EE 434
Lecture 37
Propagation Delay in Logic Circuits
Power Dissipation
Review from last time

Propagation Delay in Multiple-Levels of Logic with Stage Loading

Analysis strategy: Express delays in terms of those of reference inverter

\[ t_{\text{REF}} = t_{\text{HLREF}} + t_{\text{LHREF}} = \frac{20}{R_{\text{PDREF}} + 2R_{\text{REF}}} \]

Reference Inverter

\[ V_{\text{DD}} \]

\[ V_{\text{IN}} \]

\[ V_{\text{OUT}} \]

\[ M_1 \]

\[ M_2 \]

\[ R_{\text{PDREF}} = \frac{L_{\text{MIN}}}{\mu_n C_{\text{OX}} W_{\text{MIN}} (V_{\text{DD}} - V_{\text{Tn}})} \]

\[ V_{\text{Tn}} = 0.8V_{\text{DD}} \]

\[ C_{\text{REF}} = C_{\text{IN}} = 4C_{\text{OX}} W_{\text{MIN}} L_{\text{MIN}} \]

Assume \( \mu_n / \mu_p = 3 \quad L_n = L_p = L_{\text{MIN}} \)

\( W_n = W_{\text{MIN}}, \quad W_p = 3 W_{\text{MIN}} \)
Review from last time

Propagation Delay in Multiple-Levels of Logic with Stage Loading

**Capacitive Loading**

\[ V_{IN} \rightarrow \quad V_{OUT} \]

\[ C_L \]

Define the Fan In loading on the stage to be the total capacitive load on the stage normalized to \( C_{REF} \)

\[ F_{IL} = \frac{C_L}{C_{REF}} \]

If inverter sized for equal rise/fall

\[ t_{HL} = t_{LH} = R_{PD} C_L = R_{PD} C_{REF} F_{IL} \]

\[ t_{PROP} = t_{LH} + t_{HL} = 2 R_{PD} C_{REF} F_{IL} \]

If inverter is the reference inverter

\[ t_{PROP} = t_{REF} F_{IL} \]
Propagating Delay in Multiple-Levels of Logic with Stage Loading

Overdrive

Define the Overdrive Factor of the stage to be the factor by which PU and PD resistors are scaled relative to those of the reference inverter.

\[ R_{PDEFF} = \frac{R_{PDREF}}{OD_{HL}} \]

\[ R_{PUEFF} = \frac{R_{PUREF}}{OD_{LH}} \]

If inverter sized for equal rise/fall, \( OD_{HL} = OD_{LH} = OD \)

\[ t_{HL} = t_{LH} = \frac{R_{PDREF}}{OD} \]

\[ C_L = R_{PDREF} C_{REF} \frac{F_{IL}}{OD} \]

\[ t_{PROP} = t_{LH} + t_{HL} = t_{REF} F_{IL} \frac{O}{OD} \]

OD may be larger or smaller than 1

Review from last time
Propagation Delay in Multiple-Levels of Logic with Stage Loading

Overdrive

\[ V_{IN} \quad \text{\rightarrow} \quad V_{OUT} \]

\[ C_L \]

If inverter is not equal rise/fall

\[ t_{PROP} = t_{HL} + t_{LH} = t_{REF} \frac{F_{IL}}{OD} \]

\[ t_{HL} = R_{PDREF} \frac{C_L}{OD_{HL}} \cdot t_{REF} \frac{F_{IL}}{OD_{HL}} \]

\[ t_{LH} = R_{PUREF} \frac{C_L}{OD_{LH}} \cdot \frac{1}{2} t_{REF} \frac{F_{IL}}{OD_{LH}} \]

\[ t_{PROP} = t_{HL} + t_{LH} = \frac{1}{2} t_{REF} F_{IL} \left( \frac{1}{OD_{HL}} + \frac{1}{OD_{LH}} \right) \]
Propagation Delay in Multiple-Levels of Logic with Stage Loading

Overdrive Notation

Equal Rise/Fall with overdrive OD

Examples

Equal Rise/Fall with overdrive of 8

If $W_n - W_{MIN}$, minimum sized inverter
Propagation Delay in Multiple-Levels of Logic with Stage Loading

Example:

\[ t_{\text{PROP}} = t_{\text{HL}} + t_{\text{IL}} = t_{\text{REF}} \frac{F_{\text{IL}}}{OD} \]

If inverter is not equal rise/fall

\[ t_{\text{HL}} = \frac{R_{\text{PDREF}}}{OD_{\text{HL}}} C_L = \frac{1}{2} t_{\text{REF}} \frac{F_{\text{IL}}}{OD_{\text{HL}}} \]

\[ t_{\text{IL}} = \frac{R_{\text{PUREF}}}{OD_{\text{HL}}} C_L = \frac{1}{2} t_{\text{REF}} \frac{F_{\text{IL}}}{OD_{\text{IL}}} \]

\[ t_{\text{PROP}} = t_{\text{HL}} + t_{\text{IL}} = \frac{1}{2} t_{\text{REF}} F_{\text{IL}} \left( \frac{1}{OD_{\text{HL}}} + \frac{1}{OD_{\text{IL}}} \right) \]
Propagation Delay in Multiple-Levels of Logic with Stage Loading

\[ t_{\text{prop}} = \sum_{i=1}^{m} t_{\text{prop}_i} = t_{\text{REF}} \sum_{i=1}^{m} \frac{F1_{i+1}}{OD_i} \]
Propagation Delay in Multiple-Levels of Logic with Stage Loading

\[
C_{IN} = \frac{3+k}{4} C_{RB} \\
F_{IL} = \frac{3+k}{4}
\]

\[
C_{IN} = \frac{3k+1}{4} C_{RB} \\
F_{IL} = \frac{3k+1}{4}
\]

NAND gates cause considerably less loading than NOR gates for large number of inputs with equal rise & fall times

\[
F_{IL} = m(\frac{3+k}{4})
\]

\[
F_{IL} = m(\frac{3k+1}{4})
\]
Propagation delay with non-ideal wire & a fall time

Recall \( t_{\text{prop}} = t_{\text{REF}} \sum_{i=1}^{m} \frac{F_{IL,i}}{OD_i} = t_{\text{REF}} \sum_{i=1}^{m} \frac{F_{IL,i}}{OD} + t_{\text{REF}} \sum_{i=1}^{m} \frac{F_{XL,i}}{OD} \)

\( t_k = t_{\text{REF}} \frac{F_{IL,k}}{2} \frac{1}{OD_{HL}} + t_{\text{REF}} \frac{F_{IL,k+1}}{2} \frac{1}{OD_{LM}} \)

\( t_{\text{prop}} = \sum_{k=1}^{n} t_k = t_{\text{REF}} \sum_{k=1}^{n} F_{IL,k} \left( \frac{1}{OD_{HL,k}} + \frac{1}{OD_{LM,k}} \right) \)
Propagation Delay in Multiple-Levels of Logic with Stage Loading

\[ t_{\text{total}} = t_1 + t_2 + t_3 \]

\[ t_1 = \frac{t_{\text{rev}}}{2} \left( \frac{C_{\text{in,nor}}}{C_{\text{reset}}} + \frac{1}{2} + \left( \frac{k}{4} \right)(4) \right) \]

\[ t_2 = \frac{t_{\text{rev}}}{2} \left[ \frac{1}{2} + \frac{1}{2} \right] \left[ \frac{1}{1} + \frac{1}{\frac{1}{3}} \right] \]

\[ t_3 = \frac{t_{\text{rev}}}{2} \left[ \frac{C_{\text{in,ana}}}{C_{\text{reset}}} \right] \left[ \frac{1}{1} + \frac{1}{\frac{1}{3}} \right] \]

Recall:

\[ C_{\text{in,nor}} = \frac{3(k+1)}{4} C_{\text{reset}} \]

\[ C_{\text{in,ana}} = \frac{k+3}{4} C_{\text{reset}} \]

\[ C_{\text{in, min}} = \frac{C_{\text{reset}}}{2} \]
Propagation Delay in Multiple-Levels of Logic with Stage Loading
How can large loads be rapidly driven.

\[ t_{\text{ref}} = 10000 \]

\[ t_{\text{prop}} = (t_{\text{ref}}) \times 10000 \]

\[ t_{\text{prop}} = t_{\text{ref}} \]

\[ t_{\text{prop}} = (1001) t_{\text{ref}} \]

- Multiple loads of logic do not necessarily increase overall delay.
\[ C_L = \Theta^n \text{ CREF} \]

\[ t_{\text{CASCADE}} = t_{\text{CREF}} \left( \frac{\Theta}{1} + \frac{\Theta^2}{\Theta} + \frac{\Theta^3}{\Theta^2} + \ldots \right) \]

\[ = t_{\text{CREF}} (\Theta + \Theta + \Theta + \ldots + \Theta) \]

\[ = t_{\text{CREF}} \ n \Theta \]

\[ r = \frac{t_{\text{CASCADE}}}{t_{\text{DIRECT}}} = \frac{n \Theta \ t_{\text{CREF}}}{\Theta n \ t_{\text{CREF}}} = n \Theta \]

\[ r = \ln \left( \frac{C_L}{C_L \text{ REF}} \right) \Theta \]

\[ = \left[ \ln \left( \frac{C_L}{C_L \text{ REF}} \right) \right] \left( \frac{\Theta}{\ln \Theta} \right) \]

\[ t_{\text{PROP}} = \Theta \ t_{\text{CREF}} \]

\[ n \ln \Theta = n \left( \frac{C_L}{C_L \text{ REF}} \right) \]
\[ \hat{r} = \frac{\Theta}{\ln \Theta} \]

\[ \frac{\partial \hat{r}}{\partial \Theta} = \frac{\ln \Theta - \Theta \left( \frac{1}{\Theta} \right)}{(\ln \Theta)^2} = 0 \Rightarrow \ln \Theta = 1 \]

\[ \Theta = e \]

Practically: as \( \Theta \) goes from 2 to 3