EE 435

Lecture 28

Data Converter Characterization

• Monotonicity
• Missing Codes
• Spectral Performance
Integral Nonlinearity (DAC)

Nonideal DAC

INL often expressed in LSB

\[ \text{INL}_k = \frac{\mathcal{X}_{\text{OUT}}(k) - \mathcal{X}_{\text{OF}}(k)}{\mathcal{X}_{\text{LSB}}} \]

\[ \text{INL} = \max_{0 \leq k \leq N-1} \left\{ |\text{INL}_k| \right\} \]

- INL is often the most important parameter of a DAC
- \( \text{INL}_0 \) and \( \text{INL}_{N-1} \) are 0 (by definition)
- There are \( N-2 \) elements in the set of \( \text{INL}_k \) that are of concern
- INL is almost always nominally 0 (i.e. designers try to make it 0)
- INL is a random variable at the design stage
- \( \text{INL}_k \) is a random variable for \( 0 < k < N-1 \)
- \( \text{INL}_k \) and \( \text{INL}_{k+j} \) are almost always correlated for all \( k, j \) (not incl 0, \( N-1 \))
- Fit Line is a random variable
- INL is the \( N-2 \) order statistic of a set of \( N-2 \) correlated random variables
Integral Nonlinearity (DAC)

Nonideal DAC

- At design stage, INL characterized by standard deviation of the random variable
- Closed-form expressions for INL almost never exist because PDF of order statistics of correlated random variables is extremely complicated
- Simulation of INL very time consuming if n is very large (large sample size required to establish reasonable level of confidence)
  - Model parameters become random variables
  - Process parameters affect multiple model parameters causing model parameter correlation
  - Simulation times can become very large
- INL can be readily measured in laboratory but often dominates test costs because of number of measurements needed when n is large
- Expected of INL\(_k\) at \(k=(N-1)/2\) is largest for many architectures
- Major effort in DAC design is in obtaining acceptable yield!
Integral Nonlinearity (ADC)

Nonideal ADC

Continuous-input based INL definition

Often expressed in LSB

\[ \text{INL}(x_{IN}) = \frac{\tilde{x}_{IN}(x_{IN}) - x_{INF}(x_{IN})}{x_{LSB}} \]

\[ \text{INL} = \max_{0 \leq x_{IN} \leq x_{REF}} \left\{ |\text{INL}(x_{IN})| \right\} \]
Integral Nonlinearity (ADC)

Nonideal ADC

With this definition of INL, the INL of an ideal ADC is $\chi_{\text{LSB}}/2$ (for $\chi_{T1}=\chi_{\text{LSB}}$)

This is effective at characterizing the overall nonlinearity of the ADC but does not vanish when the ADC is ideal and the effects of the breakpoints is not explicit
Integral Nonlinearity (ADC)

Nonideal ADC

Break-point INL definition

\[ \text{INL}_k = \frac{x_{T_k} - x_{F_T}}{x_{	ext{LSB}}} \quad 1 \leq k \leq N-2 \]

\[ \text{INL} = \max_{2 \leq k \leq N-2} \{|\text{INL}_k|\} \]

- \( \text{INL} \) is often the most important parameter of an ADC
- \( \text{INL}_1 \) and \( \text{INL}_{N-1} \) are 0 (by definition)
- There are \( N-3 \) elements in the set of \( \text{INL}_k \) that are of concern
- \( \text{INL} \) is a random variable at the design stage
- \( \text{INL}_k \) is a random variable for \( 0<k<N-1 \)
- \( \text{INL}_k \) and \( \text{INL}_{k+j} \) are correlated for all \( k,j \) (not incl 0, N-1) for most architectures
- Fit Line (for cont INL) and uniformly spaced break pts (breakpoint INL) are random variables
- \( \text{INL} \) is the \( N-3 \) order statistic of a set of \( N-3 \) correlated random variables (breakpoint INL)
Integral Nonlinearity (ADC)

Nonideal ADC

Break-point INL definition

\[ \text{INL}_k = \frac{x_{Tk} - x_{FTl}}{x_{LSB}} \quad 1 \leq k \leq N-2 \]

\[ \text{INL} = \max_{2 \leq k \leq N-2} \{ |\text{INL}_k| \} \]

- At design stage, INL characterized by standard deviation of the random variable
- Closed-form expressions for INL almost never exist because PDF of order statistics of correlated random variables is extremely complicated
- Simulation of INL very time consuming if n is very large (large sample size required to establish reasonable level of confidence)
  - Model parameters become random variables
  - Process parameters affect multiple model parameters causing model parameter correlation
  - Simulation times can become very large
INL-based ENOB

Consider initially the continuous INL definition for an ADC where the INL of an ideal ADC is $X_{\text{LSB}}/2$

Assume $\text{INL} = \theta X_{\text{REF}} = \nu X_{\text{LSBR}}$

where $X_{\text{LSBR}}$ is the LSB based upon the defined resolution

Define the LSB by $X_{\text{LSB}} = \frac{X_{\text{REF}}}{2^n_{\text{EQ}}}$

Thus $\text{INL} = \theta 2^{n_{\text{EQ}}} X_{\text{LSB}}$

Since an ideal ADC has an INL of $X_{\text{LSB}}/2$, express INL in terms of ideal ADC

$\text{INL} = \left[ \theta 2^{(n_{\text{EQ}}+1)} \right] \left( \frac{X_{\text{LSB}}}{2} \right)$

Setting term in $[ ]$ to 1, can solve for $n_{\text{EQ}}$ to obtain

$\text{ENOB} = n_{\text{EQ}} = \log_2 \left( \frac{1}{2\theta} \right) = n_R - 1 - \log_2 (\nu)$

where $n_R$ is the defined resolution
INL-based ENOB

$$\text{ENOB} = n_R - 1 - \log_2(\nu)$$

Consider an ADC with specified resolution of $n_R$ and INL of $\nu$ LSB

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>ENOB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>$n$</td>
</tr>
<tr>
<td>1</td>
<td>$n-1$</td>
</tr>
<tr>
<td>2</td>
<td>$n-2$</td>
</tr>
<tr>
<td>4</td>
<td>$n-3$</td>
</tr>
<tr>
<td>8</td>
<td>$n-4$</td>
</tr>
<tr>
<td>16</td>
<td>$n-5$</td>
</tr>
</tbody>
</table>
Differential Nonlinearity (DAC)

Nonideal DAC

Increment at code k is a signed quantity and will be negative if $X_{\text{OUT}(k)} < X_{\text{OUT}(k-1)}$

\[
DNL(k) = \frac{X_{\text{OUT}(k)} - X_{\text{OUT}(k-1)} - X_{\text{LSB}}}{X_{\text{LSB}}}
\]

\[
DNL = \max_{1 \leq k \leq N-1} \{|DNL(k)|\}
\]

DNL=0 for an ideal DAC
Differential Nonlinearity (ADC)

Nonideal ADC

\[
DNL(k) = \frac{x_{T(k+1)} - x_{Tk} - x_{LSB}}{x_{LSB}}
\]

\[
DNL = \max_{2 \leq k \leq N-1} \{|DNL(k)|\}
\]

DNL=0 for an ideal ADC

Note: In some nonideal ADCs, two or more break points could cause transitions to the same code \(C_k\) making the definition of DNL ambiguous
Performance Characterization of Data Converters

- **Static characteristics**
  - Resolution
  - Least Significant Bit (LSB)
  - Offset and Gain Errors
  - Absolute Accuracy
  - Relative Accuracy
  - Integral Nonlinearity (INL)
  - Differential Nonlinearity (DNL)
  - Monotonicity (DAC)
  - Missing Codes (ADC)
  - Low-f Spurious Free Dynamic Range (SFDR)
  - Low-f Total Harmonic Distortion (THD)
  - Effective Number of Bits (ENOB)
  - Power Dissipation
Definition: An ADC is monotone if the
\[ \bar{X}_{\text{OUT}}(x_k) \geq \bar{X}_{\text{OUT}}(x_m) \quad \text{whenever} \quad x_k \geq x_m \]

Note: Have used \( x_{Bk} \) instead of \( x_{Tk} \) since more than one transition point to a given code

Note: Some authors do not define monotonicity in an ADC.
Missing Codes (ADC)

Definition: An ADC has no missing codes if there are N-1 transition points and a single LSB code increment occurs at each transition point. If these criteria are not satisfied, we say the ADC has missing code(s).

Note: With this definition, all codes can be present but we still say it has “missing codes”

Note: Some authors claim that missing codes in an ADC are the counterpart to nonmonotonicity in a DAC. This association is questionable.
**Missing Codes (ADC)**

**Nonideal ADCs**

- Missing codes
- Missing code with all codes present
Weird Things Can Happen

Nonideal ADCs

• Multiple outputs for given inputs
• All codes present but missing codes

Be careful on definition and measurement of linearity parameters to avoid having weird behavior convolute analysis, simulation or measurements

Most authors (including manufacturers) are sloppy with their definitions of data converter performance parameters and are not robust to some weird operation
Linearity Measurements (testing)

Consider ADC

\[ V_{IN}(t) \rightarrow \text{DUT} \rightarrow X_{IOUT} \]

\[ V_{REF} \]

Linearity testing often based upon code density testing

Code density testing:

Ramp or multiple ramps often used for excitation

Linearity of test signal is critical (typically 3 or 4 bits more linear than DUT)
Linearity Measurements (testing)

Code density testing:

- First and last bins generally have many extra counts (and thus no useful information)
- Typically average 16 or 32 hits per code
Linearity Measurements (testing)

Code density testing:

\[ \bar{C} = \frac{\sum_{i=1}^{N-2} \hat{C}_i}{N-2} \]

\[ DNL_i = \frac{\hat{C}_i - \bar{C}}{\bar{C}} \]

\[ \begin{cases} 0 & i=0, N-2 \\ \left[ \sum_{k=1}^{i} \hat{C}_k \right] - i\bar{C} & 1 \leq i \leq N-3 \end{cases} \]

\[ INL = \text{max} \ \{ |DNL_i| \} \]

\[ DNL = \text{max} \ \{ |DNL_i| \} \]

\[ INL = \text{max} \ \{ |DNL_i| \} \]
Performance Characterization of Data Converters

• Static characteristics
  – Resolution
  – Least Significant Bit (LSB)
  – Offset and Gain Errors
  – Absolute Accuracy
  – Relative Accuracy
  – Integral Nonlinearity (INL)
  – Differential Nonlinearity (DNL)
  – Monotonicity (DAC)
  – Missing Codes (ADC)
  – Low-f Spurious Free Dynamic Range (SFDR)
  – Low-f Total Harmonic Distortion (THD)
  – Effective Number of Bits (ENOB)
  – Power Dissipation
Spectral Characterization
INL Often Not a Good Measure of Linearity

Four identical INL with dramatically different linearity
Linearity Issues

- INL is often not adequate for predicting the linearity performance of a data converter
- Distortion (or lack thereof) is of major concern in many applications
- Distortion is generally characterized in terms of the harmonics that may appear in a waveform
Spectral Analysis

If $f(t)$ is periodic

$$f(t) = A_0 + \sum_{k=1}^{\infty} A_k \sin(k\omega t + \theta_k)$$

alternately

$$f(t) = A_0 + \sum_{k=1}^{\infty} a_k \sin(k\omega t) + \sum_{k=1}^{\infty} b_k \cos(k\omega t)$$

$$\omega = \frac{2\pi}{T}$$

$$A_k = \sqrt{a_k^2 + b_k^2}$$

Termed the Fourier Series Representation of $f(t)$
Often the system of interest is ideally linear but practically it is weakly nonlinear.

Often the input is nearly periodic and often sinusoidal and in latter case desired output is also sinusoidal.

Weak nonlinearity will cause distortion of signal as it is propagated through the system.

Spectral analysis often used to characterize effects of the weak nonlinearity.
Spectral Analysis

\[ X_{IN}(t) \xrightarrow{\text{Nonlinear System}} X_{OUT}(t) \]

If

\[ X_{IN}(t) = X_m \sin(\omega t + \theta) \]

\[ X_{OUT}(t) = A_0 + \sum_{k=1}^{\infty} A_k \sin(k \omega t + \theta_k) \]

All spectral performance metrics depend upon the sequence \( \langle A_k \rangle_{k=0}^{\infty} \)

Spectral performance metrics of interest: SNDR, SDR, THD, SFDR, IMOD
Distortion Analysis

\[ |A_k| \]

\[ \langle A_k \rangle_{k=0}^{\infty} \]

Often termed the DFT coefficients (will show later)
Spectral lines, not a continuous function

\( A_1 \) is termed the fundamental
\( A_k \) is termed the kth harmonic
Distortion Analysis

Often ideal response will have only fundamental present and all remaining spectral terms will vanish
For a low distortion signal, the 2\textsuperscript{nd} and higher harmonics are generally much smaller than the fundamental

The magnitude of the harmonics generally decrease rapidly with $k$ for low distortion signals.
Distortion Analysis

\[ f(t) \text{ is band-limited to frequency } 2\pi f_k \text{ if } A_k = 0 \text{ for all } k > k_x \]
Distortion Analysis

Total Harmonic Distortion, THD

$$\text{THD} = \frac{\text{RMS voltage in harmonics}}{\text{RMS voltage of fundamental}}$$

$$\text{THD} = \sqrt{\left(\frac{A_2}{\sqrt{2}}\right)^2 + \left(\frac{A_3}{\sqrt{2}}\right)^2 + \left(\frac{A_4}{\sqrt{2}}\right)^2 + \ldots}$$

$$\text{THD} = \frac{\frac{A_1}{\sqrt{2}}}{\sqrt{\sum_{k=2}^{\infty} A_k^2}}$$
Distortion Analysis

Spurious Free Dynamic Range, SFDR

The SFDR is the difference between the fundamental and the largest harmonic

\[ |A_k| \]

SFDR is usually determined by either the second or third harmonic
Distortion Analysis

In a fully differential symmetric circuit, all even harmonics are absent in the differential output!
**Theorem:** In a fully differential symmetric circuit, all even harmonics are absent in the differential output for symmetric differential excitations!

**Proof:** Expanding in a Taylor’s series around $V_{ID}=0$, we obtain

$$V_{OD} = f(V_{ID}) = \sum_{k=0}^{\infty} h_k V_{ID}^k$$

Assume $V_{ID}=K\sin(\omega t)$  
W.L.O.G. assume $K=1$

$$V_{O1} = \sum_{k=0}^{\infty} h_k [\sin(\omega t)]^k$$
$$V_{O2} = \sum_{k=0}^{\infty} h_k [-\sin(\omega t)]^k$$

$$V_{OD} = V_{O1} - V_{O2} = \sum_{k=0}^{\infty} h_k \left( [\sin(\omega t)]^k - [-\sin(\omega t)]^k \right) = \sum_{k=0}^{\infty} h_k \left( [\sin(\omega t)]^k - (-1)^k [\sin(\omega t)]^k \right)$$

Observe the even-ordered harmonics are absent in this last sum
Distortion Analysis

How are spectral components determined?

By integral

\[
A_k = \frac{1}{\omega T} \left( \int_{t_1}^{t_1+T} f(t)e^{-jk\omega t} \, dt + \int_{t_1}^{t_1+T} f(t)e^{jk\omega t} \, dt \right)
\]

or

\[
a_k = \frac{2}{\omega T} \int_{t_1}^{t_1+T} f(t)\sin(kt\omega) \, dt \quad b_k = \frac{2}{\omega T} \int_{t_1}^{t_1+T} f(t)\cos(kt\omega) \, dt
\]

Integral is very time consuming, particularly if large number of components are required

By DFT (with some restrictions that will be discussed)

By FFT (special computational method for obtaining DFT)
Distortion Analysis

How are spectral components determined?

Consider sampling $f(t)$ at uniformly spaced points in time $T_s$ seconds apart.

This gives a sequence of samples $\left\langle f(kT_s) \right\rangle_{k=1}^{N}$.
Distortion Analysis

NOTATION:

T: Period of Excitation
T_S: Sampling Period
N_P: Number of periods over which samples are taken
N: Total number of samples

\[ N_P = \frac{N T_S}{T} \]

Note: \( N_P \) is not an integer unless a specific relationship exists between \( N, T_S \) and \( T \)
Distortion Analysis

THEOREM: If $N_p$ is an integer and $x(t)$ is band limited to $f_{\text{MAX}}$, then

$$|A_m| = \frac{2}{N} |X(mN_p + 1)|$$

for all $m$ not defined above

where $\left\langle X(k) \right\rangle_{k=0}^{N-1}$ is the DFT of the sequence $\left\langle x(kT_s) \right\rangle_{k=0}^{N-1}$

and $X(k) = 0$ for all $k$ not defined above

with $f = 1/T$, and $f_{\text{MAX}} = \frac{f}{2} \cdot \left\lceil \frac{N}{N_p} \right\rceil$
If the hypothesis of the theorem are satisfied, we thus have
Distortion Analysis

If the hypothesis of the theorem are satisfied, we thus have

FFT is a computationally efficient way of calculating the DFT, particularly when N is a power of 2
Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required:

1. The sampling window be an integral number of periods
2. \[ N > \frac{2 f_{\text{max}}}{f_{\text{SIGNAL}}} N_p \]
Considerations for Spectral Characterization

• Tool Validation
• FFT Length
• Importance of Satisfying Hypothesis
• Windowing
Considerations for Spectral Characterization

• Tool Validation
• FFT Length
• Importance of Satisfying Hypothesis
• Windowing
FFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1. The sampling window be an integral number of periods
2. \[ N > \frac{2 f_{\text{max}}}{f_{\text{SIGNAL}}} N_p \]
Example

WLOG assume \( f_{\text{SIG}} = 50 \text{Hz} \)

\[
V_{\text{IN}} = \sin(\omega t) + 0.5 \sin(2\omega t)
\]

\[
\omega = 2\pi f_{\text{SIG}}
\]

Consider \( N_p = 20 \) \( N = 512 \)

Recall \( 20\log_{10}(0.5) = -6.0205999 \)
Input Waveform
Input Waveform
Input Waveform

Location of First Point if Extended Into Periodic Function
Spectral Response

Rect. Window N=512  Np =20

Mag(dB) vs Frequency
Spectral Response

DFT Horizontal Axis Converter to Frequency: 

\[ f_{\text{AXIS}} = f_{\text{SIGNAL}} \frac{n - 1}{N_p} \]
Spectral Response
**Fundamental will appear at position 1+Np = 21**

Columns 1 through 5

-316.1458 -312.9517 -329.5203 -311.1473 -314.2615

Columns 6 through 10

-315.2584 -330.6258 -317.2896 -312.2316 -311.6335

Columns 11 through 15


Columns 16 through 20

-314.0088 -302.6391 -306.6650 -311.3733 -308.3689

Columns 21 through 25

-0.0000 307.7012 -312.9902 -312.8737 -305.4320

Observe system noise floor due to both spectral limitations of signal generator and numerical limitations in FFT are below -300db
Second Harmonic at $1+2N_p = 41$

Columns 26 through 30

-307.8301 -309.0737 -305.8503 -312.2772 -315.7544

Columns 31 through 35

-311.9316 -316.0581 -318.3454 -306.4977 -308.6679

Columns 36 through 40

-309.9702 -305.9809 -322.1270 -310.6723 -310.3506

Columns 41 through 45

-6.0206  -309.6071 -314.1026 -307.6405 -302.9277

Columns 46 through 50

-313.0745 -304.2330 -310.8487 -317.7966 -316.3385
Third Harmonic at $1+3Np = 61$

Columns 51 through 55

-307.0529 -312.7787 -312.9340 -323.2969 -314.9297

Columns 56 through 60

-318.7605 -303.5929 -305.2994 -310.6430 -306.7613

Columns 61 through 65

-304.8298 -301.4463 -301.1410 -303.1784 -317.8343

Columns 66 through 70

-308.6310 -307.0135 -321.6015 -316.6548 -309.8946

Columns 71 through 75

-306.3472 -323.0110 -319.3267 -314.7873 -310.4085
Fourth Harmonic at $1 + 4Np = 81$

Columns 76 through 80


Columns 81 through 85


Columns 86 through 90

-313.4988 -303.4513 -310.4969 -317.9652 -312.5846

Columns 91 through 95

-309.8121 -311.6403 -312.8374 -310.5414 -308.7807

Columns 96 through 100

-316.7549 -316.3395 -308.4113 -307.3766 -311.0358
Question: How much noise is in the computational environment?

Is this due to quantization in the computational environment or to numerical rounding in the FFT?
Question: How much noise is in the computational environment?

Observation: This noise is nearly uniformly distributed. The level of this noise at each component is around -310dB.
Question: How much noise is in the computational environment?

Assume $A_k = -310$ dB for $0 \leq k \leq N$

$$A_{kDB} = 20\log_{10} A_k$$

$$A_k \approx 10^{\frac{-310}{20}} = 10^{-15.5}$$

$$V_{\text{Noise,RMS}} \approx \sqrt{\sum_{k=1}^{N-1} A_k^2} = \sqrt{N \bar{A}}$$

$$V_{\text{Noise,RMS}} \approx \sqrt{512} 10^{-15.5} = 1.8 \times 10^{-14} = 18 \text{fV}$$

Note: This computational environment has a very low total computational noise and does not become significant until the 45-bit resolution level is reached !!
Considerations for Spectral Characterization

• Tool Validation
• FFT Length
• Importance of Satisfying Hypothesis
• Windowing
Example

WLOG assume $f_{SIG}=50\text{Hz}$

$$V_{IN} = \sin(\omega t) + 0.5 \sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

Consider $N_P=20$ $N=4096$
Spectral Response

Rect. Window N=4096 Np=20

Mag(dB)

Frequency
Fundamental will appear at position $1+N_p = 21$

Columns 1 through 7


Columns 8 through 14

-319.7032 -317.4419 -327.4933 -321.1968 -318.2241 -312.7300 -316.8359

Columns 15 through 21

-315.5166 -316.1801 -307.8072 -304.3414 -301.3326 -301.7993 0

Columns 22 through 28


Columns 29 through 35

k^{th} harmonic will appear at position 1+k\cdot N_p

Columns 36 through 42

-319.0051 -309.4219 -305.5698 -302.8625 -303.2207 \quad -6.0206 \quad -302.3437

Columns 43 through 49

-300.8222 -301.6722 -304.8150 -313.0288 -313.5963 -312.1136 -310.7740

Columns 50 through 56


Columns 57 through 63

-320.2843 -320.9910 -316.8320 -318.3531 \quad -318.4341 \quad -322.1619 -321.6183

Columns 64 through 70

Example

WLOG assume $f_{SIG}=50\text{Hz}$

$V_{IN} = \sin(\omega t) + 0.5 \sin(2\omega t)$

$\omega = 2\pi f_{SIG}$

Consider $N_P=50$ $N=4096$
Spectral Response
Fundamental will appear at position $1+N_p = 51$

Columns 1 through 7

-322.4309 -325.5445 -322.2645 -321.6226 -319.5894 -323.4895 -327.3216

Columns 8 through 14

-321.2981 -316.1855 -312.3071 -310.4889 -309.6790 -309.9436 -309.3734

Columns 15 through 21


Columns 22 through 28

-310.1735 -311.1633 -308.9079 -312.0709 -310.6683 -310.6908 -307.6761

Columns 29 through 35

-312.9440 -310.5706 -316.2098 -318.9565 -327.6885 -326.4021 -322.3135
Fundamental will appear at position 1+Np = 51

Columns 36 through 42

Columns 43 through 49

Columns 50 through 56
-309.5231 0 -308.8842 -316.1343 -314.5406 -333.4024 -313.7342

Columns 57 through 63
-319.6023 -314.9029 -316.6932 -314.7123 -311.9567 -312.4610 -322.7229

Columns 64 through 70
-308.7103 -309.8064 -314.9393 -312.4610 -322.7229 -328.0350 -326.6767
\( k^{th} \) harmonic will appear at position 1 + k\( \cdot \)Np

Columns 71 through 77


Columns 78 through 84


Columns 85 through 91


Columns 92 through 98

-313.6855  -313.3882  -330.4962  -324.4762  -333.2237  -325.8694  -313.9127

Columns 99 through 105

-315.4869  -308.6364  -6.0206  -309.2723  -314.4098  -316.3311  -328.2626
The $k$th harmonic will appear at position $1+k\cdot N_p$

Columns 106 through 112


Columns 113 through 119

-319.9292 -325.4840 -318.0998 -328.0000 -321.7632 -326.5097 -328.5867

Columns 120 through 126


Columns 127 through 133

-315.0684 -308.6315 -312.9640 -309.5056 -311.6251 -316.1369 -316.1064

Columns 134 through 140

-320.4989 -331.2686 -314.3479 -310.0891 -308.0023 -308.1556 -309.0616
k^{th} harmonic will appear at position 1+k\cdot Np

Columns 141 through 147

-311.2372 -312.6180 -319.0565 -325.6750 -323.7759 -320.7444 -318.0752

Columns 148 through 154


Columns 155 through 161


Columns 162 through 168


Columns 169 through 175

Considerations for Spectral Characterization

**FFT Length**

- FFT Length does not affect the computational noise floor
- Although not shown here yet, FFT length does reduce the quantization noise floor coefficients

If we assume $E_{\text{QUANT}}$ is fixed

$$E_{\text{QUANT}} \approx \sqrt{\sum_{k=1}^{2^{n_{\text{DFT}}}} A_k^2}$$

If the $A_k$’s are constant and equal

$$E_{\text{QUANT}} \approx A_k \frac{2^{n_{\text{DFT}}}}{2}$$

Solving for $A_k$, obtain

$$A_k \approx \frac{E_{\text{QUANT}}}{2^{n_{\text{DFT}}}}$$

If input is full-scale sinusoid with only amplitude quantization with $n$-bit res,

$$E_{\text{QUANT}} \approx \frac{X_{\text{LSB}}}{\sqrt{12}} = \frac{X_{\text{REF}}}{\sqrt{3} \cdot 2^{n+1}}$$
Considerations for Spectral Characterization

FFT Length

\[ E_{\text{QUANT}} \approx \frac{X_{\text{LSB}}}{\sqrt{12}} = \frac{X_{\text{REF}}}{\sqrt{3} \cdot 2^{n+1}} \]

Substituting for \( E_{\text{QUANT}} \), obtain

\[ A_k \approx \frac{X_{\text{REF}}}{\sqrt{3} \cdot 2^{n+1} 2^{n_{\text{DFT}}/2}} \]

This value for \( A_k \) thus decreases with the length of the DFT window.
Considerations for Spectral Characterization

- Tool Validation
- FFT Length
- Importance of Satisfying Hypothesis
  - NP is an integer
  - Band-limited excitation
- Windowing
Example

\[ \text{WLOG assume } f_{\text{SIG}} = 50\text{Hz} \]

\[ V_{\text{IN}} = \sin(\omega t) + 0.5 \sin(2\omega t) \]

\[ \omega = 2\pi f_{\text{SIG}} \]

Consider \( N_P = 20.2 \) \( N = 4096 \)

Recall \( 20\log_{10}(0.5) = -6.0205999 \)
Input Waveform
Input Waveform
Input Waveform
Input Waveform
Spectral Response
Fundamental will appear at position $1+N_p = 21$

Columns 1 through 7

-35.0366  -35.0125  -34.9400  -34.8182  -34.6458  -34.4208  -34.1403

Columns 8 through 14


Columns 15 through 21


Columns 22 through 28


Columns 29 through 35

-34.1902  -35.2163  -35.9043  -36.1838  -35.9965  -35.3255  -34.1946

Note there is a dramatic increase in the noise floor and a significant change in and spreading of the fundamental!!
\textbf{k}^{\text{th}} \text{ harmonic will appear at position } 1+k\cdot N_p

<table>
<thead>
<tr>
<th>Columns 36 through 42</th>
</tr>
</thead>
<tbody>
<tr>
<td>-32.6350         -30.6397</td>
</tr>
<tr>
<td>-28.1125         -24.7689</td>
</tr>
<tr>
<td>-19.7626         \textbf{-8.5639}</td>
</tr>
<tr>
<td>-11.7825</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Columns 43 through 49</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20.0158         -23.9648</td>
</tr>
<tr>
<td>-26.5412         -28.4370</td>
</tr>
<tr>
<td>-29.9279         -31.1519</td>
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<tr>
<td>-32.1874</td>
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</tbody>
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<tr>
<th>Columns 50 through 56</th>
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<tbody>
<tr>
<td>-33.0833         -33.8720</td>
</tr>
<tr>
<td>-34.5759         -35.2113</td>
</tr>
<tr>
<td>-35.7902         -36.3218</td>
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<tr>
<td>-36.8133</td>
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<table>
<thead>
<tr>
<th>Columns 57 through 63</th>
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<tbody>
<tr>
<td>-37.2703         -37.6974</td>
</tr>
<tr>
<td>-38.0984         -38.4762</td>
</tr>
<tr>
<td>-38.8336         -39.1725</td>
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<tr>
<td>-39.4949</td>
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<tr>
<th>Columns 64 through 70</th>
</tr>
</thead>
<tbody>
<tr>
<td>-39.8024         -40.0963</td>
</tr>
<tr>
<td>-40.3778         -40.6479</td>
</tr>
<tr>
<td>-40.9076         -41.1576</td>
</tr>
<tr>
<td>-41.3987</td>
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</tbody>
</table>
\( k^{th} \) harmonic will appear at position \( 1+k\cdot N_p \)

<table>
<thead>
<tr>
<th>Columns 36 through 42</th>
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</thead>
<tbody>
<tr>
<td>-33.0833  -33.8720  -34.5759  -35.2113  -35.7902  -36.3218  -36.8133</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 57 through 63</th>
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<table>
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<tr>
<th>Columns 64 through 70</th>
</tr>
</thead>
</table>
Observations

• Modest change in sampling window of 0.2 out of 20 periods (1%) results in a big error in both fundamental and harmonic

• More importantly, dramatic raise in the noise floor !!! (from over -300dB to only -12dB)
Example

WLOG assume $f_{SIG} = 50$Hz

$$V_{IN} = \sin(\omega t) + 0.5 \sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

Consider $N_p = 20.01$  $N = 4096$

Deviation from hypothesis is .05% of the sampling window
Input Waveform
Input Waveform
Input Waveform
Spectral Response
Fundamental will appear at position 1+Np = 21

Columns 1 through 7

-89.8679  -83.0583  -77.7239  -74.2607  -71.6830  -69.5948  -67.8044

Columns 8 through 14

-66.2037  -64.7240  -63.3167  -61.9435  -60.5707  -59.1642  -57.6859

Columns 15 through 21

-56.0866  -54.2966  -52.2035  -49.6015  -46.0326  -40.0441  -0.0007

Columns 22 through 28


Columns 29 through 35

-62.2078  -65.1175  -69.1845  -76.9560  -81.1539  -69.6230  -64.0636
$k^{th}$ harmonic will appear at position $1+k\times N_p$

Columns 36 through 42

Observations

• Modest change in sampling window of 0.01 out of 20 periods (.05%) still results in a modest error in both fundamental and harmonic
• More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -40dB)
• Errors at about the 6-bit level !
Example

WLOG assume $f_{\text{SIG}}=50\text{Hz}$

$$V_{\text{IN}} = \sin(\omega t) + 0.5 \sin(2\omega t)$$

$$\omega = 2\pi f_{\text{SIG}}$$

Consider $N_P=20.001$ $N=4096$

Deviation from hypothesis is .005% of the sampling window
Spectral Response
Fundamental will appear at position $1+N_p = 21$

<table>
<thead>
<tr>
<th>Columns 1 through 7</th>
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</thead>
<tbody>
<tr>
<td>-112.2531</td>
<td>-103.4507</td>
</tr>
<tr>
<td>-97.8283</td>
<td>-94.3021</td>
</tr>
<tr>
<td>-91.7015</td>
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<td>-89.6024</td>
<td>-87.8059</td>
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<tr>
<th>Columns 8 through 14</th>
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<tbody>
<tr>
<td>-86.2014</td>
<td>-84.7190</td>
</tr>
<tr>
<td>-83.3097</td>
<td>-81.9349</td>
</tr>
<tr>
<td>-80.5605</td>
<td>-79.1526</td>
</tr>
<tr>
<td>-79.1526</td>
<td>-77.6726</td>
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<table>
<thead>
<tr>
<th>Columns 15 through 21</th>
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<tbody>
<tr>
<td>-76.0714</td>
<td>-74.2787</td>
</tr>
<tr>
<td>-72.1818</td>
<td>-69.5735</td>
</tr>
<tr>
<td>-65.9919</td>
<td>-59.9650</td>
</tr>
<tr>
<td>0.0001</td>
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<table>
<thead>
<tr>
<th>Columns 22 through 28</th>
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</tr>
</thead>
<tbody>
<tr>
<td>-60.0947</td>
<td>-66.2917</td>
</tr>
<tr>
<td>-70.0681</td>
<td>-72.9207</td>
</tr>
<tr>
<td>-75.3402</td>
<td>-77.5767</td>
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<tr>
<td>-79.8121</td>
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<table>
<thead>
<tr>
<th>Columns 29 through 35</th>
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<tbody>
<tr>
<td>-82.2405</td>
<td>-85.1651</td>
</tr>
<tr>
<td>-89.2710</td>
<td>-97.2462</td>
</tr>
<tr>
<td>-101.0487</td>
<td>-89.5195</td>
</tr>
<tr>
<td>-83.9851</td>
<td></td>
</tr>
</tbody>
</table>
$k^{th}$ harmonic will appear at position $1+k\cdot Np$

Columns 36 through 42

-79.8472  -76.1160  -72.2601  -67.6621  -60.7642  -6.0220  -59.3448

Columns 43 through 49

-64.8177  -67.8520  -69.9156  -71.4625  -72.6918  -73.7078  -74.5718

Columns 50 through 56

-75.3225  -75.9857  -76.5796  -77.1173  -77.6087  -78.0613  -78.4809

Columns 57 through 63

-78.8721  -79.2387  -79.5837  -79.9096  -80.2186  -80.5125  -80.7927
Observations

- Modest change in sampling window of 0.01 out of 20 periods (.005%) results in a small error in both fundamental and harmonic.
- More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -60dB)
- Errors at about the 10-bit level!
Spectral Response

Rect. Window  N=4096   Np =20.0001

Frequency

Mag(dB)
Fundamental will appear at position $1+N_p = 21$

<table>
<thead>
<tr>
<th>Columns 1 through 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>-130.4427 -123.1634 -117.7467 -114.2649 -111.6804 -109.5888 -107.7965</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Columns 8 through 14</th>
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</thead>
<tbody>
<tr>
<td>-106.1944 -104.7137 -103.3055 -101.9314 -100.5575 - 0.0000 -97.6702</td>
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<table>
<thead>
<tr>
<th>Columns 15 through 21</th>
</tr>
</thead>
<tbody>
<tr>
<td>-96.0691 -94.2764 -92.1793 -89.5706 -85.9878 -79.9571 0.0000</td>
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<table>
<thead>
<tr>
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<table>
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<tr>
<th>Columns 29 through 35</th>
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</table>
**k**th harmonic will appear at position 1+k•Np

<table>
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<table>
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<tr>
<th>Columns 43 through 49</th>
</tr>
</thead>
<tbody>
<tr>
<td>-84.8247  -87.8566  -89.9190  -91.4652  -92.6940  -93.7098  -94.5736</td>
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<table>
<thead>
<tr>
<th>Columns 50 through 56</th>
</tr>
</thead>
<tbody>
<tr>
<td>-95.3241  -95.9872  -96.5810  -97.1187  -97.6100  -98.0625  -98.4821</td>
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<table>
<thead>
<tr>
<th>Columns 57 through 63</th>
</tr>
</thead>
<tbody>
<tr>
<td>-98.8732  -99.2398  -99.5847  <strong>-99.9107</strong>  -100.2197  -100.5135  -100.7937</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 64 through 70</th>
</tr>
</thead>
<tbody>
<tr>
<td>-100.7937</td>
</tr>
</tbody>
</table>
Observations

• Modest change in sampling window of 0.001 out of 20 periods (.0005%) results in a small error in both fundamental and harmonic.

• More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -80dB)

• Errors at about the 13-bit level !
Lecture 33

Spectral Characterization

Distortion Analysis

• Time Quantization Effects
• Spectral Characteristic of DAC
  – Time and Amplitude Quantization
Distortion Analysis

THEOREM: If $N_p$ is an integer and $x(t)$ is band limited to $hf$, then

$$|A_m| = \frac{2}{N} |X(mN_p + 1)| \quad 0 \leq m \leq h - 1$$

and

$$X(k) = 0 \quad \text{for all } k \text{ not defined above}$$

where $\left<X(k)\right>_{k=0}^{N-1}$ is the DFT of the sequence $\left<x(kT_s)\right>_{k=0}^{N-1}$

and $f = 1/T$
Spectral Response
FFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required:

1. The sampling window be an integral number of periods

2. \[ N > \frac{2f_{\text{max}}}{f_{\text{SIGNAL}}} N_p \]
Spectral Response
Observations

• Modest change in sampling window of 0.001 out of 20 periods (.0005%) results in a small error in both fundamental and harmonic.

• More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -80dB).

• Errors at about the 13-bit level!
Considerations for Spectral Characterization

- Tool Validation
- FFT Length
- Importance of Satisfying Hypothesis
  - NP is an integer
  - Band-limited excitation
- Windowing
FFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1. The sampling window be an integral number of periods

2. $N > \frac{2f_{\text{max}}}{f_{\text{SIGNAL}}} N_P$
Example

If $f_{SIG}=50\text{Hz}$

and $N_P=20$  $N=512$

\[ N > \frac{2f_{max}}{f_{SIGNAL}} N_P \quad \rightarrow \quad f_{max} < 640\text{Hz} \]
Example

Consider $N_p=20$ $N=512$

If $f_{SIG}=50$Hz

$$V_{IN} = \sin(\omega t) + 0.5 \sin(2\omega t) + 0.5 \sin(14\omega t)$$

$$\omega = 2\pi f_{SIG}$$

(i.e. a component at 700 Hz which violates the band limit requirement)

Recall $\quad 20\log_{10}(0.5)=-6.0205999$
Effects of High-Frequency Spectral Components

![Graph showing high-frequency spectral components](image-url)
Effects of High-Frequency Spectral Components
Effects of High-Frequency Spectral Components

Rect. Window N=512 Np =20

$\text{f}_{\text{high}} = 14f_0$
Effects of High-Frequency Spectral Components

\[ f_{\text{high}} = 14f_0 \]

Columns 1 through 7


Columns 8 through 14

-299.0778 -292.3045 -297.0529 -301.4639 -297.3332 -309.6947 -308.2308

Columns 15 through 21

-297.3710 -316.5113 -293.5661 -294.4045 -293.6881 -292.6872 -0.0000

Columns 22 through 28


Columns 29 through 35

Effects of High-Frequency Spectral Components

\[ f_{\text{high}} = 14f_0 \]

Columns 36 through 42


Columns 43 through 49

-298.9215 -309.4829 -306.7363 -293.0808 -300.0882 -306.5530 -302.9962

Columns 50 through 56

-318.4706 -294.8956 -304.4663 -300.8919 -298.7732 -301.2474 -293.3188
Effects of High-Frequency Spectral Components

Aliased components at

\[ f_{\text{alias}} = 2f_{\text{sample}} - f \]

\[ f_{\text{alias}} = 2 \cdot 12.8f_{\text{sig}} - 14f_{\text{sig}} = 11.6f_{\text{sig}} \]

thus position in sequence \( = 1 + N_p \frac{f_{\text{alias}}}{f_{\text{sig}}} = 1 + 20 \cdot 11.6 = 233 \)

Columns 225 through 231

-296.8883 -292.8175 -295.8882 -286.7494 -300.3477 -284.4253 -282.7639

Columns 232 through 238


Columns 239 through 245

-299.1299 -305.8361 -295.1772 -295.1670 -300.2698 -293.6406 -304.2886

Columns 246 through 252

-302.0233 -306.6100 -297.7242 -305.4513 -300.4242 -298.1795 -299.0956
Effects of High-Frequency Spectral Components

Rect. Window  N=512  Np =20

fhigh=24 fo
Effects of High-Frequency Spectral Components

Rect. Window  N=512  Np =20

f_{high}=25 \, fo
Effects of High-Frequency Spectral Components

![Graph showing effects of high-frequency spectral components]

Rect. Window  N=512  Np =20

fhigh=25 fo
Effects of High-Frequency Spectral Components

Rect. Window N=512 Np =20

fhigh=24.4fo
Effects of High-Frequency Spectral Components

Rect. Window  N=512  Np =20

$f_{high}=24.5 f_0$
Considerations for Spectral Characterization

- Tool Validation
- FFT Length
- Importance of Satisfying Hypothesis
  - $NP$ is an integer
  - Band-limited excitation
- Windowing
Are there any strategies to address the problem of requiring precisely an integral number of periods to use the FFT?

Windowing is sometimes used

Windowing is sometimes misused
Windowing

Windowing is the weighting of the time domain function to maintain continuity at the end points of the sample window.

Well-studied window functions:

- Rectangular
- Triangular
- Hamming
- Hanning
- Blackman
Rectangular Window

Sometimes termed a boxcar window

Uniform weight

Can append zeros

Without appending zeros equivalent to no window
Rectangular Window

Assume $f_{\text{SIG}}=50\text{Hz}$

$$V_{\text{IN}} = \sin(\omega t) + 0.5 \sin(2\omega t)$$

$$\omega = 2\pi f_{\text{SIG}}$$

Consider $N_p=20.1$  $N=512$
## Rectangular Window

Columns 1 through 7


Columns 8 through 14

-44.4065  -43.4052  -42.3602  -41.2670  -40.1146  -38.8851  -37.5520

Columns 15 through 21

-36.0756  -34.3940  -32.4043  -29.9158  -26.5087  -20.9064  -0.1352

Columns 22 through 28


Columns 29 through 35

Rectangular Window

Columns 1 through 7


Columns 8 through 14

-44.4065  -43.4052  -42.3602  -41.2670  -40.1146  -38.8851  -37.5520

Columns 15 through 21

-36.0756  -34.3940  -32.4043  29.9158  -26.5087  -20.9064  -0.1352

Columns 22 through 28


Columns 29 through 35


Energy spread over several frequency components
Triangular Window

Triangular Window  N=512  Np =20.1

Graph showing the triangular window with N=512 and Np=20.1.
Triangular Window

![Graph of Triangular Window with N=512 and Np=20.1]

![Graph of a signal with a circled point highlighted]
Triangular Window  N=512  Np =20.1

Frequency

Mag(dB)

0  50  100  150  200  250

0  -20  -40  -60  -80  -100  -120
Triangular Window
## Triangular Window

### Columns 1 through 7

-100.8530  -72.0528  -99.1401  -68.0110  -95.8741  -63.9944  -92.5170

### Columns 8 through 14

-60.3216  -88.7000  -56.7717  -85.8679  -52.8256  -82.1689  -48.3134

### Columns 15 through 21

-77.0594  -42.4247  -70.3128  -33.7318  -58.8762  -15.7333  -6.0918

### Columns 22 through 28

-12.2463  -57.0917  -32.5077  -68.9492  -41.3993  -74.6234  -46.8037

### Columns 29 through 35

-77.0686  -50.1054  -77.0980  -51.5317  -75.1218  -50.8522  -71.2410
Hamming Window
Hamming Window
Comparison with Rectangular Window
### Hamming Window

Columns 1 through 7

-70.8278  -70.6955  -70.3703  -69.8555  -69.1502  -68.3632  -67.5133

Columns 8 through 14

-66.5945  -65.6321  -64.6276  -63.6635  -62.6204  -61.5590  -60.4199

Columns 15 through 21

-59.3204  -58.3582  -57.8735  -60.2994  -52.6273  -14.4702  -5.4343

Columns 22 through 28

-11.2659  -45.2190  -67.9926  -60.1662  -60.1710  -61.2796  -62.7277

Columns 29 through 35

-64.3642  -66.2048  -68.2460  -70.1835  -71.1529  -70.2800  -68.1145
Hanning Window
Hanning Window
Comparison with Rectangular Window
## Hanning Window

Columns 1 through 7


Columns 8 through 14

-92.4519 -90.4372 -87.7977 -84.9554 -81.8956 -79.3520 -75.8944

Columns 15 through 21

-72.0479 -67.4602 -61.7543 -54.2042 -42.9597 -13.4511 -6.0601

Columns 22 through 28

-10.8267 -40.4480 -53.3906 -61.8561 -68.3601 -73.9966 -79.0757

Columns 29 through 35

-84.4318 -92.7280 -99.4046 -89.0799 -83.4211 -78.5955 -73.9788
Comparison of 4 windows

Rect. Window $N=512$, $N_p=20.1$

Hamming Window $N=512$, $N_p=20.1$

Hanning Window $N=512$, $N_p=20.1$

Triangular Window $N=512$, $N_p=20.1$
Comparison of 4 windows
Preliminary Observations about Windows

- Provide separation of spectral components
- Energy can be accumulated around spectral components
- Simple to apply
- Some windows work much better than others

But – windows do not provide dramatic improvement and ...
Comparison of 4 windows when sampling hypothesis are satisfied
Comparison of 4 windows

- Rectangular Window: N=512, Np=20
- Hamming Window: N=512, Np=20
- Hanning Window: N=512, Np=20
- Triangular Window: N=512, Np=20

The diagrams show the frequency response of the windows with respect to magnitude in dB.
Preliminary Observations about Windows

• Provide separation of spectral components
• Energy can be accumulated around spectral components
• Simple to apply
• Some windows work much better than others

But – windows do not provide dramatic improvement and can significantly degrade performance if sampling hypothesis are met
Issues of Concern for Spectral Analysis

An integral number of periods is critical for spectral analysis. Not easy to satisfy this requirement in the laboratory. Windowing can help but can hurt as well. Out of band energy can be reflected back into bands of interest. Characterization of CAD tool environment is essential. Spectral Characterization of high-resolution data converters requires particularly critical consideration to avoid simulations or measurements from masking real performance.
Spectral Characterization

• Distortion Analysis

→ Time Quantization Effects

• Spectral Characteristic of DAC
  – Time and Amplitude Quantization
Quantization Effects on Spectral Performance and Noise Floor in DFT

- Assume the effective clock rate (for either an ADC or a DAC) is arbitrarily fast
- Without Loss of Generality it will be assumed that $f_{SIG}=50\text{Hz}$
- Index on DFT will be listed in terms of frequency (rather than index number)

Matlab File: afft_Quantization.m
Quantization Effects

16,384 pts  res = 4bits  N_p=25

20 msec
Quantization Effects

16,384 pts  res = 4bits  N_p=25

20 msec
Quantization Effects

16,384 pts  res = 4bits
Quantization Effects

Simulation environment:

\[ N_P = 23 \]
\[ f_{SIG} = 50\text{Hz} \]
\[ V_{REF}: -1\text{V}, 1\text{V} \]
Res: will be varied
\[ N = 2^n \text{ will be varied} \]
Quantization Effects

Res = 4 bits

Rect. Window N=512  Np =23
Quantization Effects

Res = 4 bits

Rect. Window N=4096 Np =23

Axis of Symmetry
Quantization Effects

Res = 4 bits

Some components very small
Quantization Effects

Res = 4 bits

Set lower display limit at -120dB
Quantization Effects

Res = 4 bits

Rect. Window N=16384 Np =23
Quantization Effects

Res = 4 bits
Quantization Effects

Res = 4 bits

Rect. Window N=65536 Np = 23
Quantization Effects

Res = 4 bits

Rect. Window N=65536 Np =23

Mag(dB) vs Frequency
Quantization Effects

Res = 4 bits
Quantization Effects
Res = 4 bits

Rect. Window N=65536  Np =23
Quantization Effects

Res = 4 bits

Fundamental
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits

Rect. Window \( N=256 \)  \( N_p=23 \)

Diagram showing the magnitude response (dB) vs frequency, with peaks and a general noise-like pattern.
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits

Rect. Window N=4096 Np =23
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits

Res 10  No. points 256  fsig= 50.00  No. Periods 23.00
Rectangular Window

Columns 1 through 5
-55.7419  -120.0000  -85.1461  -106.1614  -89.2395

Columns 6 through 10
-102.3822  -99.5653  -85.7335  -89.1227  -83.0851
<table>
<thead>
<tr>
<th>Columns 11 through 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>-87.5203    -78.5459  -93.9801  -89.8324  -94.5461</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Columns 16 through 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>-77.6478    -80.8867 -100.8153  -78.7936  -86.2954</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 21 through 25</th>
</tr>
</thead>
<tbody>
<tr>
<td>-85.8697    -79.5073 -101.6929  -0.0004  -83.6600</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Columns 26 through 30</th>
</tr>
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<tbody>
<tr>
<td>-83.3148    -74.8410  -89.7384  -91.5556  -86.9109</td>
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<table>
<thead>
<tr>
<th>Columns 31 through 35</th>
</tr>
</thead>
<tbody>
<tr>
<td>-93.0155    -82.1062  -78.4561  -98.7568  -109.4766</td>
</tr>
</tbody>
</table>
Columns 36 through 40
-98.2999  -84.9383  -115.7328  -100.0758  -77.1246

Columns 41 through 45
-86.6455  -82.5379  -98.8707  -111.1638  -85.9572

Columns 46 through 50
-85.7575  -92.6227  -83.7312  -83.4865  -82.4473

Columns 51 through 55
-77.4085  -88.0611  -84.5256  -98.4813  -82.7990

Columns 56 through 60
-86.0396  -83.8284  -87.2621  -97.6189  -94.7694
<table>
<thead>
<tr>
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<th>-86.9239</th>
<th>-89.5881</th>
<th>-82.8701</th>
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<tbody>
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<td>-74.9482</td>
<td>-83.4468</td>
<td>-94.0629</td>
<td>-95.3199</td>
<td>-95.4482</td>
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<td>-107.0215</td>
<td>-98.3102</td>
<td>-87.4623</td>
<td>-82.4935</td>
<td>-98.6972</td>
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<td>Columns 76 through 80</td>
<td>-83.1902</td>
<td>-82.2598</td>
<td>-103.0396</td>
<td>-87.2043</td>
<td>-79.1829</td>
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<tr>
<td>Columns 81 through 85</td>
<td>-76.6723</td>
<td>-87.0770</td>
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<td>Columns 86 through 90</td>
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<tr>
<td>-82.9621  -93.0224  -116.8549  -93.7327  -75.6231</td>
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<table>
<thead>
<tr>
<th>Columns 91 through 92</th>
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<tbody>
<tr>
<td>-94.4914  -81.0819</td>
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</table>
Res  10  No. points 4096  fsig=  50.00  No. Periods  23.00

Rectangular Window

Columns 1 through 5

-55.6060  -97.9951  -107.4593  -103.4508  -120.0000

Columns 6 through 10

-96.7808  -105.2905  -96.7395  -104.5281  -90.7582
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<tr>
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<td>-86.0534</td>
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<td>-105.9608</td>
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<td>-91.7843</td>
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<td>-98.7668</td>
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<td>-99.6057</td>
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<td>-101.5798</td>
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<td>-94.1031</td>
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<td>Columns 61 through 65</td>
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<tr>
<td>-93.2650 -103.4274 -103.9702 -98.4092 -91.1825</td>
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<table>
<thead>
<tr>
<th>Columns 66 through 70</th>
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<tbody>
<tr>
<td>-98.0638 -93.7989 -107.7453 -93.4277 -88.0409</td>
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<table>
<thead>
<tr>
<th>Columns 71 through 75</th>
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<tr>
<td>-107.3584 -102.5984 -95.3312 -102.9342 -108.5206</td>
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<table>
<thead>
<tr>
<th>Columns 76 through 80</th>
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<table>
<thead>
<tr>
<th>Columns 81 through 85</th>
</tr>
</thead>
<tbody>
<tr>
<td>-96.5194 -85.8129 -95.1970 -94.8699 -104.9224</td>
</tr>
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Quantization Effects

Res = 10 bits

With Vin=2v pp
With $V_{in}=1^{\ast}.99$ and $V_{os}=.25\text{LSB}$
With $\text{Vin} = 1.9999999 \text{ pp}$
With $V_{in}=1*0.99$ and $V_{os}=.35$ LSB
Res 10  No. points 4096  fsig= 50.00  No. Periods 25.00  Tstep 1.220703e-004
Magnitude of Fundamental 1.000  2nd Harmonic 0.000

Columns 1 through 7


Columns 8 through 14


Columns 15 through 21


Columns 22 through 28


Columns 29 through 35

Quantization Effects

Res = 10 bits
Quantization Effects

Res = 10 bits
Quantization Effects

Res = 10 bits
Quantization Effects

Res = 10 bits
Quantization Effects

Res = 5 bits
Quantization Effects

Res = 4 bits
Quantization Effects

Res = 4 bits
Quantization Effects

16,384 pts  res = 4bits
Quantization Effects

16,384 pts  res = 4bits
Quantization Effects

Res = 10 bits
Spectral Characterization

- Distortion Analysis
- Time Quantization Effects
  - Spectral Characteristic of DAC
    - Time and Amplitude Quantization
Spectral Characteristics of DACs and ADCs
Spectral Characteristics of DAC

Periodic Input Signal

Sampling Clock

Sampled Input Signal (showing time points where samples taken)
Spectral Characteristics of DAC

Quantized Sampled Input Signal (with zero-order sample and hold)

Quantization Levels

$T_{SIG}$

$T_{PERIOD}$
Spectral Characteristics of DAC

$T_{DFT\ WINDOW}$

$T_{PERIOD}$

$T_{SIG}$

$T_{CLOCK}$

Sampling Clock

$T_{DFT\ CLOCK}$

DFT Clock
Spectral Characteristics of DAC

$T_{DFT \ \text{WINDOW}}$

$T_{PERIOD}$

$T_{SIG}$

Sampling Clock

$T_{CLOCK}$

DFT Clock
Spectral Characteristics of DAC

Sampling Clock

DFT Clock
Spectral Characteristics of DAC

Sampled Quantized Signal (zoomed)

DFT Clock

Sampling Clock
Spectral Characteristics of DAC

Consider the following example
- \( f_{\text{SIG}} = 50\text{Hz} \)
- \( k_1 = 230 \)
- \( k_2 = 23 \)
- \( N_p = 1 \)
- \( n_{\text{res}} = 8\text{bits} \)
- \( X_{\text{in}(t)} = 0.95\sin(2\pi f_{\text{SIG}} t) \) (-.4455dB)

Thus
- \( N_{p1} = 23 \)
- \( \theta_{\text{SR}} = 5 \)
- \( f_{CL}/f_{\text{SIG}} = 10 \)

Matlab File: afft_Quantization_DAC.m
DFT Simulation from Matlab

\[ n_{\text{sam}} = 142.4696 \]
DFT Simulation from Matlab

Expanded View

Rect. Window N=32768 Np =1

nres=8 bits

\[ n_{\text{sam}} = 142.4696 \]

Width of this region is \( f_{\text{CL}} \)

Analogous to the overall DFT window when directly sampled but modestly asymmetric
DFT Simulation from Matlab

Expanded View

Rect. Window  N=32768  Np =1

$n_{\text{sam}} = \text{142.4696}$
DFT Simulation from Matlab

Expanded View

Rect. Window  N=32768  Np = 1

\( n_{\text{sam}} = 142.4696 \)
DFT Simulation from Matlab
Expanded View

Rect. Window  N=32768   Np =1

nres=8 bits

\[ n_{\text{sam}} = 142.4696 \]
\[ f_{\text{SIG}} = 50\text{Hz}, \ k_1 = 23, \ k_2 = 23, \ N_P = 1, \ n_{\text{res}} = 8\text{bits} \quad \text{Xin}(t) = \sin(2\pi f_{\text{SIG}}t) \quad N = 32768 \]

Columns 1 through 7

-44.0825  -84.2069  -118.6751  -89.2265 -120.0000  -76.0893 -120.0000

Columns 8 through 14

-90.3321  -120.0000  -69.9163 -120.0000  -88.9097 -120.0000  -85.1896

Columns 15 through 21

-120.0000  -83.0183 -109.4722  -89.4980 -120.0000  -79.6110 -120.0000

Columns 22 through 28

-90.2992 -120.0000  \boxed{-0.5960}  -120.0000  -88.5446 -120.0000  -86.0169

Columns 29 through 35

-120.0000  -81.5409 -109.6386  -89.7275 -120.0000  -81.8340 -120.0000
\[ f_{\text{SIG}} = 50\text{Hz}, \; k_1 = 23, \; k_2 = 23, \; N_p = 1, \; n_{\text{res}} = 8\text{bits} \]
\[ X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t) \]

Columns 36 through 42

-90.2331 -120.0000 -69.4356 -120.0000 -88.1400 -120.0000 -86.7214

Columns 43 through 49

-120.0000 -79.6273 -119.1428 -89.9175 -56.7024 -83.0511 -120.0000

Columns 50 through 56

-90.1331 -120.0000 -75.1821 -120.0000 -87.5706 -120.0000 -87.3205

Columns 57 through 63

-120.0000 -76.9769 -120.0000 -90.0703 -119.0588 -83.2950 -113.3964

Columns 64 through 70

-89.9982 -120.0000 -78.4288 -120.0000 -87.0328 -120.0000 -64.5409
\[ f_{\text{SIG}} = 50 \text{Hz}, \ k_1 = 23, \ k_2 = 23, \ N_P = 1, \ n_{\text{res}} = 8 \text{bits} \quad \text{Xin}(t) = \sin(2\pi f_{\text{SIG}} t) \]

\[ N = 32768 \]

Columns 71 through 77

-120.0000 -72.8111 -120.0000 -90.1876 -120.0000 -82.5616 -114.0867

Columns 78 through 84

-89.8269 -115.6476 -80.6553 -120.0000 -86.3818 -120.0000 -88.3454

Columns 85 through 91

-120.0000 -63.5207 -120.0000 -90.2704 -120.0000 -80.8524 -120.0000

Columns 92 through 98

-89.6174 -58.5435 -82.3253 -120.0000 -85.6188 -120.0000 -88.7339

Columns 99 through 100

-120.0000 -63.8165
DFT Simulation from Matlab

Rect. Window \( N=131072 \), \( N_p = 1 \)

- \( n_{\text{res}} = 8 \) bits

\[ n_{\text{sam}} = 569.8783 \]
n_{\text{sam}} = 569.8783
DFT Simulation from Matlab

Expanded View

Rect. Window  N=131072  Np =1

nres=8 bits

n_{sam} = 569.8783
DFT Simulation from Matlab

Expanded View

Rect. Window N=131072  Np =1

nres=8 bits

nsam =  569.8783
\[ f_{\text{SIG}} = 50 \text{Hz}, \ k_1 = 23, \ k_2 = 23, \ N_P = 1, \ n_{\text{res}} = 8 \text{bits} \]
\[ X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t) \]
\[ N = 131072 \]

Columns 1 through 7

\[
-44.0824 \ -97.0071 \ -120.0000 \ -110.6841 \ -120.0000 \ -76.0276 \ -120.0000
\]

Columns 8 through 14

\[
-103.5227 \ -120.0000 \ -109.7590 \ -120.0000 \ -89.7127 \ -120.0000 \ -107.6334
\]

Columns 15 through 21

\[
-120.0000 \ -107.8772 \ -120.0000 \ -90.3300 \ -120.0000 \ -109.5748 \ -120.0000
\]

Columns 22 through 28

\[
-104.0809 \ -120.0000 \ -0.5960 \ -120.0000 \ -110.6201 \ -120.0000 \ -98.0920
\]

Columns 29 through 35

\[
-120.0000 \ -95.8006 \ -120.0000 \ -110.7338 \ -120.0000 \ -82.3448 \ -120.0000
\]
$$f_{\text{SIG}} = 50\text{Hz}, \ k_1 = 23, \ k_2 = 23, \ N_P = 1, \ n_{\text{res}} = 8\text{bits} \quad X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t)$$

\[N = 131072\]

<table>
<thead>
<tr>
<th>Columns 36 through 42</th>
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<tbody>
<tr>
<td>-102.9185  -120.0000  -109.9276  -120.0000  -88.8778  -120.0000  -107.5734</td>
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<table>
<thead>
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<tbody>
<tr>
<td>-120.0000  -108.1493  -120.0000  -90.7672  -56.7029  -109.3748  -120.0000</td>
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<thead>
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</tr>
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<table>
<thead>
<tr>
<th>Columns 57 through 63</th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.0000  -94.4432  -120.0000  -110.7692  -120.0000  -86.1442  -120.0000</td>
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<table>
<thead>
<tr>
<th>Columns 64 through 70</th>
</tr>
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<tbody>
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<td>-102.2661  -120.0000  -110.0806  -120.0000  -87.7635  -120.0000  -64.4072</td>
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</table>
\[ f_{SIG} = 50\text{Hz} \, , \, k_1 = 23, \, k_2 = 23, \, N_p = 1, \, n_{res} = 8\text{bits} \quad X_{in}(t) = \sin(2\pi f_{SIG} t) \]

\[ N = 131072 \]

<table>
<thead>
<tr>
<th>Columns 71 through 77</th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.0000 -108.4202 -120.0000 -91.0476 -120.0000 -109.1589 -120.0000</td>
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<thead>
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<tbody>
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<td>-101.5617 -58.5437 -110.2183 -120.0000 -86.2629 -120.0000 -105.5980</td>
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<table>
<thead>
<tr>
<th>Columns 99 through 100</th>
</tr>
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<tbody>
<tr>
<td>-120.0000 -108.6808</td>
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</table>
Consider the following example
- \( f_{\text{SIG}} = 50 \text{Hz} \)
- \( k_1 = 50 \)
- \( k_2 = 5 \)
- \( N_P = 2 \)
- \( n_{\text{res}} = 8 \text{bits} \)
- \( Xin(t) = .95 \sin(2\pi f_{\text{SIG}}t) \) (\(-.4455\text{dB}\))

Thus
- \( N_{P1} = 5 \)
- \( \theta_{SR} = 5 \)
- \( N_{P2} = 10 \)
DFT Simulation from Matlab

Rect. Window $N=32768$  $N_p=2$

$nsam = 327.6800$

$n_{res} = 8$
DFT Simulation from Matlab
Expanded View

Rect. Window $N=32768$  $N_p=2$

$$n_{sam} = 327.6800$$

$$n_{res}=8$$
DFT Simulation from Matlab

Rect. Window N=256  Np =2

nsam = 2.5600

n_{res}=8
DFT Simulation from Matlab

Expanded View

Rect. Window N=256  Np =2

\[ n_{\text{res}} = 8 \]
\[ f_{\text{SIG}} = 50\text{Hz}, \; k_1 = 50, \; k_2 = 5, \; N_P = 2, \; n_{\text{res}} = 8\text{bits}, \; X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}}t) \]\n\[ N = 131072 \]

<table>
<thead>
<tr>
<th>Columns 1 through 7</th>
</tr>
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<tbody>
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<td>-50.4484</td>
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<tr>
<td>-120.0000</td>
</tr>
<tr>
<td>-47.9795</td>
</tr>
<tr>
<td>-120.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 29 through 35</th>
</tr>
</thead>
<tbody>
<tr>
<td>-78.0140</td>
</tr>
<tr>
<td>-120.0000</td>
</tr>
<tr>
<td>-47.7412</td>
</tr>
<tr>
<td>-120.0000</td>
</tr>
<tr>
<td>-85.9233</td>
</tr>
<tr>
<td>-120.0000</td>
</tr>
<tr>
<td>-27.8207</td>
</tr>
</tbody>
</table>
$f_{SIG} = 50\text{Hz},\ k_1 = 50,\ k_2 = 5,\ N_P = 2,\ n_{res} = 8\text{bits},\ X_{in}(t) = \sin(2\pi f_{SIG}t)$

$N = 131072$

Columns 36 through 42

-120.0000  -75.9471  -120.0000  -49.8914  -120.0000  -58.4761  -120.0000

Columns 43 through 49

-41.7535  -120.0000  -91.4791  -120.0000  -28.1314  -120.0000  -79.7024

Columns 50 through 56

-120.0000  -50.5858  -120.0000  -78.7241  -120.0000  -31.9459  -120.0000

Columns 57 through 63

-91.9095  -120.0000  -40.4010  -120.0000  -62.1214  -120.0000  -50.1249

Columns 64 through 70

-120.0000  -78.2678  -120.0000  -24.9258  -120.0000  -87.6235  -120.0000
\[ f_{\text{SIG}} = 50 \text{Hz}, \ k_1 = 50, \ k_2 = 5, \ N_P = 2, \ n_{\text{res}} = 8 \text{bits}, \ Xin(t) = \sin(2\pi f_{\text{SIG}} t) \]
\[ N = 131072 \]

<table>
<thead>
<tr>
<th>Columns 71 through 77</th>
</tr>
</thead>
<tbody>
<tr>
<td>-45.3926 -120.0000 -77.2183 -120.0000 -48.4567 -120.0000 -76.6666</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 78 through 84</th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.0000 -30.9406 -120.0000 -69.1777 -120.0000 -48.8912 -120.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 85 through 91</th>
</tr>
</thead>
<tbody>
<tr>
<td>-75.7581 -120.0000 -44.8212 -120.0000 -88.9694 -120.0000 -19.1255</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 92 through 98</th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.0000 -79.5390 -120.0000 -50.3103 -120.0000 -70.6123 -120.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 99 through 105</th>
</tr>
</thead>
<tbody>
<tr>
<td>-38.8332 -120.0000 -92.1633 -120.0000 -34.7560 -120.0000 -77.1229</td>
</tr>
</tbody>
</table>
DFT Simulation from Matlab

Rect. Window N=1024  Np =2

\[ n_{\text{sam}} = 10.2400 \]

\[ n_{\text{res}} = 8 \]
DFT Simulation from Matlab

Expanded View

Rect. Window N=1024 Np =2

nsam = 10.2400

nres = 8
\[ f_{\text{SIG}}=50\text{Hz}, \ k_1=50, \ k_2=5, \ N_P=2, \ n_{\text{res}}=\text{8bits}, \ Xin(t) = \sin(2\pi f_{\text{SIG}}t) \]

\[ N=1024 \]

Columns 1 through 7

| -44.0739 | -120.0000 | -53.8586 | -120.0000 | -91.9997 | -120.0000 | -50.3884 |

Columns 8 through 14

| -120.0000 | -91.3235 | -120.0000 | -0.6017 | -120.0000 | -89.9100 | -120.0000 |

Columns 15 through 21

| -41.0786 | -120.0000 | -86.6863 | -120.0000 | -48.5379 | -120.0000 | -56.7320 |

Columns 22 through 28

| -120.0000 | -53.4112 | -120.0000 | -103.7582 | -120.0000 | -54.1209 | -120.0000 |

Columns 29 through 35

| -98.4283 | -120.0000 | -51.2204 | -120.0000 | -92.1630 | -120.0000 | -39.9145 |
\[ f_{\text{SIG}} = 50 \text{Hz}, \quad k_1 = 50, \quad k_2 = 5, \quad N_P = 2, \quad n_{\text{res}} = 8\text{bits}, \quad X_{in}(t) = \sin(2\pi f_{\text{SIG}} t) \]

\[ N = 1024 \]

Columns 36 through 42

\[
\begin{array}{cccccc}
-120.0000 & -86.0994 & -120.0000 & -46.4571 & -120.0000 & -58.5568 & -120.0000 \\
\end{array}
\]

Columns 43 through 49

\[
\begin{array}{cccccc}
-45.7332 & -120.0000 & -88.7034 & -120.0000 & -52.7530 & -120.0000 & -102.0744 \\
\end{array}
\]

Columns 50 through 56

\[
\begin{array}{cccccc}
-120.0000 & -54.2124 & -120.0000 & -101.8321 & -120.0000 & -52.6742 & -120.0000 \\
\end{array}
\]

Columns 57 through 63

\[
\begin{array}{cccccc}
-89.3186 & -120.0000 & -45.3675 & -120.0000 & -62.0430 & -120.0000 & -46.7029 \\
\end{array}
\]

Columns 64 through 70

\[
\begin{array}{cccccc}
-120.0000 & -85.3723 & -120.0000 & -40.6886 & -120.0000 & -92.0718 & -120.0000 \\
\end{array}
\]
\( f_{\text{SIG}}=50\text{Hz}, \ k_1=50, \ k_2=5, \ N_P=2, \ n_{\text{res}}=8\text{bits}, \ Xin(t) = \sin(2\pi f_{\text{SIG}} t) \)

\[ \begin{array}{ccccccccc}
N=1024 \\
\text{Columns 71 through 77} \\
-51.9029 & -120.0000 & -98.8650 & -120.0000 & -54.1376 & -120.0000 & -103.6450 \\
\text{Columns 78 through 84} \\
-120.0000 & -53.3554 & -120.0000 & -68.6244 & -120.0000 & -48.3107 & -120.0000 \\
\text{Columns 85 through 91} \\
-85.8692 & -120.0000 & -41.9049 & -120.0000 & -89.7301 & -120.0000 & -19.6301 \\
\text{Columns 92 through 98} \\
-120.0000 & -91.5501 & -120.0000 & -50.5392 & -120.0000 & -92.8884 & -120.0000 \\
\text{Columns 99 through 105} \\
-53.8928 & -120.0000 & \textbf{-104.2832} & -120.0000 & -53.8225 & -120.0000 & -91.0209 \\
\end{array} \]
Spectral Characteristics of DAC

Consider the following example

- $f_{\text{SIG}} = 50\text{Hz}$
- $k_1 = 11$
- $k_2 = 1$
- $N_P = 2$
- $n_{\text{res}} = 12\text{bits}$
- $X_{\text{in}}(t) = 0.95\sin(2\pi f_{\text{SIG}} t)$ (-0.4455dB)

Thus

- $N_{P1} = 1$
- $\theta_{\text{SR}} = 11$
- $N_{P2} = 2$
DFT Simulation from Matlab
DFT Simulation from Matlab

Rec Win  N=4096  Np =2  Nsam = 186.181818  nres = 12  fCL/fsig = 11  fDFT/fsig = 2048
DFT Simulation from Matlab

- Frequency: 50, 100, 150, 200, 250, 300, 350
- Magnitude (dB): -20, -40, -60, -80, -100, -120
- **Rec Win**: N=4096, Np =2, Nsam = 186.181818
- **nres = 12**
- **fCL/fsig = 11**
- **fDFT/fsig = 2048**

Graph showing the magnitude response over frequency.
DFT Simulation from Matlab

Rec Win  N=65536 Np =2  Nsam = 2978.90909 nres = 12  fCL/fsig = 11  fDFT/fsig = 32768
DFT Simulation from Matlab
DFT Simulation from Matlab

Rec Win  N=65536 Np =2  Nsam = 2978.90909 nres = 12  fCL/fsig = 11  fDFT/fsig = 32768

Mag(dB)

0  50  100  150  200  250  300  350  400  450
Frequency
Consider the following example

- \( f_{\text{SIG}} = 50\text{Hz} \)
- \( k_1 = 230 \)
- \( k_2 = 23 \)
- \( N_P = 1 \)
- \( n_{\text{res}} = 12\text{bits} \)
- \( X_{\text{in}}(t) = 0.95 \sin(2\pi f_{\text{SIG}} t) \) (-0.4455\,\text{dB})

Thus

- \( N_{P1} = 23 \)
- \( \theta_{\text{SR}} = 10 \)
- \( N_{P2} = 23 \)
DFT Simulation from Matlab

![Graph showing frequency vs. magnitude in dB]
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab

![Graph of DFT Simulation from Matlab]
DFT Simulation from Matlab
DFT Simulation from Matlab

$f_{\text{SIG}}=50\text{Hz}$  $k_1=230$  $k_2=23$  $N_p=1$  $n_{\text{res}}=12\text{bits}$  $X_{\text{in}}(t)=.95\sin(2\pi f_{\text{SIG}} t)$  (-.4455dB)  $N_{p_1}=23$  $\theta_{\text{SR}}=10$

$N_{p_2}=23$

Columns 1 through 7

-68.1646  -94.7298 -120.0000  -90.8893 -120.0000  -75.8402 -120.0000

Columns 8 through 14

-97.7128 -120.0000 -69.7549 -120.0000 -90.5257 -120.0000 -95.1113

Columns 15 through 21

-120.0000 -94.3119 -120.0000 -91.2004 -120.0000 -79.4167 -120.0000

Columns 22 through 28

-97.6931 -120.0000 -0.5886 -120.0000 -90.1044 -120.0000 -95.4585

Columns 29 through 35

-120.0000 -93.8547 -120.0000 -91.4631 -120.0000 -81.9608 -120.0000
DFT Simulation from Matlab

Columns 36 through 42

-97.6535 -120.0000  -69.6068 -120.0000  -89.6188 -120.0000  -95.7721

Columns 43 through 49

-120.0000  -93.3545 -120.0000  -91.6806  -80.7859  -83.9353 -120.0000

Columns 50 through 56

-97.5940 -120.0000  -75.5346 -120.0000  -89.0602 -120.0000  -96.0458

Columns 57 through 63

-120.0000  -92.8067 -120.0000  -91.8555 -120.0000  -85.5462 -120.0000

Columns 64 through 70

-97.5144 -120.0000  -78.9551 -120.0000  -88.4176 -120.0000  -88.0509
## DFT Simulation from Matlab

### Columns 71 through 77

-120.0000  -92.2056  -120.0000  -91.9896  -120.0000  -86.9037  -120.0000

### Columns 78 through 84

-97.4143  -120.0000  -81.3430  -120.0000  -87.6762  -120.0000  -96.6112

### Columns 85 through 91

-120.0000  -91.5441  -120.0000  -92.0844  -120.0000  -88.0732  -120.0000

### Columns 92 through 98

-97.2936  **-82.6264**  -83.1604  -120.0000  -86.8155  -120.0000  -96.8068

### Columns 99 through 100

-120.0000  -90.8133
Spectral Characteristics of DAC

Consider the following example

- \( f_{\text{SIG}} = 50\text{Hz} \)
- \( k_1 = 230 \)
- \( k_2 = 23.1 \)
- \( N_P = 1 \)
- \( n_{\text{res}} = 12\text{bits} \)
- \( X_{\text{in}}(t) = .95\sin(2\pi f_{\text{SIG}} t) \) (-.4455dB)

Thus

- \( N_{P1} = 23.1 \)
- \( \theta_{\text{SR}} = 9.957 \)
- \( N_{P2} = 23.1 \)
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab

![Graph showing frequency response with a peak at 0 Hz and decreasing magnitude with increasing frequency.]
Spectral Characteristics of DAC

Consider the following example

- $f_{\text{SIG}}=50\text{Hz}$
- $k_1=230$
- $k_2=23$
- $N_p=1$
- $n_{\text{res}}=12\text{bits}$
- $X_{\text{in}}(t) = 0.88\sin(2\pi f_{\text{SIG}} t) + 0.1\sin(2\pi f_{\text{SIG}} t)$
- (-1.11db fundamental, -20dB 2nd harmonic)

Thus

- $N_{p1}=23$
- $\theta_{SR}=10$
- $N_{p2}=23$
DFT Simulation from Matlab

![DFT Simulation Graph](image)
DFT Simulation from Matlab
### DFT Simulation from Matlab

$$f_{SIG}=50\text{Hz} \quad k_1=230 \quad k_2=23 \quad N_p=1 \quad n_{res}=12\text{bits} \quad X_\text{in}(t)=.88\sin(2\pi f_{SIG}t)+0.1\sin(2\pi f_{SIG}t) \quad (-1.11\text{dB fundamental, -20dB 2nd harmonic})$$

- Columns 1 through 7
  - $-68.2448$ $-95.4048$ $-103.0624$ $-91.5534$ $-94.3099$ $-76.5052$ $-107.8586$

- Columns 8 through 14
  - $-98.3634$ $-107.7150$ $-70.4198$ $-97.2597$ $-91.1898$ $-103.5449$ $-95.7898$

- Columns 15 through 21

- Columns 22 through 28
  - $-98.3435$ $-107.5614$ $\text{boxed} -1.2534$ $-99.6919$ $-90.7685$ $-103.9860$ $-96.1429$

- Columns 29 through 35
### DFT Simulation from Matlab

Columns 36 through 42

\[
\begin{array}{cccccc}
-98.3035 & -107.3983 & -70.2715 & -101.8108 & -90.2829 & -104.3909 & -96.4685 \\
\end{array}
\]

Columns 43 through 49

\[
\begin{array}{cccccc}
\end{array}
\]

Columns 50 through 56

\[
\begin{array}{cccccc}
\end{array}
\]

Columns 57 through 63

\[
\begin{array}{cccccc}
-108.8537 & -93.4756 & -100.5602 & -92.5195 & -83.3389 & -86.2119 & -108.3343 \\
\end{array}
\]

Columns 64 through 70

\[
\begin{array}{cccccc}
-98.1627 & -107.0564 & -79.6196 & -105.4341 & -89.0818 & -105.1065 & \textbf{-82.5417} \\
\end{array}
\]
### DFT Simulation from Matlab

Columns 71 through 77


Columns 78 through 84


Columns 85 through 91


Columns 92 through 98

-97.9383  **-82.1713**  -83.8248  -108.1091  -87.4797  -105.7091  -97.4305

Columns 99 through 105