Switched-Capacitor Filters and Amplifiers
Review from last time

Observation:

• The integrator is the key building block in most filters
• Accuracy of $I_0$ and $\alpha$ is important!
Challenges in Integrated Filter Design

• Accuracy of components is not good enough (orders of magnitude)
• Area too large for audio frequencies (orders of magnitude)
Challenges in Integrated Filter / Integrated Integrator Design

- Accuracy of R and C difficult to accurately control – particularly in integrated applications
- Size of R and C unacceptably large if $I_0$ is in audio frequency range
- Amplifier GB limits performance

\[ T(s) = -\frac{1}{RCs} \]
\[ I_0 = \frac{1}{RC} \]
Consider the following circuit

\[ Q_{RC} = - \int_{t_1}^{t_1+T} i(t) \, dt = - \frac{1}{R} \int_{t=t_1}^{t_1+T} V_{IN}(t) \, dt \]

\[ V_{OUT}(t_1+T) = V_{OUT}(t_1) - \frac{Q_{RC}}{C} \]

If \( T << T_{SIG} \), \( V_{IN}(t) \approx V_{IN}(t_1) \) for \( t_1 < t < t_1 + T \)
Consider the following circuit

If $T \ll T_{SIG}$

\[
Q_{RC} = - \int_{t=t_1}^{t_1+T} i(t)\,dt = -\frac{1}{R} \int_{t=t_1}^{t_1+T} V_{IN}(t)\,dt = -\frac{V_{IN}(t_1)}{R} \int_{t=t_1}^{t_1+T} 1\,dt = -\frac{V_{IN}(t_1)}{R} T
\]

\[
V_{OUT}(t_1+T) = V_{OUT}(t_1) - \frac{Q_{RC}}{C} = V_{OUT}(t_1) - \frac{V_{IN}(t_1)}{RC} T
\]
Consider the following circuit

If $T << T_{SIG}$

$$Q_{SC} = -C_1 \cdot v_{IN}(t_1)$$

$$v_{OUT}(t_1+T) = v_{OUT}(t_1) - \frac{Q_{SC}}{C} \approx v_{OUT}(t_1) - v_{IN}(t_1) \frac{C_1}{C}$$
Consider the following two circuits

Both transfer charge proportional to $V_{IN}(t_1)$
Consider the following two circuits

\[ Q_{RC} = -\frac{v_{IN}(t_1)}{R}T \]

\[ Q_{SC} = -C_1 \cdot v_{IN}(t_1) \]

Thus equating charges

\[ Q_{RC} = Q_{SC} \]

Obtain the equivalent resistance of the switched-capacitor circuit

\[ R_{EQ} = \frac{T}{C_1} \]
Thus, if \textbf{clocked} with a clock period of T, for $T \ll T_{SIG}$, we have the following equivalence:

$$R_{EQ} \approx \frac{1}{C_1}$$

Equivalence of a switched capacitor and a resistor has been known for over 100 years. Up until 1977, this knowledge was of little practical significance.
The switched-capacitor integrator

But what about the challenges of integrating the active RC integrator?

- Accuracy of R and C difficult to accurately control – particularly in integrated applications
- Size of R and C unacceptably large if $I_0$ is in audio frequency range
- Amplifier GB limits performance

Large resistors require small capacitors (since $C_1$ appears in denominator)!
The switched-capacitor integrator

- Capacitor ratios can be maintained to the 0.1% accuracy level or better
- Very accurate clocks can be inexpensively generated
- GB requirements for Op Amp actually relaxed with SC integrator compared to active RC integrator
The switched-capacitor integrator

![Switched-capacitor integrator diagram]

But what about the challenges of integrating the active RC integrator?

- Accuracy of R and C difficult to accurately control – particularly in integrated applications
- Size of R and C unacceptably large if \( I_0 \) is in audio frequency range
- Amplifier GB limits performance

Switched-Capacitor Integrators offers orders of magnitude improvement in first two major challenges!
The switched-capacitor integrator

\[ I_0 = \frac{1}{R_{EQ}C} \rightarrow I_0 = T \cdot \left( \frac{C_1}{C} \right) \rightarrow I_0 = \frac{1}{f_{CLK} \left( \frac{C_1}{C} \right)} \]

\[ R_{EQ} \approx \frac{T}{C_1} \]
The switched-capacitor (SC) integrator

\[ i_0 = \frac{1}{f_{CLK}} \left( \frac{C_1}{C} \right) \]

Non-overlap of clocks is critical in SC circuits
Switched-Capacitor Filter

\[ T_{CLK} \ll T_{SIG} \]

\[ - \]

\[ T_{SIG} \]

\[ V_{IN} \]
\[ \varphi_1 \]
\[ \varphi_2 \]
\[ C \]
\[ C_1 \]
\[ V_{OUT} \]

\[ \varphi_1 \]
\[ \varphi_2 \]

\[ t_1 \]
\[ t_1 + T_{CLK} \]
Basic Building Blocks in Both Cascaded Biquads and Multiple Feedback Structures

- Developed from observations from feedback implementations
  1. Integrators
  2. Summers
  3. First-order filter blocks
  4. Biquads
  5. Switches

- Same building blocks used in open-loop applications as well
Switched-Capacitor Filters

What if $T_{CLK}$ is not much-much smaller than $T_{SIG}$?

For $T_{CLK} \ll T_{SIG}$

![Diagram showing waveforms and time intervals $T_{CLK}$ and $T_{SIG}$]
Switched-Capacitor Filters

What if $T_{CLK}$ is not much-much smaller than $T_{SIG}$?

For $T_{CLK} \ll T_{SIG}$
Switched-Capacitor Filters

What if $T_{CLK}$ is not much much smaller than $T_{SIG}$?

For $T_{CLK} \ll T_{SIG}$

$V(nT)$

$V((n+1)T)$

$\varphi_1$

$\varphi_2$

$nT_{CLK}$

$(n+1)T_{CLK}$

Define

$T = T_{CLK}$
Switched-Capacitor Filters

What if $T_{CLK}$ is not much-much smaller than $T_{SIG}$?

For $T_{CLK} \ll T_{SIG}$

\[ V(nT) \rightarrow V((n+1)T) \]

Considerable change in $V(t)$ in clock period

For $T_{CLK} < T_{SIG}$

Considerable change in $V(t)$ in clock period
Switched-Capacitor Filters

What if $T_{CLK}$ is not much much smaller than $T_{SIG}$?

For $T_{CLK} < T_{SIG}$

- $T_{CLK}$
- $T_{SIG}$
- $\phi_1$
- $\phi_2$
Switched-Capacitor Filters

What if $T_{CLK}$ is not much much smaller than $T_{SIG}$?

For $T_{CLK} < T_{SIG}$
Switched-Capacitor Filters

What if $T_{\text{CLK}}$ is not much much smaller than $T_{\text{SIG}}$?

For $T_{\text{CLK}} < T_{\text{SIG}}$

$V(nT)$

$V((n+1)T)$

$\phi_1$

$\phi_2$

$T_{\text{CLK}}$

$nT_{\text{CLK}}$  $(n+1)T_{\text{CLK}}$

Define $T = T_{\text{CLK}}$

$\phi_1$

$\phi_2$

$nT$  $(n+1)T$

$V_0(nT+T) = V_0(nT) + \frac{\Delta Q}{C}$

but $-\Delta Q$ is the charge on $C_1$ at the time $\phi_1$ opens

$-\Delta Q \approx C_1 V_{\text{IN}}(nT+T/2)$

$\therefore V_{\text{OUT}}(nT+T) = V_{\text{OUT}}(nT) - \frac{C_1}{C} V_{\text{IN}}(nT+T/2)$

If an input S/H, $V_{\text{IN}}$ constant over periods of length $T$

thus, assume $V_{\text{IN}}(nT+T/2) \approx V_{\text{IN}}(nT)$

So obtain

$V_{\text{OUT}}(nT+T) = V_{\text{OUT}}(nT) - \frac{C_1}{C} V_{\text{IN}}(nT)$
Switched-Capacitor Filters

What if $T_{CLK}$ is not much-much smaller than $T_{SIG}$?

$$V_{OUT}(nT+T) = V_{OUT}(nT) - \frac{C_1}{C} V_{IN}(nT)$$

This is a difference equation relating $V_{OUT}(nT)$ to $V_{IN}(nT)$

Coefficients in difference equation are all accurately controlled!

Difference equation is highly insensitive to process variations and temperature variations!
Switched-Capacitor Filters

What if $T_{CLK}$ is not much much smaller than $T_{SIG}$?

\[ V_{OUT}(nT+T) = V_{OUT}(nT) - \left( \frac{C_1}{C} \right) V_{IN}(nT) \]

Taking the z-transform we obtain

\[ zV_O = V_O - \left( \frac{C_1}{C} \right) V_{IN} \]

\[ I(z) = \frac{V_O}{V_{IN}} = \frac{-\left( \frac{C_1}{C} \right)}{z-1} \]
Switched-Capacitor Filters

What if $T_{\text{CLK}}$ is not much-much smaller than $T_{\text{SIG}}$?

$V_{\text{OUT}}(nT+T) = V_{\text{OUT}}(nT) - \frac{C_1}{C}V_{\text{IN}}(nT)$

$I(z) = -\left(\frac{C_1}{C}\right) \frac{1}{z-1}$

- Switched-capacitor circuits are analyzed in the z-domain rather than in the s-domain.
- Coefficients are precisely controlled with small area even if $T_{\text{CLK}}$ is not much smaller than $T_{\text{SIG}}$.
- Assumption of input S/H is really not necessary.
- Often no underlying Active RC circuit (direct synthesis in the discrete domain).
- SC circuits are discrete-time continuous-amplitude circuits.
Switched-Capacitor Filters

Parasitic Capacitances

Parasitic capacitances are large, do not match, and most are nonlinear!
Switched-Capacitor Filters

Parasitic Capacitances

Parasitics affecting charge transfer are indicated

\[ C_P = C_{s1} + C_{d2} + C_{T1} \]

\[ V_{\text{OUT}}(nT + T) = V_{\text{OUT}}(nT) - \left[ \frac{C_1 + C_P}{C} \right] V_{\text{IN}}(nT) \]

• Can affect the ratio \( C_1 / C_P \) by 30% or more
• Most of the accuracy improvements offered by SC technique lost in parasitics!
Switched-Capacitor Filters

Consider:

\[ V_{OUT}(nT+T) = V_{OUT}(nT) + \left(\frac{C_1}{C}\right)V_{IN}(nT) \]

with input S/H

\( I(z) = \frac{C_1}{C} \cdot \frac{1}{z-1} \)

- Performs as a noninverting integrator
- Note simple noninverting integrator function without need for extra op amp
- Requires extra switches
- Has many more parasitics
Switched-Capacitor Filters

Parasitic Capacitances
Switched-Capacitor Filters

Parasitics affecting charge transfer are indicated

- Effects of all parasitic capacitances have been essentially eliminated!
- Termed a stray-insensitive or parasitic-insensitive structure
- Widely used as a noninverting SC integrator
Switched-Capacitor Filters

\[ V_{\text{OUT}}(nT+T) = V_{\text{OUT}}(nT) - \left( \frac{C_1}{C} \right) V_{\text{IN}}(nT+T) \]

without input S/H

\[ I(z) = - \frac{z \left( \frac{C_1}{C} \right)}{z-1} \]

Serves as inverting stray-insensitive SC integrator
Switched-Capacitor Filters

Summing inputs  (any number of summing inputs can be used)

(Shown stray-sensitive to reduce schematic complexity only)
Switched-Capacitor Filters

Lossy inverting integrator

\[ V_{\text{OUT}}(nT+T) = V_{\text{OUT}}(nT) - \frac{C_2}{C} V_{\text{IN}}(nT) - \frac{C_1}{C} V_{\text{IN}}(nT) \]

\[
\frac{V_{\text{OUT}}(z)}{V_{\text{IN}}(z)} = l(z) = - \frac{z \left( \frac{C_1}{C} \right)}{z - \left( 1 - \frac{C_2}{C} \right)}
\]

(Shown stray-sensitive to reduce schematic complexity only)
Switched-Capacitor Filters

Summing Lossy inverting integrator

(Shown stray-sensitive to reduce schematic complexity only)
Switched-Capacitor Filters

Stray Insensitive Lossy Integrator with inverting and noninverting summing inputs

\[ V_{OUT}(nT+T) = V_{OUT}(nT) - \left(\frac{C_3}{C}\right) V_{OUT}(nT+T) + \left(\frac{C_1}{C}\right) V_{IN1}(nT+T) - \left(\frac{C_2}{C}\right) V_{IN2}(nT+T) \]

\[
V_{OUT}(z) = \frac{z \left(\frac{C_1}{C}\right) V_{IN1} - z \left(\frac{C_2}{C}\right) V_{IN2}}{\left(1 + \frac{C_3}{C_1}\right) z - 1}
\]