

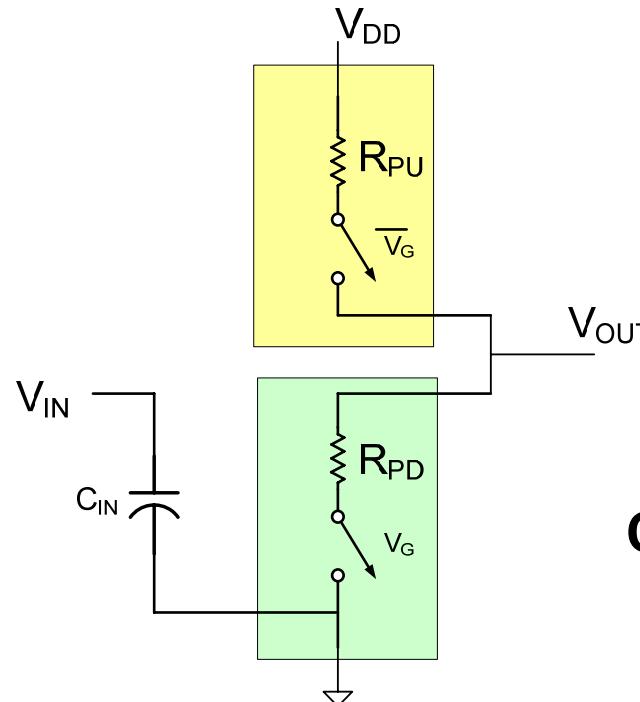
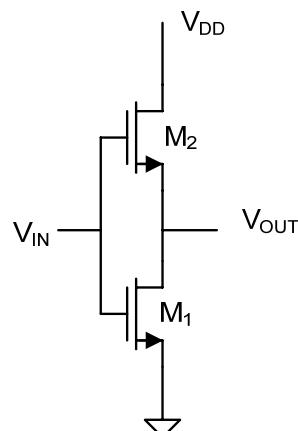
EE 434

## Lecture 35

Device Sizing

Propagation Delay in Logic Circuits

# Propagation Delay in Static CMOS Family



$$R_{PD} = \frac{L_1}{\mu_n C_{ox} W_1 (V_{DD} - V_{Tn})}$$

$$R_{PU} = \frac{L_2}{\mu_p C_{ox} W_2 (V_{DD} + V_{Tp})}$$

$$C_{IN} = C_{ox} (W_1 L_1 + W_2 L_2)$$

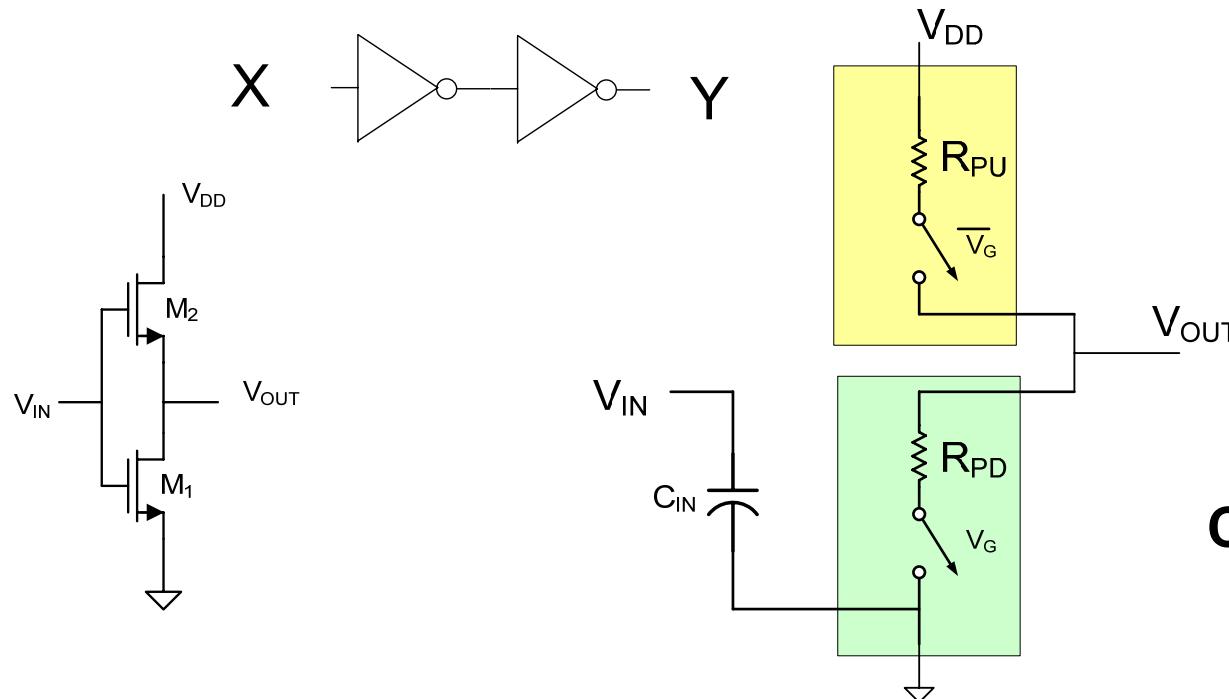
If  $\mu_n C_{ox} = 100 \mu A V^{-2}$ ,  $C_{ox} = 4 fF \mu^{-2}$ ,  $V_{Tn} = V_{DD}/5$ ,  $V_{Tp} = -V_{DD}/5$ ,  $\mu_n/\mu_p = 3$ ,  $L_1 = W_1 = L_{MIN}$ ,  $L_2 = W_2 = L_{MIN}$ ,  $L_{MIN} = 0.5 \mu$  and  $V_{DD} = 5V$

$$R_{PD} = \frac{1}{10^{-4} \cdot 0.8 V_{DD}} = 2.5 K\Omega$$

$$C_{IN} = 4 \cdot 10^{-15} \cdot 2 L_{MIN}^2 = 2 fF$$

$$R_{PU} = \frac{1}{10^{-4} \cdot \frac{1}{3} \cdot 0.8 V_{DD}} = 7.5 K\Omega$$

# Propagation Delay in Static CMOS Family



$$R_{PD} = \frac{L_1}{\mu_n C_{ox} W_1 (V_{DD} - V_{Tn})}$$

$$R_{PU} = \frac{L_2}{\mu_p C_{ox} W_2 (V_{DD} + V_{Tp})}$$

$$C_{IN} = C_{ox} (W_1 L_1 + W_2 L_2)$$

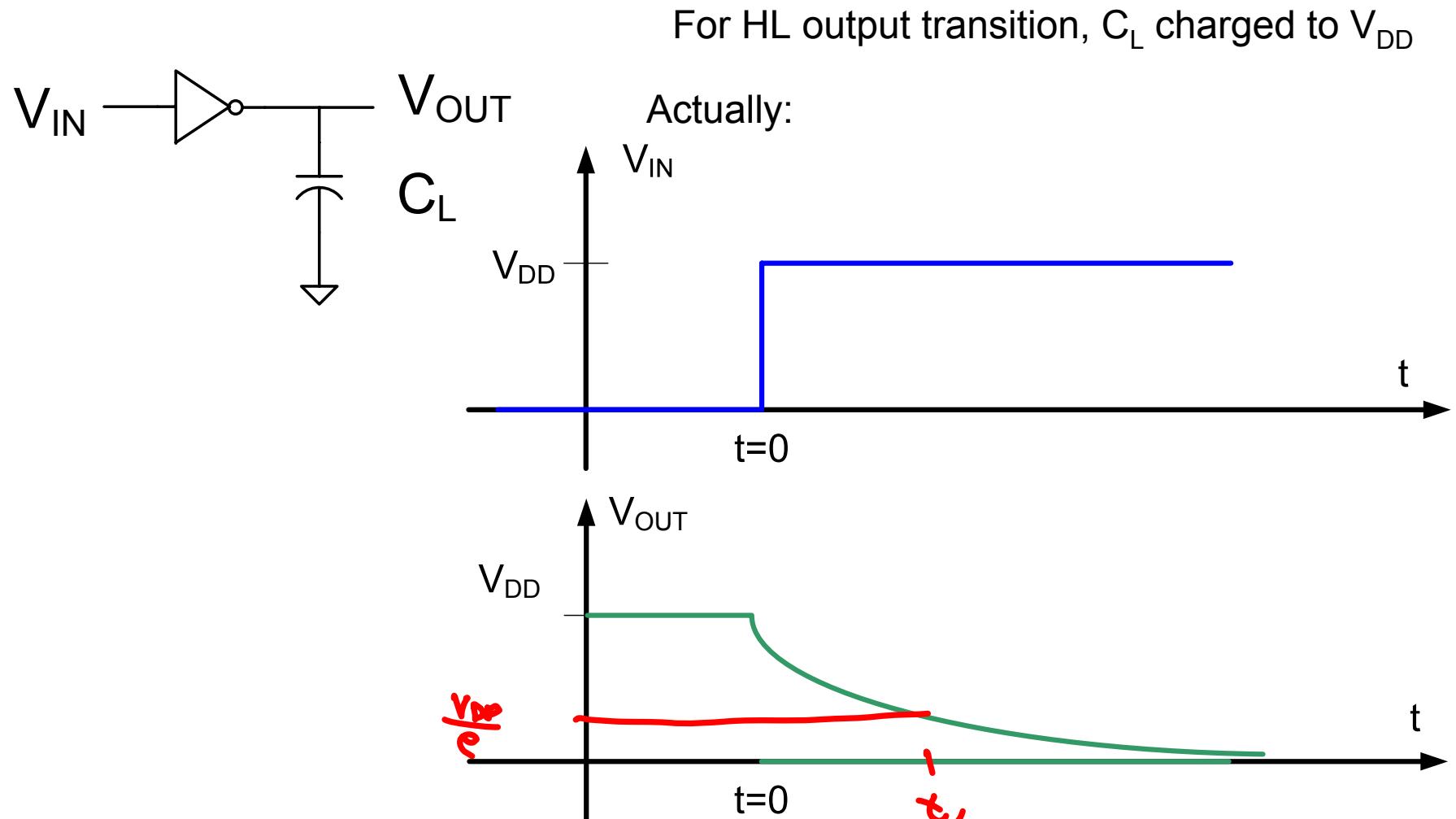
If  $\mu_n C_{ox} = 100 \mu A V^{-2}$ ,  $C_{ox} = 4 fF \mu^{-2}$ ,  $V_{Tn} = V_{DD}/5$ ,  $V_{Tp} = -V_{DD}/5$ ,  $\mu_n/\mu_p = 3$ ,  $L_1 = W_1 = L_{MIN}$ ,  $L_2 = W_2 = L_{MIN}$ ,  $L_{MIN} = 0.5 \mu$  and  $V_{DD} = 5V$

$$R_{PD} = \frac{1}{10^{-4} \cdot 0.8 V_{DD}} = 2.5 K\Omega$$

$$C_{IN} = 4 \cdot 10^{-15} \cdot 2 L_{MIN}^2 = 2 fF$$

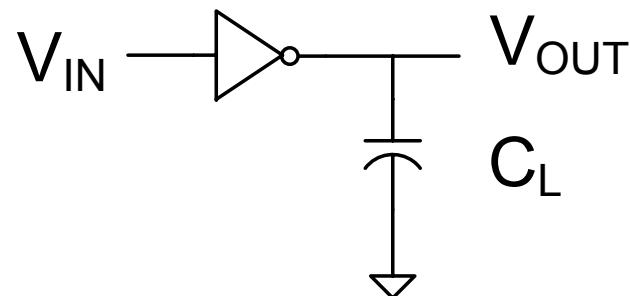
$$R_{PU} = \frac{1}{10^{-4} \cdot \frac{1}{3} \cdot 0.8 V_{DD}} = 7.5 K\Omega$$

# Propagation Delay in Static CMOS Family



What is the transition time  $t_{HL}$  ?

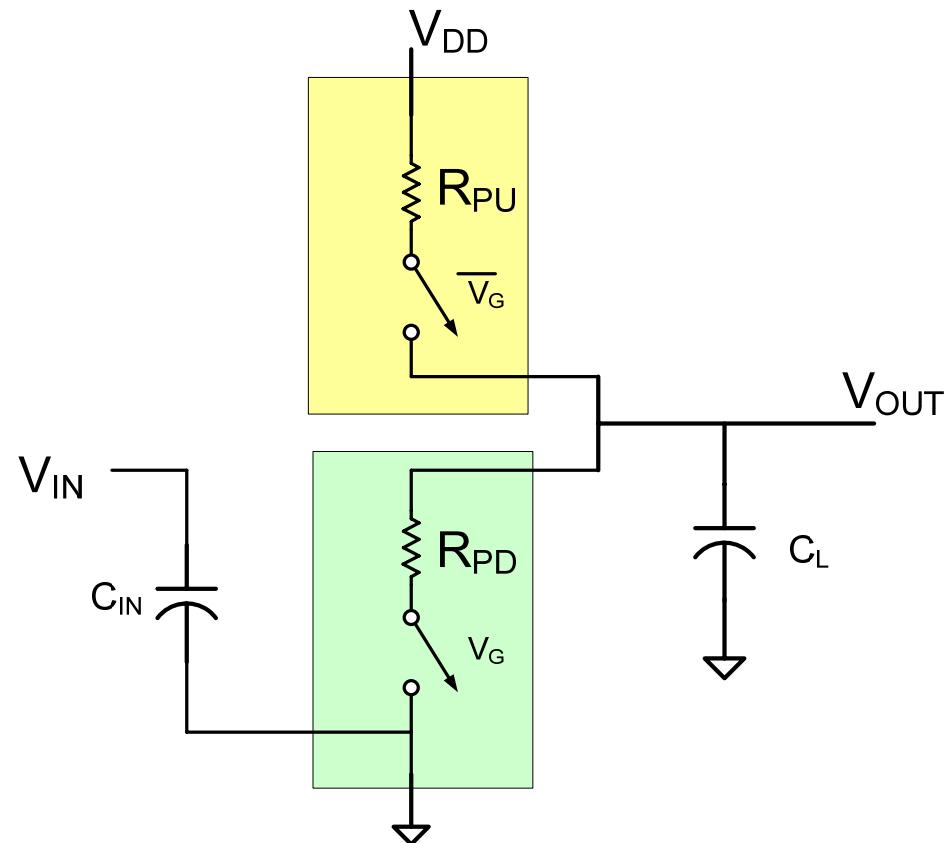
# Propagation Delay in Static CMOS Family



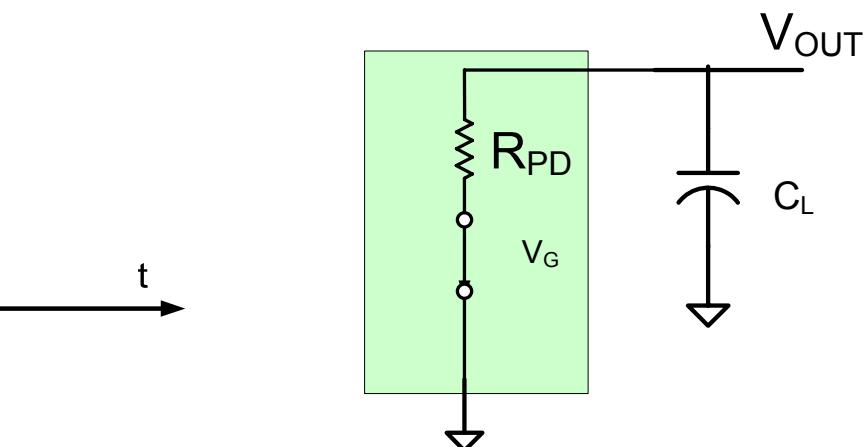
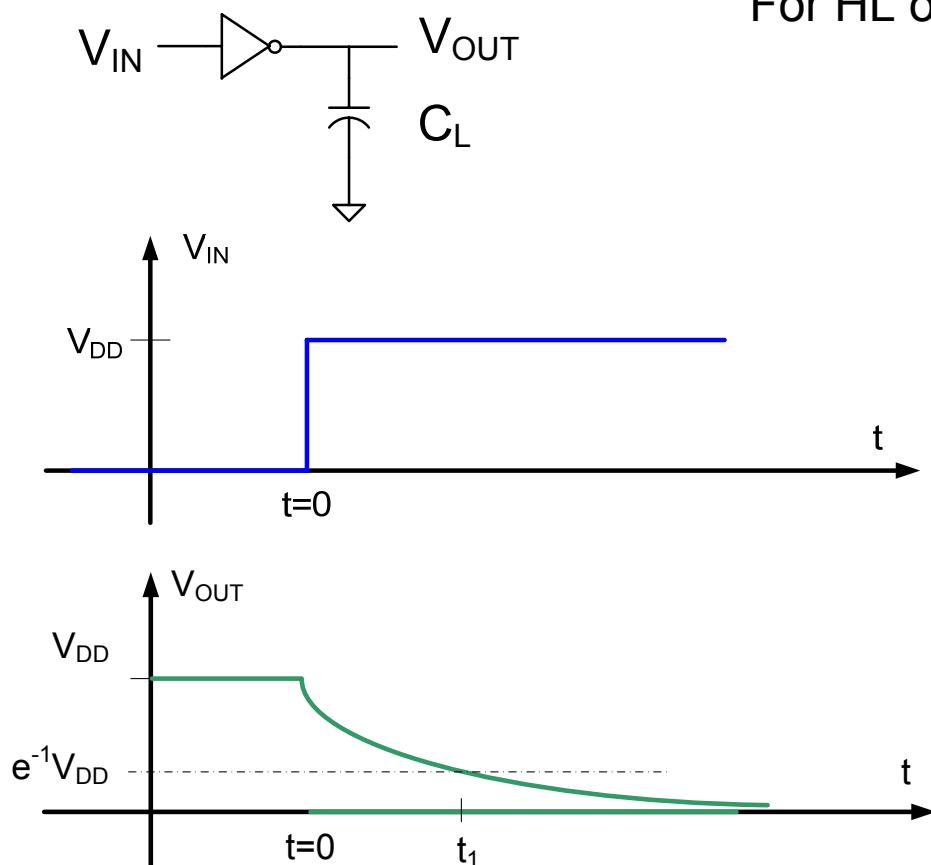
$$t_{HL} = R_{PD} C_L$$

$$t_{LH} = R_{PU} C_L$$

↓



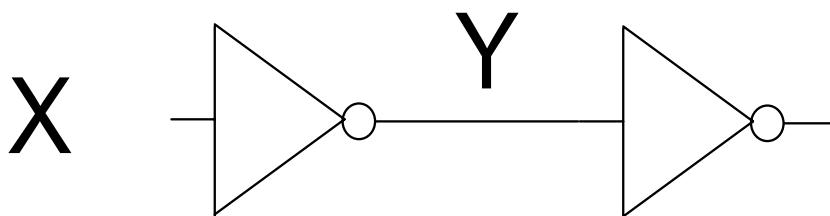
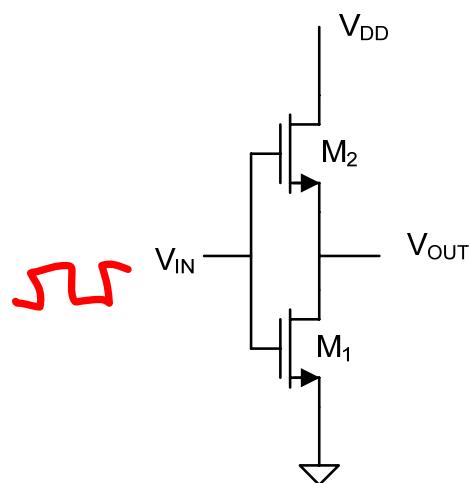
# Propagation Delay in Static CMOS Family



$$t_{HL} \approx R_{PD} C_L$$

$$t_{LH} \approx R_{PU} C_L$$

# Propagation Delay in Static CMOS Family

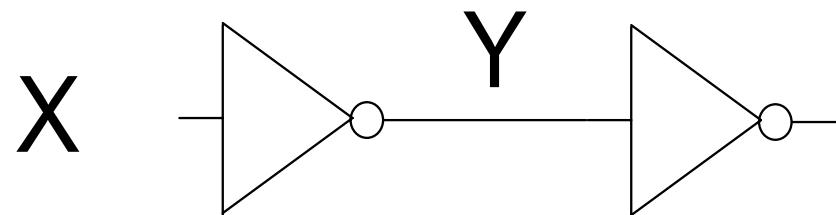
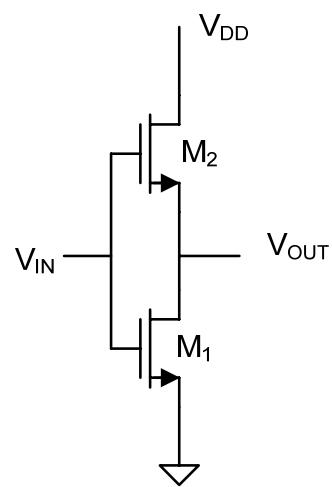


$$t_{H_L} \approx (7.5k)(2f_F) = 5 \mu\text{sec}$$

$$t_{L_H} \approx (7.5k)(2f_F) \approx 15 \mu\text{sec}$$

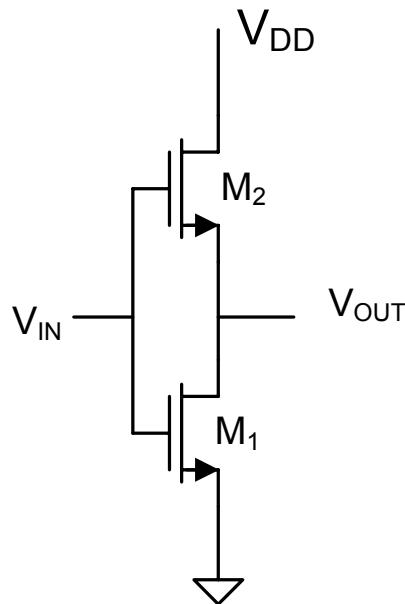
$$T_{total} \approx t_{H_L} + t_{L_H} \approx 20 \mu\text{sec} \sim 50 \text{ GHz}$$

# Propagation Delay in Static CMOS Family



# Device Sizing

- Since not ratio logic,  $V_H$  and  $V_L$  are independent of device sizes for this inverter
- With  $L_1=L_2=L_{\min}$ , there are 2 degrees of freedom ( $W_1$  and  $W_2$ )

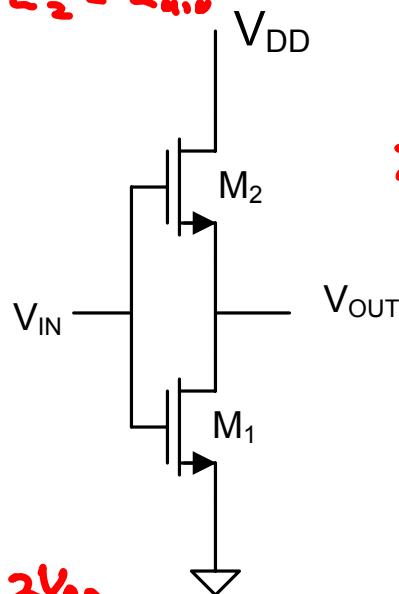


## Sizing Strategies

- Minimum Size
- Fixed  $V_{TRIP}$
- Equal rise-fall times  
(equal worst-case rise and fall times)
- Minimum power dissipation
- Minimum time required to drive a given load
- Minimum input capacitance

# Device Sizing

$$L_1 = L_2 = L_{min}$$



- Minimum Size

2 D.O.F.

$$W_1 = W_2 = W_{MIN}$$

also provides minimum input capacitance

✓  $t_{LH}$  is longer than  $t_{HL}$

by  $\frac{M_n}{M_p} \leq 3$

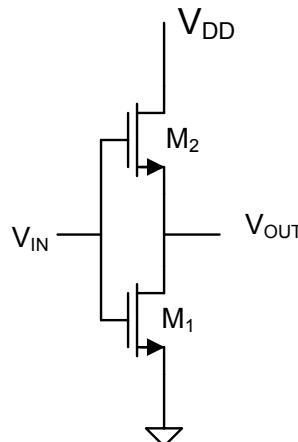
$$V_{TH} = .2V_{DD}$$

$$V_{PP} = -.2V_{DD}$$

$$V_{IN^{TRIP}} = \frac{(0.2V_{DD}) + (V_{DD} - 0.2V_{DD})\sqrt{\frac{1}{3}}}{1 + \sqrt{\frac{1}{3}}} = .42V_{DD}$$

~ ~ ~ ~ ~

$$L_1 = L_2 = L_{min}$$



~~$$\omega_2 = \omega_{min}$$~~
~~$$\omega_1 \neq \frac{\omega_{min}}{3}$$~~

$$\omega_1 = \omega_{min}$$

$$\omega_2 = 3\omega_{min}$$

# Device Sizing

$$V_{TRIP} = -V_{TN} = -\frac{V_{DD}}{3}$$

$$L_1 = L_2 = L_{min}$$

Equal Rise-Fall Times

~~$$\frac{t_{LH}}{t_{HL}} = \frac{R_{PU} C_{OX}}{R_{PD} C_{OX}} = \frac{\mu_n}{\mu_p}$$~~

~~$$\frac{L_1}{L_2} \propto \frac{\omega_2}{\omega_1} (V_{DD} + V_{TRIP})$$~~
~~$$\frac{L_1}{L_2} \propto \frac{\omega_1}{\omega_2} (V_{DD} - V_{TN})$$~~

Thus if  $t_{HL} = t_{LH}$ , must have

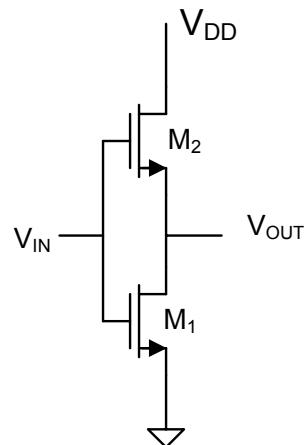
$$\frac{W_2}{W_1} = \frac{\mu_n}{\mu_p} = 3$$

What about the second degree of freedom?

$$V_{TRIP} = ?$$

$$\text{Want } V_{TRIP} \approx \frac{V_{DD}}{2}$$

# Device Sizing



Equal Rise-Fall Times

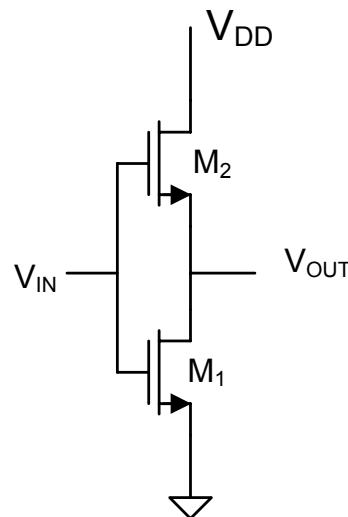
$$\frac{t_{LH}}{t_{HL}} = \frac{R_{PU} C_{OX}}{R_{PD} C_{OX}} = \frac{\mu_p}{\mu_n}$$

Thus if  $t_{HL} = t_{LH}$ , must have

$$\frac{W_2}{W_1} = \frac{\mu_n}{\mu_p}$$

$$V_{IN} = \frac{(V_{Tn}) + (V_{DD} + V_{Tp}) \sqrt{\frac{\mu_p}{\mu_n} \frac{W_2}{W_1} \frac{L_1}{L_2}}}{1 + \sqrt{\frac{\mu_p}{\mu_n} \frac{W_2}{W_1} \frac{L_1}{L_2}}} = \frac{0.2V_{DD} + 0.8V_{DD}}{2} = \frac{V_{DD}}{2}$$

# Device Sizing



Fixed V<sub>TRIP</sub>

Set V<sub>TRIP</sub>=V<sub>DD</sub>/2

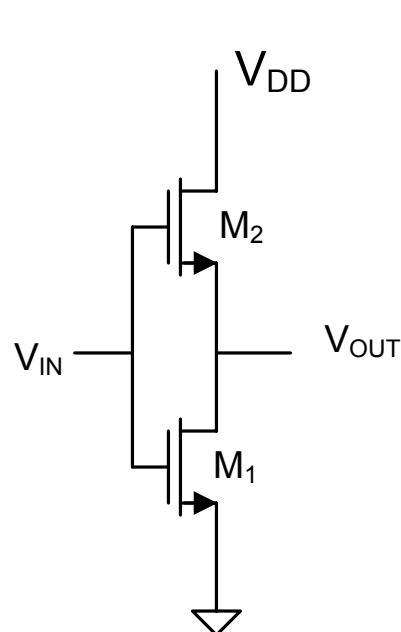
$$W_1 = W_{min}$$
$$W_2 = 3W_{min}$$

$$\frac{(.2V_{DD}) + (V_{DD} - .2V_{DD}) \sqrt{\frac{\mu_p}{\mu_n} \frac{W_2}{W_1} \frac{L_1}{L_2}}}{1 + \sqrt{\frac{\mu_p}{\mu_n} \frac{W_2}{W_1} \frac{L_1}{L_2}}} = \frac{V_{DD}}{2}$$

Solving, obtain  $W_2/W_1 = \mu_n/\mu_p$

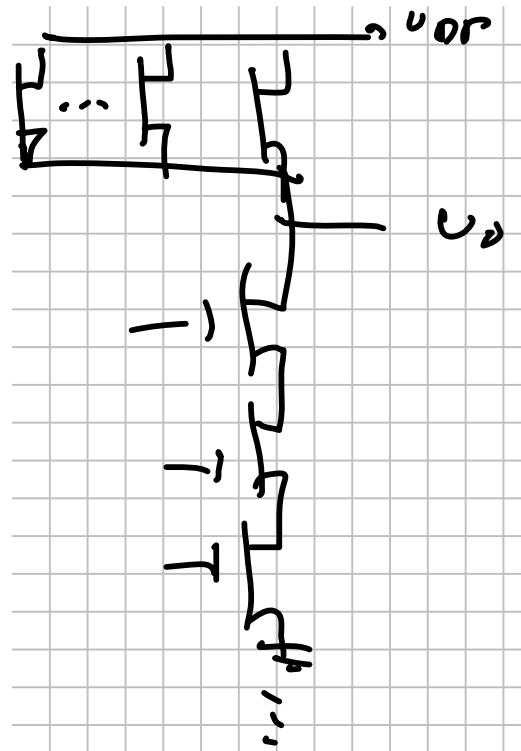
$$V_{DD} + V_F = .8 V_{DD}$$

# Device Sizing



## Sizing Strategies

- Minimum Size
- Fixed  $V_{TRIP}$   $\approx \frac{V_{DD}}{2}$
- Equal rise-fall times  
(equal worst-case rise and fall times)
- Minimum power dissipation
- Minimum time required to drive a given load
- Minimum input capacitance



for min size of

$$t_{HL} = K R_{Ref} C_L = K t_{HL} R_{Ref}, \quad t_{LH} = 3 \Rightarrow R_{Ref}$$

$$FI = 2(\alpha_{Wmin} L_{min}) = \frac{C_{Ref}}{\Sigma}$$

for equal work-time rise & fall

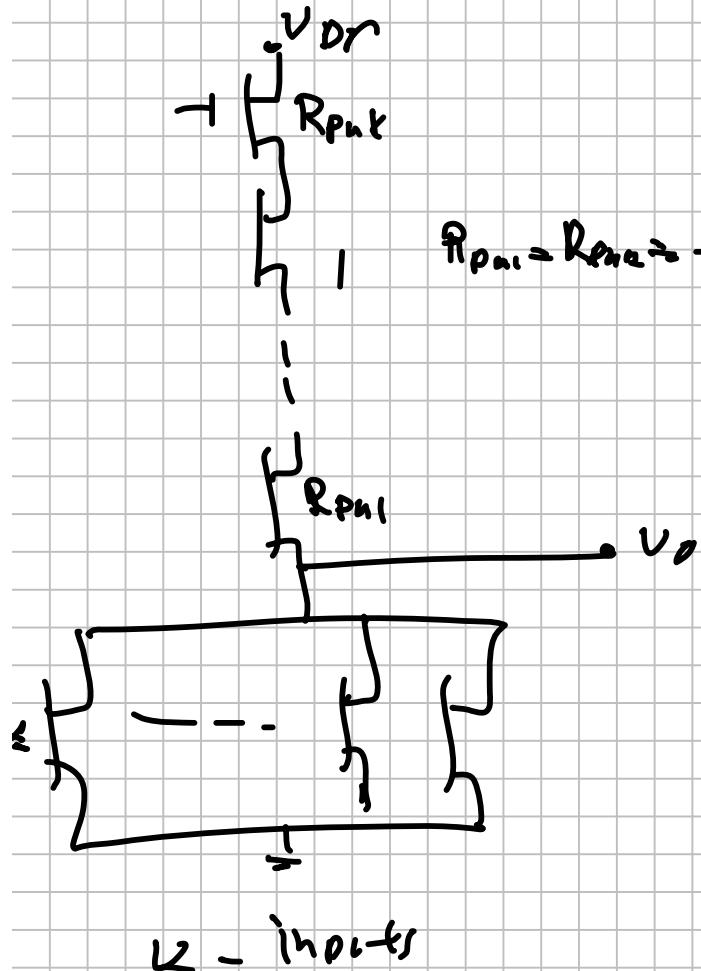
$$\omega_n = (\omega_{min})^k$$

$$\omega_p = 3\omega_{min}$$

~~$$FI = \alpha_{Wmin} L_{min} + 3\alpha_{Wmin} L_{min}$$~~

$$= (3+k)(\alpha_{Wmin} L_{min})$$

$$= \frac{3+k}{4} C_{Ref}$$



T6 Inverter with  $t_{THL} = t_{LH}$

& all delays as small as possible  
 $(W_1 = W_{in}, W_2 = 3W_{in}, L_1 = L_2 = L_{in})$

called Ref. Inverter

- Equal worst case rise & fall times
- Set equal to that of an inverter with equal rise & fall times

$$R_{PA1} = R_{PA2} = \dots = R_{PAk} = R_{PA}$$

$$t_{THL} =$$

$$\left\{ \begin{array}{l} R_{PA} C_L \\ \vdots \\ R_{PA} C_L \end{array} \right.$$

$$C_{in RER} =$$

4 (of minimum)

$$FanIn = \frac{3k+1}{4} C_{RER}$$

"worst" is toward

$$R_{PA} = R_{PA \text{REF}}$$

$$t_{LH} = k R_{PA} \tau_E = R_{PA} \tau_E$$

$$R_{PA} = \frac{R_{PA}}{k}$$

$$R_{PA} = \frac{\mu_n}{\mu_p} R_{PA}$$

$$\frac{\mu_n}{\mu_p}$$

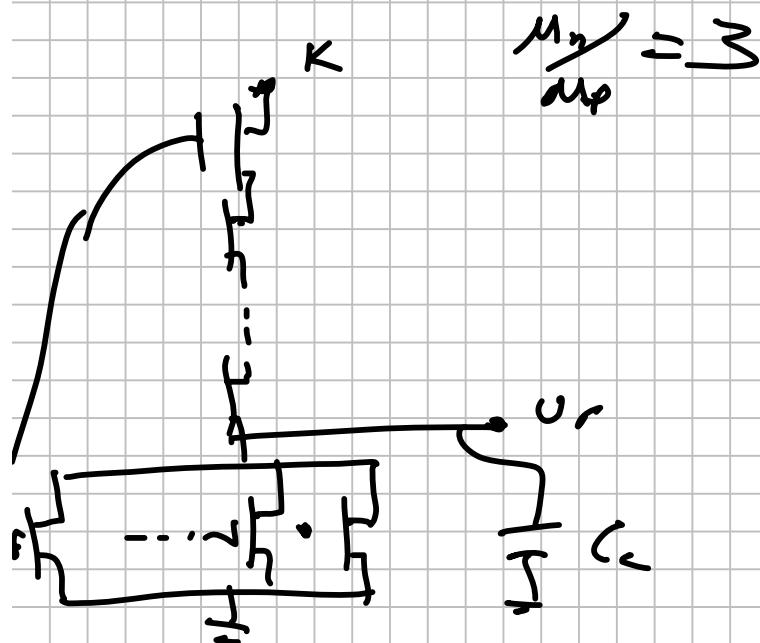
$$W_P = \frac{\mu_n}{\mu_p} k$$

$$\approx \frac{3k}{3k}$$

$$FanIn((C_{in W_{in} L_1} + C_{in W_{in} (3k) L_{in}}))$$

$$= (3k+1) C_{in W_{in} L_{in}}$$

## Sizing multiple-input gates



$$L_1 = L_2 = L_{min}$$

$$w_1 = w_2 = 0.5 \mu m$$

will trans only if all inputs are "0"  
prior to clock, 1, 2, ..., K go high  
at clock

minimum

- equal rise & fall  
(worst case)

$$t_{HL} = \left\{ \begin{array}{l} R_{PD} C_L \\ \frac{R_{PD}}{2} C_L \\ \vdots \\ \frac{R_{PD}}{K} C_L \end{array} \right.$$

for 1 input chg

for 2 inputs

for all K  
changes

worst case  $R_{PD} C_L$

$$t_{LH} = K R_{PD} C_L = 3 K R_{PD} C_L$$

Fan in on each input =  $C_0 w_1 L_1 + C_0 w_2 L_2 \approx 2 C_0 w_{min} L_{min}$