EE 435

Lecture 16

Compensation
Simple pole calculations for 2-stage op amp

Since the poles of the 2-stage op amp must be widely separated, a simple calculation of the poles from the characteristic polynomial is possible.

Assume $p_1$ and $p_2$ are the poles and $p_1 << p_2$

$D(s)=s^2+a_1s+a_0$

but

$D(s)=(s+p_1)(s+p_2)=s^2+s(p_1+p_2)+p_1p_2 \approx s^2 + p_2s + p_1p_2$

determines $p_1$

determines $p_2$

thus

$p_2=-a_1$ and $p_1 = -a_0/a_1$
Basic Two-Stage Op Amp

\[
A_{FB}(s) \equiv \frac{g_{md}(g_{m0} - sC_c)}{s^2C_cC_L + sC_c(g_{mo} - \beta g_{md}) + \beta g_{md}g_{mo}}
\]

It can be shown that

\[
Q = \sqrt{\frac{C_L}{C_c}} \sqrt{\beta} \frac{\sqrt{g_{mo}g_{md}}}{g_{mo} - \beta g_{md}}
\]

\[
C_c = \frac{C_L \beta}{Q^2} \frac{g_{mo}g_{md}}{(g_{mo} - \beta g_{md})^2}
\]

where \( g_{md} = g_{m1} \) \( g_{mo} = g_{m5} \)

\( g_{oo} = g_{o5} + g_{o6} \) and \( g_{od} = g_{o2} + g_{o4} \)

Right Half-Plane Zero Limits Performance
Compensation

What is “compensation” or “frequency compensation” and what is the goal of compensation?

Nobody defines it or defines it correctly but everybody tries to do it!
Compensation

Compensation (alt Frequency Compensation) is the manipulation of the poles and/or zeros of the open-loop amplifier so that the closed-loop amplifier will perform acceptably.

Note this definition does not mention stability, positive feedback, negative feedback, phase margin, or oscillation.

Note that acceptable performance is strictly determined by the user in the context of the specific application.
Compensation

Compensation requirements usually determined by closed-loop pole locations:

\[ D_{FB}(s) = D(s) + \beta(s)N(s) \]

- Often Phase Margin or Gain Margin criteria are used instead of pole Q criteria when compensating amplifiers (for historical reasons but must still be conversant with this approach).
- Nyquist plots are an alternative stability criteria that is used some in the design of amplifiers.
- Phase Margin and Gain Margin criteria are directly related to the Nyquist Plots.
- Compensation requirements are strongly dependent on \( \beta \).

Characteristic Polynomial obtained from denominator term of basic feedback equation

\[ 1 + A(s)\beta(s) \]

\( A(s)\beta(s) \) defined to be the “loop gain” of a feedback amplifier.
Pole Locations and Stability

Theorem: A system is stable iff all closed-loop poles lie in the open left half-plane.

Note: Practically want to avoid having closed-loop amplifier poles close to the imaginary axis to provide reasonable stability margin, to minimize ringing in the time-domain, and to minimize peaking in the frequency domain.

45° pole-pair angle corresponds to $Q = \frac{1}{\sqrt{2}} = .707$

90° pole angle (on pole pair) corresponds to $Q = \frac{1}{2}$
Nyquist Plots

The Nyquist Plot is a plot of the Loop Gain \( A\beta \) versus \( j\omega \) in the complex plane for \(-\infty < \omega < \infty\).

**Theorem:** A system is stable iff the Nyquist Plot does not encircle the point \(-1+j0\).

**Note:** If there are multiple crossings of the real axis by the Nyquist Plot, the term encirclement requires a formal definition that will not be presented here.
Nyquist Plots

$D_{FB}(s) = 1 + A(s)\beta(s)$

$s$-plane

$A(s)\beta$

-1+j0 is the image of ALL poles

The Nyquist Plot is the image of the entire imaginary axis and separates the image complex plane into two parts

Everything outside of the Nyquist Plot is the image of the LHP

Nyquist plot can be generated with pencil and paper
Conceptually would like to be sure Nyquist Plot does not get too close to -1+j0
Conceptually would like to be sure Nyquist Plot does not get too close to -1+j0

But identification of a suitable neighborhood is not natural
Conceptually would like to be sure Nyquist Plot does not get too close to -1+j0.

But identification of a suitable neighborhood is not natural.
Nyquist Plots

Phase margin is $180^\circ$ – angle of $A\beta$ when the magnitude of $A\beta = 1$
Gain margin is $1 - \text{magnitude of } A\beta$ when the angle of $A\beta = 180^\circ$. 
Nyquist Plots

Theorem: A system is stable iff the phase margin is positive
Theorem: A system is stable iff the gain margin is positive

The phase margin is often the parameter that is specified when compensating operational amplifiers

Phase margins of 45° to 60° or sometimes even 75° are often used

The definition of phase margin does not depend upon the order of the system and is affected by the location of the zeros of the system

The phase margin is a function of β
Review of Basic Concepts

Nyquist Plots

Engineers have some comfort in how far an amplifier is from becoming stable when specifying phase margin criteria (but this is often not mathematically justifiable)

Pole Q criteria are generally much better to use than phase margin criteria but industry is heavily “phase-margin” entrenched!

Separate magnitude and phase plots are often used rather than Nyquist Plots when assessing phase margins or gain margins

The magnitude and phase plots convey exactly the same information as Nyquist Plots but have a linear (or logarithmic) axis rather than the highly skewed imaginary axis of the Nyquist Plot
Example

A feedback amplifier has a characteristic polynomial of

\[ D(s) = s^2 + 9000s + 1.8\times10^3 \]

Without using the quadratic equation, determine the poles by inspection and determine the ratio of the two poles.
solution

A feedback amplifier has a characteristic polynomial of

\[ D(s) = s^2 + 9000s + 1.8 \times 10^3 \]

\[ P_h = -9000 \]

\[ P_L = -2 \]

Ratio = 4500
Review from last time

Analysis of two-stage op amps is very systematic and can be done by inspection if characteristics of quarter circuit are known.

Compensation is essential for stability when applying feedback.

Miller compensation is very useful for decreasing size of internal compensation capacitor but it does not act as a shunting capacitor at high frequencies.

Nyquist plots are a viable alternative for determining stability from the loop gain.

Nyquist plot is a mapping by the function $A\beta$ from the s-plane to the s-plane and the image of the imaginary axis is the Nyquist plot.

Phase margin (and sometimes gain margin) are widely used to specify compensation expectations but probably not as useful as pole-Q compensation criterion however legacy will keep these concepts around for a long time.
Nyquist and Gain-Phase Plots convey identical information but gain-phase plots often easier to work with.

Note: The two plots do not correspond to the same system in this slide.
Gain and Phase Margin Examples

\[ T(s) = \frac{1000}{(s + 1)^3} \]

Phase Margin

\[ \beta^{-1} \]

Magnitude in dB

Angle in degrees

-180°
Gain and Phase Margin Examples

\[ T(s) = \frac{1000}{(s + 1)^3} \]

Gain Margin

β⁻¹

ω

Magnitude in dB

Gain Margin

ω

Angle in degrees

-180°
Gain and Phase Margin Examples

$$T(s) = \frac{1000}{(s + 1)^3}$$

Magnitude in dB

\[\beta^{-1}\]

Phase Margin

$-180^\circ$
Gain and Phase Margin Examples

\[ T(s) = \frac{1000}{(s + 1)^3} \]

- Gain Margin
- \( \beta^{-1} \)

Angle in degrees

Magnitude in dB
Gain and Phase Margin Examples

\[ T(s) = \frac{1000}{(s + 1)} \]

- **Gain Margin**
- **Phase Margin**

**Phase Margin**

\[ \beta^{-1} \]

**Angle in degrees**

\[ -180^\circ \]
Gain and Phase Margin Examples

$$T(s) = \frac{1581}{(s + 1)^2(s + 20)}$$

Be aware of the multiple values of the arctan function!
Gain and Phase Margin Examples

\[ T(s) = \frac{1581}{(s + 1)^2(s + 20)} \]

Angle in degrees

-300
-250
-200
-150
-100
-50
0
50
100
150
200
250
300

Magnitude in dB

-80
-60
-40
-20
0
20
40
60
80

\[ \beta^{-1} \]

Phase Margin
Gain and Phase Margin Examples

\[ T(s) = \frac{1581}{(s + 1)^2(s + 20)} \]
Relationship between pole Q and phase margin

In general, the relationship between the phase margin and the pole Q is dependent upon the order of the transfer function and on the location of the zeros

In the special case that the open loop amplifier is second-order low-pass, a closed form analytical relationship between pole Q and phase margin exists and this is independent of $A_0$ and $\beta$.

$$Q = \frac{\sqrt{\cos(\varphi_M)}}{\sin(\varphi_M)}$$
$$\varphi_M = \cos^{-1}\left(\sqrt{1 + \frac{1}{4Q^4}} - \frac{1}{2Q^2}\right)$$

The region of interest is invariable only for $0.5 < Q < 0.7$

larger Q introduces unacceptable ringing and settling
smaller Q slows the amplifier down too much
Phase Margin vs Q

Second-order low-pass Amplifier
Phase Margin vs Q

Second-order low-pass Amplifier
Phase Margin vs $Q$

Second-order low-pass Amplifier
Magnitude Response of 2\textsuperscript{nd}-order Lowpass Function

\[ G(\Omega) = 20 \log |A_{cl}| (\text{dB}) \]

\[ Q_{\text{MAX}} \text{ for no peaking } = \frac{1}{\sqrt{2}} = 0.707 \]

\[ \xi = \frac{1}{2Q} \]

From Laker-Sansen Text
Phase Response of 2nd-order Lowpass Function

\[ \psi(\Omega) = \arg(A_C/\Omega) \text{ (degrees)} \]

\[ \zeta = \frac{1}{2Q} \]

(b) From Laker-Sansen Text
Step Response of 2\textsuperscript{nd}-order Lowpass Function

\[ \zeta = \frac{1}{2Q} \]

\( Q_{\text{MAX}} \) for no overshoot = 1/2

From Laker-Sansen Text
Step Response of 2\textsuperscript{nd}-order Lowpass Function

\[ \xi = \frac{1}{2Q} \]

From Laker-Sansen Text
Natural Parameter Space for the Two-Stage Amplifier Design

\[ S_{\text{NATURAL}} = \{ W_1, L_1, W_3, L_3, W_5, L_5, W_6, L_6, W_7, L_7, I_T, I_{D6}, C_c, V_{B2}, V_{B3} \} \]
Design Degrees of Freedom

Total independent variables: 15

Degrees of Freedom: 15

If phase margin is considered a constraint, 15 independent Variables, 1 constraint and thus 14 degrees of freedom
Observation: \( W, L \) appear as \( W/L \) ratio in almost all characterizing equations

Implication: Degrees of Freedom are Reduced

\[ S_{\text{NATURAL-REDUCED}} = \{(W/L)_1, (W/L)_3, (W/L)_5, (W/L)_6, (W/L)_7, I_{D6}, I_T, C_C\} \]

With phase margin constraint,

Degrees of freedom: 7
## Common Performance Parameters of Operational Amplifiers

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ao</td>
<td>Open-loop DC Gain</td>
</tr>
<tr>
<td>GB</td>
<td>Gain-Bandwidth Product</td>
</tr>
<tr>
<td>$N_m$ (or Q)</td>
<td>Phase Margin (or pole Q)</td>
</tr>
<tr>
<td>SR</td>
<td>Slew Rate</td>
</tr>
<tr>
<td>$T_{SETTLE}$</td>
<td>Settling Time</td>
</tr>
<tr>
<td>$A_T$</td>
<td>Total Area</td>
</tr>
<tr>
<td>$A_A$</td>
<td>Total Active Area</td>
</tr>
<tr>
<td>P</td>
<td>Power Dissipation</td>
</tr>
<tr>
<td>$\sigma_{VOS}$</td>
<td>Standard Deviation of Input Referred Offset Voltage (often termed the input offset voltage)</td>
</tr>
<tr>
<td>CMRR</td>
<td>Common Mode Rejection Ratio</td>
</tr>
<tr>
<td>PSRR</td>
<td>Power Supply Rejection Ratio</td>
</tr>
<tr>
<td>Vimax</td>
<td>Maximum Common Mode Input Voltage</td>
</tr>
<tr>
<td>Vimin</td>
<td>Minimum Common Mode Output Voltage</td>
</tr>
<tr>
<td>Vomax</td>
<td>Maximum Output Voltage Swing</td>
</tr>
<tr>
<td>Vomin</td>
<td>Minimum Output Voltage Swing</td>
</tr>
<tr>
<td>Vnoise</td>
<td>Input Referred RMS Noise Voltage</td>
</tr>
<tr>
<td>Sv</td>
<td>Input Referred Noise Spectral Density</td>
</tr>
</tbody>
</table>
Performance Parameters

Total: 17
Performance Parameters

Total: 17

System is Generally Highly Over Constrained!