EE 435

Lecture 12

Cascaded Amplifier Structures
Review from last time

**Increasing Gain by Cascading**

Provided the stages are non-interacting

\[
\frac{X_{OUT}}{X_{IN}} = A_1 A_2
\]

\[
\frac{X_{OUT}}{X_{IN}} = A_1 A_2 A_3
\]

\[
\frac{X_{OUT}}{X_{IN}} = \prod_{i=1}^{n} A_i
\]

Gain can be easily increased to almost any desired level!
A cascade of amplifiers can result in a very high dc gain!

Characteristics of feedback amplifier (where the op amp is applied) are of ultimate concern

Some critical and fundamental issues came up with even the most basic cascades when they are used in a feedback configuration

Must understand how open-loop and closed-loop amplifier performance relate before proceeding to design amplifiers by cascading
Review from last time

An amplifier is stable iff all poles lie in the open LHP

Routh-Hurwitz Criteria is often a practical way to determine if an amplifier is stable

Although stability of an amplifier is critical, a good amplifier must not only be stable but generally must satisfy magnitude peaking and/or settling requirements thus poles need to be moved a reasonable distance from the imaginary axis

The cascade of three identical high-gain amplifiers will result in a pole-pair far in the right half plane when feedback is applied so FB amplifier will be unstable

\[
A_{FB} = \frac{A}{1 + A\beta} = \frac{A_0^3}{\left(\frac{s}{\beta} + 1\right)^3 + \beta A_0^3}
\]

For stability

\[8 > \beta A_0^3\]
Example:

Assume an amplifier has a transfer function that has a denominator polynomial that can be expressed as

\[ D(s) = s^3 + 2ks^2 + 4s + 16 \]

Determine the minimum value of \( k \) that will result in a stable amplifier.
Solution:

Assume an amplifier has a transfer function that has a denominator polynomial that can be expressed as

\[ D(s) = s^3 + 2ks^2 + 4s + 16 \]

Determine the minimum value of \( k \) that will result in a stable amplifier

Solution: Recall from the RH criteria that all roots of a third-order polynomial of the form \( s^3 + a_2s^2 + a_1s + a_0 \) will lie in the LHP provided all coefficients are positive and \( a_1a_2 > a_0 \)

Thus, for the current problem, must have

\[ (2k)^4 > 16 \]

or

\[ k > 2 \]
Consider Again the Frequency Response of the basic Feedback Amplifier

Example: If $n=3$ and stages are not identical

$$A_{FB} = \frac{A}{1 + A\beta} = \frac{A_{01}A_{02}A_{03}}{\left(\frac{s}{\tilde{p}_1} + 1\right)\left(\frac{s}{\tilde{p}_2} + 1\right)\left(\frac{s}{\tilde{p}_3} + 1\right) + \beta A_{02}A_{03}A_{03}}$$

$$D_{FB}(s) = s^3 + s^2(\tilde{p}_1 + \tilde{p}_2 + \tilde{p}_3) + s(\tilde{p}_1\tilde{p}_2 + \tilde{p}_1\tilde{p}_3 + \tilde{p}_2\tilde{p}_3) + \tilde{p}_1\tilde{p}_2\tilde{p}_3 \beta A_{0TOT}$$

where $A_{0TOT} = A_{01}A_{02}A_{03}$
Consider Again the Frequency Response of Feedback Amplifier

![Feedback Amplifier Diagram]

Example: If \( n=3 \) and stages are not identical (cont)

\[
D_{FB}(s) = s^3 + s^2(\tilde{p}_1 + \tilde{p}_2 + \tilde{p}_3) + s(\tilde{p}_1 \tilde{p}_2 + \tilde{p}_1 \tilde{p}_3 + \tilde{p}_2 \tilde{p}_3) + \tilde{p}_1 \tilde{p}_2 \tilde{p}_3 \beta A_{\text{TOT}}
\]

Routh-Hurwitz Stability Criteria: (by assuming \( 1 + \beta A_{\text{TOT}} \approx \beta A_{\text{TOT}} \))

\[
(\tilde{p}_1 + \tilde{p}_2 + \tilde{p}_3)(\tilde{p}_1 \tilde{p}_2 + \tilde{p}_1 \tilde{p}_3 + \tilde{p}_2 \tilde{p}_3) > \tilde{p}_1 \tilde{p}_2 \tilde{p}_3 \beta A_{\text{TOT}}
\]

WOLG, assume \( \tilde{p}_1 < \tilde{p}_2 < \tilde{p}_3 \) and define \( \tilde{p}_2 = k_2 \tilde{p}_1 \) and \( \tilde{p}_3 = k_3 \tilde{p}_1 \)

Thus the RH criteria can be expressed as

\[
(1 + k_2 + k_3)(k_2 + k_3 + k_2 k_3) > \beta A_{\text{TOT}}
\]
Consider Again the Frequency Response of Feedback Amplifier (cont)

Example: If $n=3$ and stages are not identical

RH criteria:

$$\left(1 + k_2 + k_3 \right) \left(k_2 + k_3 + k_2 k_3 \right) \beta A_{0TOT}$$

Since $A_{0TOT}$ will, in general, be very large for the cascade of 3 stages, a very large pole ratio is required just to maintain stability and an even larger ratio needed to avoid a close to becoming unstable situation

Practically it is difficult to obtain such a large spread in the bandwidth of the amplifiers

Problem can be viewed as one of accumulating too much phase shift before gain drops to an acceptable value

Little commercial use of the cascade of three amplifiers (each with gain) in the design of op amps though some academic groups have worked on this approach with minimal practical success
Similar implications on amplifier even if not a basic voltage feedback amplifier

\[ A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + \frac{1}{A_{V}} \left(1 + \frac{R_2}{R_1}\right)} \]

\[ \beta = \frac{R_1}{R_2 + R_1} \]

\[ A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{A_{V}}{1 + \beta A_{V}} \]
Similar implications on amplifier even if not a basic voltage feedback amplifier

\[ A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{1}{A_V} \left(1 + \frac{R_2}{R_1}\right)} \]

\[ A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{A_V \left(\frac{R_1}{R_2 + R_1}\right)}{1 + A_V \left(\frac{R_2}{R_2 + R_1}\right)} \]

These circuits have

- same β
- same dead network
- same characteristic polynomial
- same poles
- different zeros

\[ \beta = \frac{R_1}{R_2 + R_1} \]

\[ D(s) = 1 + A\beta \quad \text{(expressed as polynomial)} \]
Example: Determine the dc open-loop gain, dc closed-loop gain, the open-loop poles, the open-loop zeros, the closed-loop poles, the closed-loop zeros, and the characteristic polynomial if

\[ A(s) = 10^7 \frac{s+1}{(s+10)(s+1000)} \]
Example: Determine the dc open-loop gain, dc closed-loop gain, the open-loop poles, the open-loop zeros, the closed-loop poles, the closed-loop zeros, and the characteristic polynomial if \( A(s) = 10^7 \frac{s+1}{(s+10)(s+1000)} \)

\[ A_{OL} = \]

Open-loop zeros =

Open-loop poles =
Example: Determine the dc open-loop gain, dc closed-loop gain, the open-loop poles, the open-loop zeros, the closed-loop poles, the closed-loop zeros, and the characteristic polynomial if

\[ A(s) = 10^7 \frac{s+1}{(s+10)(s+1000)} \]

\[
A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{-\frac{R_2}{R_1}}{1 + \frac{1}{A_V}\left(1 + \frac{R_2}{R_1}\right)}
\]

\[
\beta = \frac{R_1}{R_2 + R_1}
\]

\[
A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{-\frac{R_2}{R_1}}{1 + \frac{(s+10)(s+1000)}{10^7\beta(s+1)}}
\]

\[
A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{-\frac{R_2}{R_1}10^7\beta(s+1)}{(s+1)10^7\beta + (s+10)(s+1000)}
\]
Example: Determine the dc open-loop gain, dc closed-loop gain, the open-loop poles, the open-loop zeros, the closed-loop poles, the closed-loop zeros, and the characteristic polynomial if

\[ A(s) = 10^7 \frac{s+1}{(s+10)(s+1000)} \]

\[ A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{-\frac{R_2}{R_1}10^7\beta(s+1)}{(s+1)10^7\beta + (s+10)(s+1000)} \]

\[ D_{FB}(s) = (s+1)10^7\beta + (s+10)(s+1000) \]

In integer-monic form:

\[ D_{FB}(s) = s^2 + s\left(10+1000+10^7\beta\right) + 10^7\beta \]

Closed-loop zeros =

Closed-loop poles =
Cascaded Amplifier Issues

- Three amplifier cascades - for ideally identical stages
  \[ 8 > \beta A_0^3 \]
  -- seldom used in industry!

- Four or more amplifier cascades - problems even larger than for three stages
  -- seldom used in industry!
Consider Again the Frequency Response of Feedback Amplifier

\[ A_{FB} = \frac{A}{1 + A\beta} = \frac{A_0 A_2}{\left(\frac{s}{\tilde{p}_1} + 1\right)\left(\frac{s}{\tilde{p}_2} + 1\right) + \beta A_0 A_2} \]

Consider cascade of two stages, i.e. \( n=2 \)

If we assume \( \tilde{p}_2 \geq \tilde{p}_1 \) and thus express \( \tilde{p}_2 = k \tilde{p}_1 \)

The characteristic polynomial can be expressed as

\[ D_{FB}(s) = s^2 + s\tilde{p}_1(1 + k) + k\tilde{p}_1^2\left(1 + \beta A_{TOT}\right) \]

Note this amplifier is stable !!!!
(at least based upon this analysis)
Two-stage Cascade (continued)

\[ D_{FB}(s) = s^2 + s\tilde{p}_1(1+k) + k\tilde{p}_1^2(1+\beta A_{0\text{TOT}}) \]

Consider special case of identical stages (i.e. k=1)

\[ D_{FB}(s) = s^2 + s\tilde{p}_1(2) + \tilde{p}_1^2(1+\beta A_{0\text{TOT}}) \approx s^2 + s\tilde{p}_1(2) + \tilde{p}_1^2(\beta A_{0\text{TOT}}) \]

thus the poles of the feedback amplifier are located at

\[ p_{1,2} = -\tilde{p}_1 \pm \sqrt{\tilde{p}_1^2(1-\beta A_{0\text{TOT}})} \approx -\tilde{p}_1 \left(1 \pm j\sqrt{\beta A_{0\text{TOT}}} \right) \]

- FB poles are very close to the imaginary axis
- Very highly under damped
- Not useful as an amplifier (excessive ringing)
- Other poles will make it unstable
Two-stage Cascade (continued)

\[ D_{FB}(s) = s^2 + s\tilde{p}_1(1+k) + k\tilde{p}_1^2(1+\beta A_{TOT}) \]

Thus, must make \( k >> 1 \) if there is any potential for the two-stage cascade

\[ D_{FB}(s) \cong s^2 + s\tilde{p}_1(k) + k\tilde{p}_1^2(\beta A_{TOT}) \]

thus the poles of the feedback amplifier are located at

\[ p_{1,2} \cong \frac{\tilde{p}_1}{2} \left( -k \pm j\sqrt{4A_{TOT}k\beta - k^2} \right) \]

Case 1: No complex conjugate poles; must make discriminate 0, thus

\[ k \cong 4\beta A_{TOT} \]
Two-stage Cascade (continued)

\[ p_{1,2} \approx \frac{\tilde{p}_1}{2} \left( -k \pm j \sqrt{4A_{0\text{TOT}}k\beta - k^2} \right) \]

Case 2: Maximally flat magnitude response; must make real and imaginary parts equal

\[ k = \sqrt{4A_{0\text{TOT}}k\beta - k^2} \]

\[ k \approx 2\beta A_{0\text{TOT}} \]

- Small ringing in step response
- Factor of 2 reduction in pole spread
The pole spread is quite large for the two-stage amplifier but can be achieved.

Usually will make angle of feedback poles with imaginary axis between 45° and 90°.

This results in an open loop pole spread that satisfies the relationship:

\[ 4\beta A_{0TOT} > k > 2\beta A_{0TOT} \]

“Compensation” is the modification of the pole locations of an amplifier to achieve a desired closed-loop pole angle.
Cascaded Amplifier Issues

- Single-stage amplifiers
  - widely used in industry, little or no concern about compensation

- Two amplifier cascades
  - widely used in industry but compensation is essential!
  - \[ 4\beta A_{\text{TOT}} > k > 2\beta A_{\text{TOT}} \]

- Three amplifier cascades - for ideally identical stages
  - seldom used in industry!
  - \[ 8 > \beta A_0^3 \]

- Four or more amplifier cascades - problems even larger than for three stages
  - seldom used in industry!

Note: Some amplifiers that are termed single-stage amplifiers in many books and papers are actually two-stage amplifiers and some require modest compensation. Some that are termed two-stage amplifiers are actually three-stage amplifiers. These invariably have a very small gain on the first stage and a very large bandwidth. The nomenclature on this summary refers to the number of stages that have reasonably large gain.