EE 435

Lecture 32
Quantization Noise in Data Converters
Spectral Characterization
Noise

We will define “Noise” to be the difference between the actual output and the desired output of a system.

Types of noise:

- Random noise due to movement of electrons in electronic circuits
- Interfering signals generated by other systems
- Interfering signals generated by a circuit or system itself
- Error signals associated with imperfect signal processing algorithms or circuits

Quantization noise is a significant component of this noise in ADCs and DACs and is present even if the ADC or DAC is ideal.
For large $n$, this periodic waveform behaves much like a random noise source that is uncorrelated with the input and can be characterized by its RMS value which can be obtained by integrating over any interval of length $T_1$. For notational convenience, shift the waveform by $T_1/2$ units.

$$E_{\text{RMS}} = \sqrt{\frac{1}{T_1} \int_{T_1 - T_1/2}^{T_1/2} \varepsilon_Q^2(t) dt}$$
Quantization Noise in ADC

\[ E_{\text{RMS}} = \frac{\chi_{\text{LSB}}}{\sqrt{12}} \]

The signal to quantization noise ratio (SNR) can now be determined. Since the input signal is a sawtooth waveform of period \( T \) and amplitude \( \chi_{\text{REF}} \), it follows by the same analysis that it has an RMS value of

\[ \chi_{\text{RMS}} = \frac{\chi_{\text{REF}}}{\sqrt{12}} \]

Thus the SNR is given by

\[ \text{SNR} = \frac{\chi_{\text{RMS}}}{E_{\text{RMS}}} = \frac{\chi_{\text{RMS}}}{\chi_{\text{LSB}}} = 2^n \]

or, in dB,

\[ \text{SNR}_{\text{dB}} = 20(n \cdot \log_2) = 6.02n \]

Note: dB subscript often neglected when not concerned about confusion
Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $X_{REF}$ centered at $X_{REF}/2$?

SNR $= 20(n \cdot \log2) = 6.02n$
Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $X_{\text{REF}}$ centered at $X_{\text{REF}}/2$?

![Graph of a sinusoidal input with time and amplitude quantization points]
Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $X_{\text{REF}}$ centered at $X_{\text{REF}}/2$?

Error waveform
Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $X_{\text{REF}}$ centered at $X_{\text{REF}}/2$?

For low $f_{\text{SIG}}/f_{\text{CL}}$ ratios, bounded by $\pm X_{\text{LB}}$ and at any point in time, behaves almost as if a uniformly distributed random variable

$$\varepsilon_Q \sim U[-0.5X_{\text{LSB}}, 0.5X_{\text{LSB}}]$$
Quantization Noise in ADC

Recall:

If the random variable $f$ is uniformly distributed in the interval $[A,B]$ then the mean and standard deviation of $f$ are given by

$$\mu_f = \frac{A+B}{2} \quad \sigma_f = \frac{B-A}{\sqrt{12}}$$

Theorem: If $n(t)$ is a random process, then for large $T$,

$$V_{RMS} = \sqrt{\frac{1}{T} \int_{t_1}^{t_1+T} n^2(t) \, dt} = \sqrt{\sigma_n^2 + \mu_n^2}$$
ENOB based upon Quantization Noise

$$\text{SNR} = 6.02 \, n + 1.76$$

Solving for $n$, obtain

$$\text{ENOB} = \frac{\text{SNR}_d - 1.76}{6.02}$$

Note: could have used the $\text{SNR}_d$ for a triangle input and would have obtained the expression

$$\text{ENOB} = \frac{\text{SNR}_d}{6.02}$$

But the earlier expression is more widely used when specifying the ENOB based upon the noise level present in a data converter.
## ENOB based upon Quantization Noise

\[
\text{SNR} = 6.02 \ n + 1.76
\]

<table>
<thead>
<tr>
<th>Res (n)</th>
<th>SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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<td>19.82</td>
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<tr>
<td>8</td>
<td>49.92</td>
</tr>
<tr>
<td>10</td>
<td>61.96</td>
</tr>
</tbody>
</table>

Table values in dB
Quantization Noise

Effects of quantization noise can be very significant, even at high resolution, when signals are not of maximum magnitude.

Quantization noise remains constant but signal level is reduced.

The desire to use a data converter at a small fraction of full range is one of the major reasons high resolution is required.
Quantization Noise

Effects of quantization noise can be very significant, even at high resolution, when signals are not of maximum magnitude

\[ x_{\text{REF}} \]

\[ x_{\text{IN}} \]

\[ t \]
Quantization Noise

Example: If a 14-bit audio output is derived from a DAC designed for providing an output of 100W but the normal listening level is at 50mW, what is the SNR due to quantization noise at maximum output and at the normal listening level? What is the ENOB of the audio system when operating at 50mW?
Quantization Noise

Example: If a 14-bit audio output is derived from a DAC designed for providing an output of 100W but the normal listening level is at 50mW, what is the SNR due to quantization noise at maximum output and at the normal listening level? What is the ENOB of the audio system when operating at 50mW?

At 100W

\[
\frac{V_{\text{MRMS}}^2}{R_L} = 100W
\]

\[
X_{\text{REF}} = 2\sqrt{2} \cdot V_{\text{MRMS}}
\]

SNR =
Quantization Noise

Example: If a 14-bit audio output is derived from a DAC designed for providing an output of 100W but the normal listening level is at 50mW, what is the SNR due to quantization noise at maximum output and at the normal listening level? What is the ENOB of the audio system when operating at 50mW?

At 100W

\[ \frac{V_{MRMS}^2}{R_L} = 100W \]

\[ X_{REF} = 2\sqrt{2} \cdot V_{MRMS} \]

\[ SNR = 6.02n + 1.76 = 90.6dB \]
Quantization Noise

Example: If a 14-bit audio output is derived from a DAC designed for providing an output of 100W but the normal listening level is at 50mW, what is the SNR due to quantization noise at maximum output and at the normal listening level? What is the ENOB of the audio system when operating at 50mW?

At 50mW
\[
\frac{V_1^2}{R_L} = 50\text{mW}
\]

Signal level has been reduced but noise level is same as before
\[
\frac{V_1}{V_M} = \sqrt{\frac{50\text{mW}}{100\text{W}}} = 0.0224
\]

\[
V_{1\text{dB}} = V_{M\text{dB}} - 33\text{dB}
\]

SNR =
Quantization Noise

Example: If a 14-bit audio output is derived from a DAC designed for providing an output of 100W but the normal listening level is at 50mW, what is the SNR due to quantization noise at maximum output and at the normal listening level? What is the ENOB of the audio system when operating at 50mW?

At 50mW
\[
\frac{V_1^2}{R_L} = 50\text{mW}
\]

SNR = 90.6dB - 33dB = 57.6dB

\[
X_{\text{REF}} = 2\sqrt{2} \cdot V_{\text{MRMS}}
\]

\[
\text{ENOB} = \frac{\text{SNR}_{\text{dB}} - 1.76}{6.02} = 9.3
\]
Quantization Noise

Example: If a 14-bit audio output is derived from a DAC designed for providing an output of 100W but the normal listening level is at 50mW, what is the SNR due to quantization noise at maximum output and at the normal listening level? What is the ENOB of the audio system when operating at 50mW?

At 100W output, $\text{SNR}=6.02n+1.76 = 90.6\text{dB}$

At 50mW output, SNR reduced by 33dB to 57.6dB

**ENOB = 9.3**

Note the dramatic reduction in the effective resolution of the DAC when operated at only a small fraction of full-scale.
ENOB Summary

Resolution:

\[ \text{ENOB} = \frac{\log_{10} N_{\text{ACT}}}{\log_{10} 2} = \log_2 N_{\text{ACT}} \]

INL:

\[ \text{ENOB} = n_R \cdot \log_2 (v) - 1 \]

- \( n_R \) specified res, \( v \) INL in LSB

\[ \text{ENOB} = -\log_2 (\text{INL}_{\text{REF}}) - 1 \]

- \( \text{INL}_{\text{REF}} \) INL rel to \( X_{\text{REF}} \)

DNL:

HW problem

Quantization noise:

\[ \text{ENOB} = \frac{\text{SNR}_{\text{dB}}}{6.02} \]

- rel to triangle/sawtooth

\[ \text{ENOB} = \frac{\text{SNR}_{\text{dB}} - 1.76}{6.02} \]

- rel to sinusoid
Performance Characterization of Data Converters

• Static characteristics
  – Resolution
  – Least Significant Bit (LSB)
  – Offset and Gain Errors
  – Absolute Accuracy
  – Relative Accuracy
  – Integral Nonlinearity (INL)
  – Differential Nonlinearity (DNL)
  – Monotonicity (DAC)
  – Missing Codes (ADC)
  – Quantization Noise
  – Low-f Spurious Free Dynamic Range (SFDR)
  – Low-f Total Harmonic Distortion (THD)
  – Effective Number of Bits (ENOB)
  – Power Dissipation
Absolute Accuracy

Absolute Accuracy is the difference between the actual output and the ideal or desired output of a data converter.

The ideal or desired output is in reference to an absolute standard (often maintained by the National Bureau of Standards) and could be volts, amps, time, weight, distance, or one of a large number of other physical quantities.

Absolute accuracy provides no tolerance to offset errors, gain errors, nonlinearity errors, quantization errors, or noise.

In many applications, absolute accuracy is not of a major concern.

but … scales, meters, etc. may be more concerned about Absolute accuracy than any other parameter.
Relative Accuracy

In the context of data converters, pseudo-static Relative Accuracy is the difference between the actual output and an appropriate fit-line to overall output of the data converter.

INL is often used as a measure of the relative accuracy.

In many, if not most, applications, relative accuracy is of much more concern than absolute accuracy.

Some architectures with good relative accuracy will have very small deviations in the outputs for closely-spaced inputs whereas others may have relatively large deviations in outputs for closely-spaced inputs.

DNL provides some measure of how outputs for closely-spaced inputs compare.
Considerations for Spectral Characterization

• Tool Validation
• FFT Length
• Importance of Satisfying Hypothesis
• Windowing
Question: How much noise is in the computational environment?

Observation: This noise is nearly uniformly distributed. The level of this noise at each component is around -310dB.
Question: How much noise is in the computational environment?

Assume $A_k = -310$ dB for $0 \leq k \leq N$

$$A_{kDB} = 20 \log_{10} A_k$$

$$A_k \approx 10^{\frac{-310}{20}} = 10^{-15.5}$$

$$V_{\text{Noise,RMS}} \approx \sqrt{\sum_{k=1}^{N-1} A_k^2} = \sqrt{N \overline{A}}$$

$$V_{\text{Noise,RMS}} \approx \sqrt{512} \ 10^{-15.5} = 1.8 \cdot 10^{-14} = 18 \text{ fV}$$

Note: This computational environment has a very low total computational noise and does not become significant until the 45-bit resolution level is reached!!
Considerations for Spectral Characterization

• Tool Validation
• FFT Length
• Importance of Satisfying Hypothesis
• Windowing
Spectral Response
Spectral Response

Rect. Window  N=4096  Np =50

Frequency

Mag(dB)
Considerations for Spectral Characterization

Number of Sampling Periods

- Number of sampling periods has minimal effect on FFT
- Good to use a prime number of periods

FFT Length

- FFT Length has minimal affect the computational noise floor

Quantization Noise

\[ E_{QUANT} = \frac{\chi_{\text{LSB}}}{\sqrt{12}} \]

- For most n of interest, quantization noise nearly uniformly distributed in all FFT bins
- FFT length reduces effects of quantization noise on all FFT bins
Considerations for Spectral Characterization

FFT Length

- Effect of FFT length on quantization noise floor coefficients

\[ E_{\text{QUANT}} \approx \sqrt{\frac{2^{n_{\text{DFT}}}}{\sum_{k=1}^{2^n} A_k^2}} \]

If we assume \( E_{\text{QUANT}} \) is fixed

If the \( A_k \)'s are constant and equal

\[ E_{\text{QUANT}} \approx A_k \frac{2^{n_{\text{DFT}}}}{2} \]

Solving for \( A_k \), obtain

\[ A_k \approx \frac{E_{\text{QUANT}}}{2^{n_{\text{DFT}}}/2} \]

If input is full-scale sinusoid with only amplitude quantization with n-bit res,

\[ E_{\text{QUANT}} \approx \frac{X_{\text{LSB}}}{\sqrt{12}} = \frac{X_{\text{REF}}}{\sqrt{3} \cdot 2^{n+1}} \]
Considerations for Spectral Characterization

FFT Length

\[ E_{QUANT} \approx \frac{X_{LSB}}{\sqrt{12}} = \frac{X_{REF}}{\sqrt{3 \cdot 2^{n+1}}} \]

Substituting for \( E_{QUANT} \), obtain

\[ A_k = \frac{X_{REF}}{\sqrt{3 \cdot 2^{n+1}2^{n_{DFT}/2}}} \]

This value for \( A_k \) thus decreases with the length of the DFT window.
THEOREM: If \( N_p \) is an integer and \( x(t) \) is band limited to \( f_{\text{MAX}} \), then

\[
|A_m| = \frac{2}{N} |X(mN_p + 1)| \quad 0 \leq m \leq h - 1
\]

and \( X(k) = 0 \) for all \( k \) not defined above

where \( \left< X(k) \right>_{k=0}^{N-1} \) is the DFT of the sequence \( \left< x(kT_s) \right>_{k=0}^{N-1} \)

\[
f = 1/T, \quad f_{\text{MAX}} = \frac{f}{2} \left\lceil \frac{N}{N_p} \right\rceil
\]
Considerations for Spectral Characterization

- Tool Validation
- FFT Length
- Importance of Satisfying Hypothesis
  - NP is an integer
  - Band-limited excitation
- Windowing
Example

WLOG assume $f_{SIG}=50\text{Hz}$

$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

Consider $N_P=20.2$ $N=4096$

Recall $20\log_{10}(0.5)=-6.0205999$
Recall this is the spectral response obtained if the hypothesis are satisfied

- Extreme accuracy in obtaining fundamental and all harmonics
- Computational noise floor down around -310dB in MATLAB
Input Waveform
Input Waveform
Input Waveform
Input Waveform
Spectral Response

Rect. Window N=4096 Np =20.2
<table>
<thead>
<tr>
<th>Columns 1 through 7</th>
<th>Columns 8 through 14</th>
<th>Columns 15 through 21</th>
<th>Columns 22 through 28</th>
<th>Columns 29 through 35</th>
</tr>
</thead>
<tbody>
<tr>
<td>-35.0366</td>
<td>-35.0125</td>
<td>-34.9400</td>
<td>-34.8182</td>
<td>-34.6458</td>
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<td>-34.4208</td>
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<tr>
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<td>-36.1838</td>
<td>-35.9965</td>
<td>-35.3255</td>
<td>-34.1946</td>
</tr>
</tbody>
</table>

Note there is a dramatic increase in the noise floor and a significant change in and spreading of the fundamental!!
$k^{th}$ harmonic will appear at position $1+k\cdot N_p$

Columns 36 through 42


Columns 43 through 49


Columns 50 through 56

-33.0833  -33.8720  -34.5759  -35.2113  -35.7902  -36.3218  -36.8133

Columns 57 through 63


Columns 64 through 70

The kth harmonic will appear at position 1+k•Np

Columns 36 through 42

\[-32.6350 \ -30.6397 \ -28.1125 \ -24.7689 \ -19.7626 \ -8.5639 \ -11.7825\]

Columns 43 through 49

\[-20.0158 \ -23.9648 \ -26.5412 \ -28.4370 \ -29.9279 \ -31.1519 \ -32.1874\]

Columns 50 through 56

\[-33.0833 \ -33.8720 \ -34.5759 \ -35.2113 \ -35.7902 \ -36.3218 \ -36.8133\]

Columns 57 through 63

\[-37.2703 \ -37.6974 \ -38.0984 \ -38.4762 \ -38.8336 \ -39.1725 \ -39.4949\]

Columns 64 through 70

\[-39.8024 \ -40.0963 \ -40.3778 \ -40.6479 \ -40.9076 \ -41.1576 \ -41.3987\]
Observations

• Modest change in sampling window of 0.2 out of 20 periods (1%) results in a big error in both fundamental and harmonic

• More importantly, dramatic raise in the noise floor !!! (from over -300dB to only -12dB)
Example

WLOG assume $f_{\text{SIG}} = 50\text{Hz}$

$$V_{\text{IN}} = \sin(\omega t) + 0.5 \sin(2\omega t)$$

$$\omega = 2\pi f_{\text{SIG}}$$

Consider $N_p = 20.01$ $N = 4096$

Deviation from hypothesis is .05% of the sampling window
Input Waveform
Input Waveform
Input Waveform
Input Waveform
Spectral Response
Fundamental will appear at position $1+Np = 21$

<table>
<thead>
<tr>
<th>Columns 1 through 7</th>
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<tbody>
<tr>
<td>-89.8679   -83.0583   -77.7239   -74.2607   -71.6830   -69.5948   -67.8044</td>
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<tbody>
<tr>
<td>-56.0866   -54.2966   -52.2035   -49.6015   -46.0326   -40.0441   -0.0007</td>
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<table>
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<tbody>
<tr>
<td>-62.2078   -65.1175   -69.1845   -76.9560   -81.1539   -69.6230   -64.0636</td>
</tr>
</tbody>
</table>

*Note: The last value (-0.0007) in the last column is highlighted for emphasis.*
$k^{th}$ harmonic will appear at position $1+k\cdot Np$

Columns 36 through 42

Observations

- Modest change in sampling window of 0.01 out of 20 periods (.05%) still results in a modest error in both fundamental and harmonic.
- More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -40dB).
- Errors at about the 6-bit level !
Example

WLOG assume $f_{\text{SIG}}=50\text{Hz}$

$$V_{\text{IN}} = \sin(\omega t) + 0.5 \sin(2\omega t)$$

$$\omega = 2\pi f_{\text{SIG}}$$

Consider $N_p=20.001$  $N=4096$

Deviation from hypothesis is .005% of the sampling window
Spectral Response

Rect. Window  N=4096  Np=20.001
Fundamental will appear at position $1+N_p = 21$

Columns 1 through 7

-112.2531 -103.4507 -97.8283 -94.3021 -91.7015 -89.6024 -87.8059

Columns 8 through 14

-86.2014 -84.7190 -83.3097 -81.9349 -80.5605 -79.1526 -77.6726

Columns 15 through 21

-76.0714 -74.2787 -72.1818 -69.5735 -65.9919 -59.9650 0.0001

Columns 22 through 28

-60.0947 -66.2917 -70.0681 -72.9207 -75.3402 -77.5767 -79.8121

Columns 29 through 35

-82.2405 -85.1651 -89.2710 -97.2462 -101.0487 -89.5195 -83.9851
\( k^{\text{th}} \) harmonic will appear at position \( 1 + k \cdot N_p \)

Columns 36 through 42

-79.8472  -76.1160  -72.2601  -67.6621  -60.7642  \boxed{-6.0220}  -59.3448

Columns 43 through 49

-64.8177  -67.8520  -69.9156  -71.4625  -72.6918  -73.7078  -74.5718

Columns 50 through 56

-75.3225  -75.9857  -76.5796  -77.1173  -77.6087  -78.0613  -78.4809

Columns 57 through 63

-78.8721  -79.2387  -79.5837  -79.9096  \boxed{-80.2186}  -80.5125  -80.7927
Observations

• Modest change in sampling window of 0.01 out of 20 periods (.005%) results in a small error in both fundamental and harmonic

• More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -60dB)

• Errors at about the 10-bit level !
Spectral Response
**Fundamental will appear at position $1+N_p = 21$**

Columns 1 through 7

-130.4427 -123.1634 -117.7467 -114.2649 -111.6804 -109.5888 -107.7965

Columns 8 through 14


Columns 15 through 21

-96.0691 -94.2764 -92.1793 -89.5706 -85.9878 -79.9571 0.0000

Columns 22 through 28


Columns 29 through 35

$k^{th}$ harmonic will appear at position $1+k\cdot N_p$

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<thead>
<tr>
<th>Columns 36 through 42</th>
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<tr>
<th>Columns 43 through 49</th>
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<tr>
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<td>-93.7098 -94.5736</td>
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<td>-98.0625 -98.4821</td>
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<table>
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</thead>
<tbody>
<tr>
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<td>-100.2197 -100.5135 -100.7937</td>
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</tbody>
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<thead>
<tr>
<th>Columns 64 through 70</th>
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Observations

• Modest change in sampling window of 0.001 out of 20 periods (.0005%) results in a small error in both fundamental and harmonic.

• More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -80dB).

• Errors at about the 13-bit level !
Observations

Extremely important to satisfy the hypothesis of an integral number of periods if using the DFT (or FFT) for spectral characterization

– This is relatively easy to do in a simulation environment

– Creates a real challenge in the laboratory
Considerations for Spectral Characterization

- Tool Validation
- FFT Length
- Importance of Satisfying Hypothesis
  - NP is an integer
  - Band-limited excitation
- Windowing
Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1. The sampling window be an integral number of periods

2. \[ N > \frac{2f_{\text{max}}}{f_{\text{SIGNAL}}} N_p \]
Example

If $f_{SIG}=50\text{Hz}$
and $N_p=20$  $N=512$

$$N > \frac{2f_{\text{max}}}{f_{\text{SIGNAL}}} N_p$$

$\Rightarrow f_{\text{max}} < 640\text{Hz}$
Example

Consider \( N_P=20 \) \( N=512 \)

If \( f_{\text{SIG}}=50\text{Hz} \)

\[
V_{\text{IN}} = \sin(\omega t) + 0.5 \sin(2\omega t) + 0.5 \sin(14\omega t)
\]

\[
\omega = 2\pi f_{\text{SIG}}
\]

(i.e. a component at 700 Hz which violates the band limit requirement)

Recall \( 20\log_{10}(0.5)=-6.0205999 \)
Effects of High-Frequency Spectral Components
Effects of High-Frequency Spectral Components
Effects of High-Frequency Spectral Components

Component at 700Hz appears to be at 580Hz!
### Effects of High-Frequency Spectral Components

\[ f_{\text{high}} = 14f_0 \]

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>-299.0778  -292.3045  -297.0529  -301.4639  -297.3332  -309.6947  -308.2308</td>
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<tbody>
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<td>-297.3710  -316.5113  -293.5661  -294.4045  -293.6881  -292.6872  -0.0000</td>
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<table>
<thead>
<tr>
<th>Columns 22 through 28</th>
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<tr>
<th>Columns 29 through 35</th>
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</table>
Effects of High-Frequency Spectral Components

\[ f_{\text{high}} = 14 f_0 \]

| Columns 36 through 42 |  
|----------------------|---|

| Columns 43 through 49 |  
|----------------------|---|
| -298.9215  | -309.4829  | -306.7363  | -293.0808  | -300.0882  | -306.5530  | -302.9962  |

| Columns 50 through 56 |  
|----------------------|---|
| -318.4706  | -294.8956  | -304.4663  | -300.8919  | -298.7732  | -301.2474  | -293.3188  |
Effects of High-Frequency Spectral Components

Aliased components at

$$f_{\text{alias}} = 2f_{\text{sample}} - f$$

$$f_{\text{alias}} = 2 \cdot 12.8f_{\text{sig}} - 14f_{\text{sig}} = 11.6f_{\text{sig}}$$

Thus, position in sequence

$$= 1 + N_p \frac{f_{\text{alias}}}{f_{\text{sig}}} = 1 + 20 \cdot 11.6 = 233$$

Columns 225 through 231

-296.8883 -292.8175 -295.8882 -286.7494 -300.3477 -284.4253 -282.7639

Columns 232 through 238


Columns 239 through 245

-299.1299 -305.8361 -295.1772 -295.1670 -300.2698 -293.6406 -304.2886

Columns 246 through 252

-302.0233 -306.6100 -297.7242 -305.4513 -300.4242 -298.1795 -299.0956
Effects of High-Frequency Spectral Components

Rect. Window  N=512  Np =20

f_{high}=24 \, f_{0}

Mag(dB)

Frequency
Effects of High-Frequency Spectral Components

Rect. Window N=512 Np =20

fhigh=25 fo

Mag(dB)

0 200 400 600 800 1000 1200

Frequency
Effects of High-Frequency Spectral Components
Effects of High-Frequency Spectral Components

Rect. Window N=512  Np =20

\( f_{\text{high}} = 24.4 f_0 \)
Effects of High-Frequency Spectral Components

Rect. Window  N=512  Np =20

$f_{\text{high}}=24.5f_0$
Considerations for Spectral Characterization

- Tool Validation
- FFT Length
  - Importance of Satisfying Hypothesis
    - NP is an integer
    - Band-limited excitation
- Windowing
Are there any strategies to address the problem of requiring precisely an integral number of periods to use the FFT?

Windowing is sometimes used

Windowing is sometimes misused
Windowing

Windowing is the weighting of the time domain function to maintain continuity at the end points of the sample window

Well-studied window functions:

- Rectangular
- Triangular
- Hamming
- Hanning
- Blackman
Rectangular Window

Sometimes termed a boxcar window

Uniform weight

Can append zeros

Without appending zeros equivalent to no window
Rectangular Window

Assume $f_{SIG} = 50\text{Hz}$

$$V_{IN} = \sin(\omega t) + 0.5\sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

Consider $N_P = 20.1$  $N = 512$
## Rectangular Window

### Columns 1 through 7


### Columns 8 through 14

-44.4065 -43.4052 -42.3602 -41.2670 -40.1146 -38.8851 -37.5520

### Columns 15 through 21

-36.0756 -34.3940 -32.4043 -29.9158 -26.5087 -20.9064 -0.1352

### Columns 22 through 28


### Columns 29 through 35

Rectangular Window

Columns 1 through 7

Columns 8 through 14
-44.4065  -43.4052  -42.3602  -41.2670  -40.1146  -38.8851  -37.5520

Columns 15 through 21
-36.0756  -34.3940  -32.4043  29.9158  -26.5087  -20.9064  -0.1352

Columns 22 through 28

Columns 29 through 35

Energy spread over several frequency components
Triangular Window
Triangular Window

Triangular Window \( N=512 \) \( N_p=20.1 \)
Triangular Window
## Triangular Window

Columns 1 through 7

\[-100.8530 \quad -72.0528 \quad -99.1401 \quad -68.0110 \quad -95.8741 \quad -63.9944 \quad -92.5170\]

Columns 8 through 14

\[-60.3216 \quad -88.7000 \quad -56.7717 \quad -85.8679 \quad -52.8256 \quad -82.1689 \quad -48.3134\]

Columns 15 through 21

\[-77.0594 \quad -42.4247 \quad -70.3128 \quad -33.7318 \quad -58.8762 \quad -15.7333 \quad -6.0918\]

Columns 22 through 28

\[-12.2463 \quad -57.0917 \quad -32.5077 \quad -68.9492 \quad -41.3993 \quad -74.6234 \quad -46.8037\]

Columns 29 through 35

\[-77.0686 \quad -50.1054 \quad -77.0980 \quad -51.5317 \quad -75.1218 \quad -50.8522 \quad -71.2410\]
Hamming Window
Hamming Window
Comparison with Rectangular Window
Hamming Window

Columns 1 through 7

-70.8278  -70.6955  -70.3703  -69.8555  -69.1502  -68.3632  -67.5133

Columns 8 through 14

-66.5945  -65.6321  -64.6276  -63.6635  -62.6204  -61.5590  -60.4199

Columns 15 through 21

-59.3204  -58.3582  -57.8735  -60.2994  -52.6273  -14.4702  -5.4343

Columns 22 through 28

-11.2659  -45.2190  -67.9926  -60.1662  -60.1710  -61.2796  -62.7277

Columns 29 through 35

-64.3642  -66.2048  -68.2460  -70.1835  -71.1529  -70.2800  -68.1145
Hanning Window
Hanning Window
Comparison with Rectangular Window
Hanning Window

<table>
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<table>
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<tbody>
<tr>
<td>-92.4519 -90.4372 -87.7977 -84.9554 -81.8956 -79.3520 -75.8944</td>
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<tr>
<td>-10.8267  -40.4480  -53.3906  -61.8561  -68.3601  -73.9966  -79.0757</td>
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<tbody>
<tr>
<td>-84.4318  -92.7280  -99.4046  -89.0799  -83.4211  -78.5955  -73.9788</td>
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</table>
Comparison of 4 windows
Comparison of 4 windows
Preliminary Observations about Windows

- Provide separation of spectral components
- Energy can be accumulated around spectral components
- Simple to apply
- Some windows work much better than others

But – windows do not provide dramatic improvement and …
Comparison of 4 windows when sampling hypothesis are satisfied
Comparison of 4 windows
Preliminary Observations about Windows

• Provide separation of spectral components
• Energy can be accumulated around spectral components
• Simple to apply
• Some windows work much better than others

But – windows do not provide dramatic improvement and can significantly degrade performance if sampling hypothesis are met
Issues of Concern for Spectral Analysis

An integral number of periods is critical for spectral analysis.

Not easy to satisfy this requirement in the laboratory.

Windowing can help but can hurt as well.

Out of band energy can be reflected back into bands of interest.

Characterization of CAD tool environment is essential.

Spectral Characterization of high-resolution data converters requires particularly critical consideration to avoid simulations or measurements from masking real performance.
Spectral Characterization

- Distortion Analysis
- Time Quantization Effects
- Spectral Characteristic of DAC
  - Time and Amplitude Quantization
Quantization Effects on Spectral Performance and Noise Floor in DFT

• Assume the effective clock rate (for either an ADC or a DAC) is arbitrarily fast

• Without Loss of Generality it will be assumed that $f_{\text{SIG}}=50\text{Hz}$

• Index on DFT will be listed in terms of frequency (rather than index number)

Matlab File: afft_Quantization.m
Quantization Effects

16,384 pts   res = 4bits   $N_p=25$

20 msec
Quantization Effects

16,384 pts  res = 4bits  \( N_p=25 \)

20 msec
Quantization Effects

16,384 pts   res = 4bits
Quantization Effects

Simulation environment:

\[ N_P = 23 \]
\[ f_{\text{SIG}} = 50 \text{Hz} \]
\[ V_{\text{REF}}: -1 \text{V}, 1 \text{V} \]
Res: will be varied
\[ N = 2^n \text{ will be varied} \]
Quantization Effects

Res = 4 bits

Rect. Window N=512  Np =23
Quantization Effects

Res = 4 bits

Rect. Window N=4096 Np=23

Axis of Symmetry
Quantization Effects

Res = 4 bits

Some components very small
Quantization Effects

Res = 4 bits

Set lower display limit at -120dB
Quantization Effects

Res = 4 bits
Quantization Effects
Res = 4 bits
Quantization Effects

Res = 4 bits

Rect. Window N=65536 Np =23

Mag(dB)

Frequency x 10^4
Quantization Effects

Res = 4 bits
Quantization Effects

Res = 4 bits

Rect. Window N=65536   Np =23
Quantization Effects

Res = 4 bits
Quantization Effects

Res = 4 bits

Fundamental
Quantization Effects

Res = 10 bits

Rect. Window  N=256  Np = 23
Quantization Effects
Res = 10 bits
Quantization Effects

Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits

Rect. Window N=4096   Np =23

Mag(dB)

Frequency
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits

Rectangular Window

Columns 1 through 5

-55.7419 -120.0000 -85.1461 -106.1614 -89.2395

Columns 6 through 10

-102.3822 -99.5653 -85.7335 -89.1227 -83.0851
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<th>Columns 31 through 35</th>
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<tr>
<td>-93.0155  -82.1062  -78.4561  -98.7568  -109.4766</td>
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</tbody>
</table>
### Columns 36 through 40
-98.2999  -84.9383  -115.7328  -100.0758  -77.1246

### Columns 41 through 45
-86.6455  -82.5379  -98.8707  -111.1638  -85.9572

### Columns 46 through 50
-85.7575  -92.6227  -83.7312  -83.4865  -82.4473

### Columns 51 through 55
-77.4085  -88.0611  -84.5256  -98.4813  -82.7990

### Columns 56 through 60
-86.0396  -83.8284  -87.2621  -97.6189  -94.7694
Columns 61 through 65

-86.9239  -89.5881  -82.8701  -95.5137  -82.3502

Columns 66 through 70

-74.9482  -83.4468  -94.0629  -95.3199  -95.4482

Columns 71 through 75

-107.0215  -98.3102  -87.4623  -82.4935  -98.6972

Columns 76 through 80

-83.1902  -82.2598  -103.0396  -87.2043  -79.1829

Columns 81 through 85

-76.6723  -87.0770  -91.5964  -82.1222  -78.7656
Columns 86 through 90
-82.9621  -93.0224  -116.8549  -93.7327  -75.6231

Columns 91 through 92
-94.4914  -81.0819
Rectangular Window

Columns 1 through 5

-55.6060  -97.9951  -107.4593  -103.4508  -120.0000

Columns 6 through 10

-96.7808  -105.2905  -96.7395  -104.5281  -90.7582
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<td>Columns 41 through 45</td>
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<td>Columns 46 through 50</td>
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<tbody>
<tr>
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</table>
Quantization Effects

Res = 10 bits

With Vin=2v pp
With $V_{in}=1^{*}.99$ and $V_{os}=.25$ LSB
With Vin = 1.999999 pp
With $V_{in}=1.99$ and $V_{os}=.35$ LSB
Res 10   No. points 4096   fsig= 50.00   No. Periods 25.00 Tstep 1.220703e-004
Magnitude of   Fundamental 1.000     2nd Harmonic  0.000

Columns 1 through 7

Columns 8 through 14

Columns 15 through 21

Columns 22 through 28

Columns 29 through 35
Quantization Effects

Res = 10 bits
Quantization Effects

Res = 10 bits
Quantization Effects

Res = 10 bits
Quantization Effects

Res = 10 bits
Quantization Effects

Res = 5 bits
Quantization Effects

Res = 4 bits

Rect. Window N=512 Np = 25
Quantization Effects

Res = 4 bits
Quantization Effects

16,384 pts  res = 4bits
Quantization Effects

16,384 pts   res = 4bits
Quantization Effects

Res = 10 bits
Distortion Analysis

Time Quantization Effects

Spectral Characteristic of DAC
  – Time and Amplitude Quantization
Spectral Characteristics of DACs and ADCs
Spectral Characteristics of DAC

Periodic Input Signal

Sampling Clock

Sampled Input Signal (showing time points where samples taken)
Spectral Characteristics of DAC

Quantized Sampled Input Signal (with zero-order sample and hold)
Spectral Characteristics of DAC

$T_{DFT \ WINDOW}$

$T_{PERIOD}$

$T_{SIG}$

$T_{CLOCK}$

Sampling Clock

$T_{DFT \ CLOCK}$

DFT Clock
Spectral Characteristics of DAC
Spectral Characteristics of DAC

Sampling Clock

DFT Clock
Spectral Characteristics of DAC

Sampled Quantized Signal (zoomed)

DFT Clock

Sampling Clock
Spectral Characteristics of DAC

Consider the following example

- $f_{\text{SIG}}=50\text{Hz}$
- $k_1=230$
- $k_2=23$
- $N_P=1$
- $n_{\text{res}}=8\text{bits}$
- $X_{\text{in}}(t) = 0.95\sin(2\pi f_{\text{SIG}}t)$ (-0.4455dB)

Thus

- $N_{P_1}=23$
- $\theta_{\text{SR}}=5$
- $f_{\text{CL}}/f_{\text{SIG}}=10$

Matlab File: afft_Quantization_DAC.m
DFT Simulation from Matlab

\[ n_{\text{sam}} = 142.4696 \]
DFT Simulation from Matlab

Expanded View

Width of this region is $f_{CL}$

Analogous to the overall DFT window when directly sampled but modestly asymmetric

$n_{sam} = 142.4696$
DFT Simulation from Matlab

Expanded View

Rect. Window  N=32768  Np =1

$n_{\text{res}}=8$ bits

$n_{\text{sam}} = 142.4696$
DFT Simulation from Matlab

Rect. Window N=32768 Np =1

n_{sam} = 142.4696
DFT Simulation from Matlab

Expanded View

Rect. Window  N=32768  Np =1

nres=8 bits

\( n_{\text{sam}} = 142.4696 \)
\[ f_{\text{SIG}}=50\text{Hz} , \; k_1=23, \; k_2=23, \; N_P=1, \; n_{\text{res}}=8\text{bits} \quad \text{Xin}(t) = \sin(2\pi f_{\text{SIG}} t) \]

\[ N=32768 \]

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<tr>
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<tbody>
<tr>
<td>-120.0000 -81.5409 -109.6386 -89.7275 -120.0000 -81.8340 -120.0000</td>
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</table>
$f_{\text{SIG}} = 50\text{Hz}$, $k_1 = 23$, $k_2 = 23$, $N_P = 1$, $n_{\text{res}} = 8\text{bits}$  
$X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t)$  
$N = 32768$

Columns 36 through 42

-90.2331 -120.0000 -69.4356 -120.0000 -88.1400 -120.0000 -86.7214

Columns 43 through 49

-120.0000 -79.6273 -119.1428 -89.9175 -56.7024 -83.0511 -120.0000

Columns 50 through 56

-90.1331 -120.0000 -75.1821 -120.0000 -87.5706 -120.0000 -87.3205

Columns 57 through 63

-120.0000 -76.9769 -120.0000 -90.0703 -119.0588 -83.2950 -113.3964

Columns 64 through 70

-89.9982 -120.0000 -78.4288 -120.0000 -87.0328 -120.0000 -64.5409
\[ f_{\text{SIG}} = 50\text{Hz} , \quad k_1 = 23 , \quad k_2 = 23 , \quad N_P = 1 , \quad n_{\text{res}} = 8\text{bits} \quad \text{Xin}(t) = \sin(2\pi f_{\text{SIG}} t) \]

\[ \text{N=32768} \]

Columns 71 through 77

\[-120.0000 \quad -72.8111 \quad -120.0000 \quad -90.1876 \quad -120.0000 \quad -82.5616 \quad -114.0867 \]

Columns 78 through 84

\[-89.8269 \quad -115.6476 \quad -80.6553 \quad -120.0000 \quad -86.3818 \quad -120.0000 \quad -88.3454 \]

Columns 85 through 91

\[-120.0000 \quad -63.5207 \quad -120.0000 \quad -90.2704 \quad -120.0000 \quad -80.8524 \quad -120.0000 \]

Columns 92 through 98

\[-89.6174 \quad -58.5435 \quad -82.3253 \quad -120.0000 \quad -85.6188 \quad -120.0000 \quad -88.7339 \]

Columns 99 through 100

\[-120.0000 \quad -63.8165 \]
DFT Simulation from Matlab

Rect. Window N=131072 Np =1

nres=8 bits

\[ n_{\text{sam}} = 569.8783 \]
DFT Simulation from Matlab

Expanded View

Rect. Window N=131072  Np =1

nres=8 bits

n_{sam} = 569.8783
DFT Simulation from Matlab

Expanded View

Rect. Window N=131072 Np =1

nres=8 bits

n_{\text{sam}} = 569.8783
DFT Simulation from Matlab

Expanded View

Rect. Window  N=131072  Np =1

nres=8 bits

nsam =  569.8783
\[ f_{SIG} = 50\text{Hz} , \ k_1 = 23, \ k_2 = 23, \ N_P = 1, \ n_{res} = 8\text{bits} \quad X_{in}(t) = \sin(2\pi f_{SIG}t) \]
\[ N = 131072 \]

<table>
<thead>
<tr>
<th>Columns 1 through 7</th>
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<tbody>
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<td>-44.0824 -97.0071 -120.0000 -110.6841 -120.0000 -76.0276 -120.0000</td>
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</tbody>
</table>
\[ f_{SIG} = 50 \text{Hz}, \ k_1 = 23, \ k_2 = 23, \ N_P = 1, \ n_{res} = 8\text{bits} \quad X_{in}(t) = \sin(2\pi f_{SIG} t) \]

\[ N = 131072 \]

Columns 36 through 42

\[-102.9185 -120.0000 -109.9276 -120.0000 -88.8778 -120.0000 -107.5734\]

Columns 43 through 49

\[-120.0000 -108.1493 -120.0000 -90.7672 -56.7029 -109.3748 -120.0000\]

Columns 50 through 56

\[-104.5924 -120.0000 -75.3784 -120.0000 -110.5416 -120.0000 -99.0764\]

Columns 57 through 63

\[-120.0000 -94.4432 -120.0000 -110.7692 -120.0000 -86.1442 -120.0000\]

Columns 64 through 70

\[-102.2661 -120.0000 -110.0806 -120.0000 -87.7635 -120.0000 -64.4072\]
\[ f_{\text{SIG}}=50\text{Hz} , \quad k_1=23, \quad k_2=23, \quad N_p=1, \quad n_{\text{res}}=8\text{bits} \quad X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}}t) \]

\[ N=131072 \]

Columns 71 through 77

-120.0000 -108.4202 -120.0000 -91.0476 -120.0000 -109.1589 -120.0000

Columns 78 through 84

-105.0508 -120.0000 -81.0390 -120.0000 -110.4486 -120.0000 -99.9756

Columns 85 through 91

-120.0000 -92.8919 -120.0000 -110.7904 -120.0000 -88.9028 -120.0000

Columns 92 through 98

-101.5617 \textcolor{red}{-58.5437} -110.2183 -120.0000 -86.2629 -120.0000 -105.5980

Columns 99 through 100

-120.0000 -108.6808
Spectral Characteristics of DAC

Consider the following example

- $f_{\text{SIG}}=50\text{Hz}$
- $k_1=50$
- $k_2=5$
- $N_P=2$
- $n_{\text{res}}=8\text{bits}$
- $X_{\text{in}(t)} = .95\sin(2\pi f_{\text{SIG}}t)$ ($-0.4455\text{dB}$)

Thus

- $N_{P1}=5$
- $\theta_{SR}=5$
- $N_{P2}=10$
DFT Simulation from Matlab

Rect. Window N=32768  Np =2

nsam = 327,680

n_res = 8
DFT Simulation from Matlab

Expanded View

Rect. Window N=32768  Np =2

$n_{\text{sam}} = 327.6800$

$n_{\text{res}} = 8$
DFT Simulation from Matlab

Rect. Window N=256  Np =2

\[ n_{\text{sam}} = 2.5600 \]

\[ n_{\text{res}} = 8 \]
DFT Simulation from Matlab

Expanded View

Rect. Window N=256 Np =2

\[ n_{res} = 8 \]
\( f_{\text{SIG}}=50\text{Hz}, \ k_1=50, \ k_2=5, \ N_P=2, \ n_{\text{res}}=8\text{bits}, \ Xin(t)=\sin(2\pi f_{\text{SIG}}t) \)

\[ N=131072 \]

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<tr>
<th>Columns 1 through 7</th>
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<tbody>
<tr>
<td>-44.1164 -120.0000 -36.9868 -120.0000 -74.6451 -120.0000 -50.4484</td>
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<tr>
<td>-78.0140 -120.0000 -47.7412 -120.0000 -85.9233 -120.0000 -27.8207</td>
</tr>
</tbody>
</table>
\( f_{\text{SIG}} = 50 \text{Hz}, \ k_1 = 50, \ k_2 = 5, \ N_P = 2, \ n_{\text{res}} = 8 \text{bits}, \ Xin(t) = \sin(2\pi f_{\text{SIG}} t) \)

\[ N = 131072 \]

Columns 36 through 42

\[-120.0000 \ -75.9471 \ -120.0000 \ -49.8914 \ -120.0000 \ -58.4761 \ -120.0000\]

Columns 43 through 49

\[-41.7535 \ -120.0000 \ -91.4791 \ -120.0000 \ -28.1314 \ -120.0000 \ -79.7024\]

Columns 50 through 56

\[-120.0000 \ -50.5858 \ -120.0000 \ -78.7241 \ -120.0000 \ -31.9459 \ -120.0000\]

Columns 57 through 63

\[-91.9095 \ -120.0000 \ -40.4010 \ -120.0000 \ -62.1214 \ -120.0000 \ -50.1249\]

Columns 64 through 70

\[-120.0000 \ -78.2678 \ -120.0000 \ -24.9258 \ -120.0000 \ -87.6235 \ -120.0000\]
\[ f_{\text{SIG}} = 50\text{Hz}, \quad k_1 = 50, \quad k_2 = 5, \quad N_P = 2, \quad n_{\text{res}} = 8\text{bits}, \quad X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t) \]
\[ N = 131072 \]

<table>
<thead>
<tr>
<th>Columns 71 through 77</th>
</tr>
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<tbody>
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<thead>
<tr>
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<thead>
<tr>
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<tbody>
<tr>
<td>-38.8332 -120.0000 -92.1633 -120.0000 -34.7560 -120.0000 -77.1229</td>
</tr>
</tbody>
</table>
DFT Simulation from Matlab

Rect. Window N=1024 Np =2

\[ n_{\text{sam}} = 10.2400 \]

\[ n_{\text{res}} = 8 \]
DFT Simulation from Matlab
Expanded View

Rect. Window N=1024   Np =2

\[ \text{nsam} = 10.2400 \]
\[ n_{\text{res}} = 8 \]
\[ f_{\text{SIG}} = 50\text{Hz, } k_1 = 50, k_2 = 5, N_P = 2, n_{\text{res}} = 8\text{bits, } X_{\text{in}(t)} = \sin(2\pi f_{\text{SIG}} t) \]

\[ N = 1024 \]

<table>
<thead>
<tr>
<th>Columns 1 through 7</th>
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<tbody>
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<td>-44.0739 -120.0000 -53.8586 -120.0000 -91.9997 -120.0000 -50.3884</td>
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<th>Columns 22 through 28</th>
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<tbody>
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<thead>
<tr>
<th>Columns 29 through 35</th>
</tr>
</thead>
<tbody>
<tr>
<td>-98.4283 -120.0000 -51.2204 -120.0000 -92.1630 -120.0000 -39.9145</td>
</tr>
</tbody>
</table>
\[ f_{\text{SIG}} = 50\text{Hz}, \quad k_1 = 50, \quad k_2 = 5, \quad N_P = 2, \quad n_{\text{res}} = 8\text{bits}, \quad X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t) \]

\[ N = 1024 \]

<table>
<thead>
<tr>
<th>Columns 36 through 42</th>
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</thead>
<tbody>
<tr>
<td>-120.0000 -86.0994 -120.0000 -46.4571 -120.0000</td>
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<thead>
<tr>
<th>Columns 43 through 49</th>
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<tbody>
<tr>
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<thead>
<tr>
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<tr>
<td>-120.0000 -54.2124 -120.0000 -101.8321 -120.0000 -52.6742 -120.0000</td>
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<tr>
<td>-89.3186 -120.0000 -45.3675 -120.0000</td>
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<thead>
<tr>
<th>Columns 64 through 70</th>
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<tbody>
<tr>
<td>-120.0000 -85.3723 -120.0000 -40.6886 -120.0000 -92.0718 -120.0000</td>
</tr>
</tbody>
</table>
\[ f_{\text{SIG}} = 50 \text{Hz}, \quad k_1 = 50, \quad k_2 = 5, \quad N_P = 2, \quad n_{\text{res}} = 8 \text{bits}, \quad X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}}t) \]

\[ N = 1024 \]

Columns 71 through 77

\[-51.9029 \quad -120.0000 \quad -98.8650 \quad -120.0000 \quad -54.1376 \quad -120.0000 \quad -103.6450\]

Columns 78 through 84

\[-120.0000 \quad -53.3554 \quad -120.0000 \quad -68.6244 \quad -120.0000 \quad -48.3107 \quad -120.0000\]

Columns 85 through 91

\[-85.8692 \quad -120.0000 \quad -41.9049 \quad -120.0000 \quad -89.7301 \quad -120.0000 \quad -19.6301\]

Columns 92 through 98

\[-120.0000 \quad -91.5501 \quad -120.0000 \quad -50.5392 \quad -120.0000 \quad -92.8884 \quad -120.0000\]

Columns 99 through 105

\[-53.8928 \quad -120.0000 \quad -104.2832 \quad -120.0000 \quad -53.8225 \quad -120.0000 \quad -91.0209\]
Spectral Characteristics of DAC

Consider the following example

- \( f_{\text{SIG}} = 50\text{Hz} \)
- \( k_1 = 11 \)
- \( k_2 = 1 \)
- \( N_P = 2 \)
- \( n_{\text{res}} = 12\text{bits} \)
- \( X_{\text{in}}(t) = .95\sin(2\pi f_{\text{SIG}}t) \ (\text{-.4455dB}) \)

Thus

- \( N_{P1} = 1 \)
- \( \theta_{SR} = 11 \)
- \( N_{P2} = 2 \)
DFT Simulation from Matlab

Rec Win N=4096 Np =2 Nsam = 186.181818 nres = 12 fCL/fsig = 11 fDFT/fsig = 2048

Frequency

Mag(dB)
DFT Simulation from Matlab

Rec Win  N=4096 Np =2 Nsam = 186.181818 nres = 12 fCL/fsig = 11 fDFT/fsig = 2048

![Graph showing magnitude in decibels versus frequency with specific parameters.]
DFT Simulation from Matlab

![Graph showing frequency domain analysis with labels: Rec Win N=4096 Np =2 Nsam = 186.181818 nres = 12 fCL/fsig = 11 fDFT/fsig = 2048.](image)
DFT Simulation from Matlab
DFT Simulation from Matlab

![Graph showing frequency response with labels Rec Win, N=65536, Np=2, Nsam=2978.90909, nres=12, fCL/fsig=11, fDFT/fsig=32768. The graph plots magnitude in dB against frequency.](image-url)
DFT Simulation from Matlab
Spectral Characteristics of DAC

Consider the following example

- \( f_{\text{SIG}} = 50\text{Hz} \)
- \( k_1 = 230 \)
- \( k_2 = 23 \)
- \( N_P = 1 \)
- \( n_{\text{res}} = 12\text{bits} \)
- \( X_{\text{in}}(t) = 0.95\sin(2\pi f_{\text{SIG}}t) (-0.4455\text{dB}) \)

Thus

- \( N_{P1} = 23 \)
- \( \theta_{SR} = 10 \)
- \( N_{P2} = 23 \)
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab

\[ f_{\text{SIG}}=50\text{Hz} \quad k_1=230 \quad k_2=23 \quad N_\text{p}=1 \quad n_{\text{res}}=12\text{bits} \quad X_{\text{in}}(t) = .95\sin(2\pi f_{\text{SIG}} t) \quad (-.4455\text{dB}) \quad N_{p_1}=23 \quad \theta_{\text{SR}}=10 \quad N_{p_2}=23 \]

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<thead>
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<th>Columns 1 through 7</th>
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<tr>
<td>-68.1646 -94.7298 -120.0000 -90.8893 -120.0000 -75.8402 -120.0000</td>
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</table>
## DFT Simulation from Matlab

Columns 36 through 42

-97.6535 -120.0000 -69.6068 -120.0000 -89.6188 -120.0000 -95.7721

Columns 43 through 49

-120.0000 -93.3545 -120.0000 -91.6806 -80.7859 -83.9353 -120.0000

Columns 50 through 56

-97.5940 -120.0000 -75.5346 -120.0000 -89.0602 -120.0000 -96.0458

Columns 57 through 63

-120.0000 -92.8067 -120.0000 -91.8555 -85.5462 -120.0000

Columns 64 through 70

-97.5144 -120.0000 -78.9551 -120.0000 -88.4176 -120.0000 -88.0509
DFT Simulation from Matlab

Columns 71 through 77
-120.0000 -92.2056 -120.0000 -91.9896 -120.0000 -86.9037 -120.0000

Columns 78 through 84
-97.4143 -120.0000 -81.3430 -120.0000 -87.6762 -120.0000 -96.6112

Columns 85 through 91
-120.0000 -91.5441 -120.0000 -92.0844 -120.0000 -88.0732 -120.0000

Columns 92 through 98
-97.2936 82.6264 -82.6264 -83.1604 -120.0000 -86.8155 -120.0000 -96.8068

Columns 99 through 100
-120.0000 -90.8133
Spectral Characteristics of DAC

Consider the following example

- \( f_{\text{SIG}} = 50\text{Hz} \)
- \( k_1 = 230 \)
- \( k_2 = 23.1 \)
- \( N_P = 1 \)
- \( n_{\text{res}} = 12\text{bits} \)
- \( X_{\text{in}}(t) = .95\sin(2\pi f_{\text{SIG}} t) \) \((-0.4455\text{dB})\)

Thus

- \( N_{P1} = 23.1 \)
- \( \theta_{SR} = 9.957 \)
- \( N_{P2} = 23.1 \)
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab
Spectral Characteristics of DAC

Consider the following example

- \( f_{\text{SIG}} = 50 \text{Hz} \)
- \( k_1 = 230 \)
- \( k_2 = 23 \)
- \( N_P = 1 \)
- \( n_{\text{res}} = 12 \text{bits} \)
- \( X_{\text{in}}(t) = 0.88 \sin(2\pi f_{\text{SIG}}t) + 0.1 \sin(2\pi f_{\text{SIG}}t) \)
- (-1.11db fundamental, -20dB 2\(^{nd}\) harmonic)

Thus

- \( N_{P1} = 23 \)
- \( \theta_{\text{SR}} = 10 \)
- \( N_{P2} = 23 \)
DFT Simulation from Matlab

![Graph of DFT Simulation](image)
DFT Simulation from Matlab
### DFT Simulation from Matlab

\[ f_{\text{SIG}} = 50\text{Hz} \quad k_1 = 230 \quad k_2 = 23 \quad N_p = 1 \quad n_{\text{res}} = 12\text{bits} \quad X(t) = 0.88\sin(2\pi f_{\text{SIG}}t) + 0.1\sin(2\pi f_{\text{SIG}}t) \quad (-1.11\text{dB fundamental, -20dB 2nd harmonic}) \]

\[ N_{p_1} = 23 \quad \theta_{\text{SR}} = 10 \quad N_{p_2} = 23 \]

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DFT Simulation from Matlab

Columns 36 through 42

-98.3035 -107.3983  -70.2715 -101.8108  -90.2829 -104.3909  -96.4685

Columns 43 through 49


Columns 50 through 56

-98.2433 -107.2276  -76.1993 -103.7144  -89.7244 -104.7634  -96.7781

Columns 57 through 63

-108.8537  -93.4756 -100.5602  -92.5195  -83.3389  -86.2119  -108.3343

Columns 64 through 70

-98.1627 -107.0564  -79.6196 -105.4341  -89.0818 -105.1065  -82.5417
## DFT Simulation from Matlab

Columns 71 through 77


Columns 78 through 84


Columns 85 through 91


Columns 92 through 98

|  -97.9383 | **82.1713** |  -83.8248 |  -108.1091 |  -87.4797 |  -105.7091 |    -97.4305 |

Columns 99 through 105