty$
  - Assume $g_o$ can be neglected

- $A_v = \frac{v_{OUT}}{v_{IN}} = -\infty$

- **Notes**
  - $A_v$ is unrealistically large
  - Must include more accurate small-signal model!
Comparison of MOSFET and BJT

**BJT**

$$V_{IN} \rightarrow V_{be} \rightarrow g_m V_{be} \rightarrow V_{OUT}$$

- Include $g_o$ effects

**MOSFET**

$$V_{IN} \rightarrow V_{gs} \rightarrow g_m V_{gs} \rightarrow V_{OUT}$$

- Include $g_o$ effects

$$A_v = \frac{V_{OUT}}{V_{IN}} = -\frac{g_m}{g_o}$$

Functional form of gain is the same for both circuits.
Comparison of MOSFET and BJT

**BJT**

\[ A_y = \frac{v_{OUT}}{v_{IN}} = -\frac{g_m}{g_o} \]

\[ A_{VBJT} = -\frac{I_{CQ}/V_I}{I_{CQ}/V_{AF}} = -\frac{V_{AF}}{V_I} \]

**MOSFET**

\[ A_y = \frac{v_{OUT}}{v_{IN}} = -\frac{g_m}{g_o} \]

\[ A_{VMOS} = -\frac{2I_{DQ}/V_{EB}}{\lambda I_{DQ}} = -\frac{2}{\lambda V_{EB}} \]

- BJT Gain is Very Large and Independent of Operating Point
- MOS Gain is dependent upon operating conditions (\(V_{EB}\))
- \(V_{AF}\) and \(2/\lambda\) are comparable for large MOS devices, \(V_{AF}\) considerably larger than \(2/\lambda\) for short devices
- Practically, \(V_t<<V_{EB}\)
- BJT gain typically much larger than MOS gain for this configuration too
Student Question

Can a single transistor be used to realize the current source?

Yes – it provides reasonable performance but there are some limitations

Current sources often characterized by their nominal output current, their small signal output impedance, and their output signal swing

Nominal output current:

\[ I_{\text{OUT}} \approx \mu C_{\text{OX}} \frac{W}{2L} (V_{\text{EB}})^2 \]
Student Question

Can a single transistor be used to realize the current source?

Output impedance:

\[ R_{out} = \frac{V}{i} \]
Student Question

Can a single transistor be used to realize the current source?

Output signal swing:

To maintain saturation region operation

\[ V_{DS} > V_{GS} - V_T \]

\[ V_{OUT} > V_{XX} - V_T \]
Student Question
Are there better current source circuits?

Yes – and most focus on improving either the signal swing or the output impedance

High output impedance current source:

\[ I_{\text{OUT}} \approx \mu C_{\text{OX}} \frac{W_1}{2L_1} (V_{EB1})^2 \]
Student Question
Are there better current source circuits?

Output Impedance:

\[ R_{out} = \frac{v}{i} \]
Student Question
Are there better current source circuits?

Output Impedance:

\[
R_{out} = \frac{v}{i} = \frac{g_{m2} + g_{o1} + g_{o2}}{g_{o1} + g_{o2}} \approx \left[ \frac{1}{g_{o1}} \right] \frac{g_{m2}}{g_{o2}} \gg \left[ \frac{1}{g_{o1}} \right]
\]

\[
i = (v - v_1)g_{o2} + g_{m2}v_{gs2}
\]

\[
v_1(g_{o1} + g_{o2}) = g_{m2}v_{gs2} + g_{o2}v
\]

\[
v_1 = -v_{gs2}
\]