EE 435

Lecture 5:

Fully Differential Single-Stage Amplifier Design
Review from last lecture:
Symmetric Networks

Theorem: If a linear network is symmetric, then for all differential symmetric excitations, the small signal voltage is zero at all points on the axis of symmetry.
Review from last lecture:

**Counterpart Networks**

Definition: The counterpart network of a network is obtained by replacing all n-channel devices with p-channel devices, replacing all p-channel devices with n-channel devices, replacing $V_{SS}$ biases with $V_{DD}$ biases, and replacing all $V_{DD}$ biases with $V_{SS}$ biases.
Review from last lecture:

**Counterpart Networks**

- $V_{DD}$
- $M_2$
- $M_1$
- $V_{SS}$

the counterpart network is unique

the counterpart of the counterpart is the original network
Counterpart Networks

Theorem: The parametric expressions for all small-signal characteristics, such as voltage gain, output impedance, and transconductance of a network and its counterpart network are the same.
Synthesis of fully-differential op amps from symmetric networks and counterpart networks

Theorem: If F is any network with a single input and P is its counterpart network, then the following circuits are fully differential circuits --- “op amps”.

\[ V_{d} = V_{1} - V_{2} \]
Synthesis of fully-differential op amps from symmetric networks and counterpart networks

A fully differential op amp is derived from any quarter circuit by combining it with its counterpart to obtain a half-circuit, combining two half-circuits to form a differential symmetric circuit and then biasing the symmetric differential circuit on the axis of symmetry.

Further, most of the properties of the operational amplifier can be obtained by inspection, from those of the quarter circuit.

Implications: Much Op Amp design can be reduced to designing much simpler quarter-circuits where it is much easier to get insight into circuit performance.
Review from last lecture:
Determination of op amp characteristics from quarter circuit characteristics

Small signal Quarter Circuit

\[
A_{\text{VOQC}} = -\frac{G_M}{G} \\
\text{BW} = \frac{G}{C_L} \\
\text{GB} = \frac{G_M}{C_L}
\]

Small signal differential amplifier

\[
A_{\text{VO}} = \frac{-G_{M1}}{2(G_1 + G_2)} \\
\text{BW} = \frac{G_1 + G_2}{C_L} \\
\text{GB} = \frac{G_{M1}}{2C_L}
\]

Note: Factor of 4 reduction of gain
Comparison of Tail Voltage and Tail Current Source Structures

Small signal half-circuits are identical so voltage gains, BW, and GB are all the same.
Review from last lecture:

Biasing Issues for Differential Amplifier

• Tail voltage bias not suitable for large common-mode (CM) input range but does offer good output swing

• Tail current bias provides good CM input range but at the expense of a modest reduction in output signal swing
Review from last lecture:

**Differential Output Amplifiers**

- **Single-Ended Outputs**
- **Differential Output**

- Differential Voltage Gain Double that of Single-Ended Structure
- BW is the same
- GB Doubles for the Differential Output Structure
Consider an output voltage for any linear circuit with two inputs.

By superposition

\[ V_{OUT} = A_1 V_1 + A_2 V_2 \]

where \( A_1 \) and \( A_2 \) are the gains (transfer functions) from inputs 1 and 2 to the output respectively.

Define the common-mode and difference-mode inputs by

\[ V_c = \frac{V_1 + V_2}{2} \]
\[ V_d = V_1 - V_2 \]

These two equations can be solved for \( V_1 \) and \( V_2 \) to obtain

\[ V_1 = V_c + \frac{V_d}{2} \]
\[ V_2 = V_c - \frac{V_d}{2} \]
Common-Mode and Differential-Mode Analysis

Consider an output voltage for any linear circuit with two inputs

\[ V_{\text{OUT}} = A_1 V_1 + A_2 V_2 \]

Substituting into the expression for \( V_{\text{OUT}} \), we obtain

\[ V_{\text{OUT}} = A_1 \left( v_c + \frac{v_d}{2} \right) + A_2 \left( v_c - \frac{v_d}{2} \right) \]

Rearranging terms we obtain

\[ V_{\text{OUT}} = v_c \left( A_1 + A_2 \right) + v_d \left( \frac{A_1 - A_2}{2} \right) \]

If we define \( A_c \) and \( A_d \) by

\[ A_c = A_1 + A_2 \quad A_d = \frac{A_1 - A_2}{2} \]

Can express \( V_{\text{OUT}} \) as

\[ V_{\text{OUT}} = v_c A_c + v_d A_d \]
Common-Mode and Differential-Mode Analysis

Consider any output voltage for any linear circuit with two inputs

\[ V_{\text{OUT}} = A_1 V_1 + A_2 V_2 \]
\[ V_{\text{OUT}} = V_c A_c + V_d A_d \]

Implication: Can solve a linear two-input circuit by applying superposition with \( V_1 \) and \( V_2 \) as inputs or by applying \( V_c \) and \( V_d \) as inputs

Implication: In a circuit with \( A_2 = -A_1 \), \( A_c = 0 \) so
\[ V_{\text{OUT}} = V_d A_d \]
Common-Mode and Differential-Mode Analysis

Depiction of single-ended inputs and common/difference mode inputs

\[ V_{\text{OUT}} = A_1 V_1 + A_2 V_2 \]

\[ V_{\text{OUT}} = V_c A_c + V_d A_d \]
Common-Mode and Differential-Mode Analysis

Extension to differential outputs and symmetric circuits

Theorem: The symmetric differential output voltage for any symmetric linear network excited at symmetric nodes can be expressed as

$$ V_{OUT} = A_d V_d $$

where $A_d$ is the differential voltage gain and the voltage $V_d = V_1 - V_2$.
Common-Mode and Differential-Mode Analysis

Proof for Symmetric Circuit with Symmetric Differential Output:

By superposition, the single-ended outputs can be expressed as

\[ v_{\text{OUT}+} = T_{0PA}v_1 + T_{0PB}v_2 \]
\[ v_{\text{OUT}-} = T_{0NA}v_1 + T_{0NB}v_2 \]

where \( T_{0PA}, T_{0PB}, T_{0NA} \) and \( T_{0NB} \) are the transfer functions from the A and B inputs to the single-ended + and - outputs.

taking the difference of these two equations we obtain

\[ v_{\text{OUT}} = v_{\text{OUT}+} - v_{\text{OUT}-} = (T_{0PA} - T_{0NA})v_1 + (T_{0PB} - T_{0NB})v_2 \]

by symmetry, we have

\[ T_{0PA} = T_{0NB} \text{ and } T_{0NA} = T_{0PB} \]

thus can be express \( v_{\text{OUT}} \) as

\[ v_{\text{OUT}} = (T_{0PA} - T_{0NA})(v_1 - v_2) \]

or as

\[ v_{\text{OUT}} = A_d v_d \]

where \( A_d = T_{0PA} - T_{0NA} \) and where \( v_d = v_1 - v_2 \)
Consider any output voltage for any linear circuit with two inputs:

\[ v_{\text{OUT}} = A_1 v_1 + A_2 v_2 \]
\[ v_{\text{OUT}} = v_c A_c + v_d A_d \]

**Single-Ended Superposition**

**Difference-Mode/Common-Mode Superposition**
Consider an output voltage for any linear circuit with two inputs

\[ v_{\text{OUT}} = v_c A_c + v_d A_d \]

Difference-Mode/Common-Mode Superposition is almost exclusively used for characterizing Amplifiers that are designed to have a large differential gain and a small common-mode gain.
Performance with Common-Mode Input

Single-Ended Outputs
Tail-Current Bias

Differential Output
Tail Current Bias

Single-Ended Outputs
Tail-Voltage Bias

Differential Output
Tail Voltage Bias
Performance with Common-Mode Input

Consider tail-current bias amplifier

No current flows across axis of symmetry in a symmetric circuit

Common-Mode Half-Circuit
Performance with Common-Mode Input

Consider tail-current bias amplifier

\[ v_{\text{OUTC}} = 0 \quad \text{thus } A_C = 0 \]
Performance with Common-Mode Input

Consider tail-voltage bias amplifier

No current flows across axis of symmetry in a symmetric circuit

Common-Mode Half-Circuit
Performance with Common-Mode Input

Consider tail-voltage bias amplifier

\[ \mathbf{v}_{\text{OUTC}} = \mathbf{v}_c \]

\[ \mathbf{v} \]

Solving, we obtain

\[ \frac{\mathbf{v}_{\text{OUTC}}}{\mathbf{v}_c} = \mathbf{A}_C = \frac{-\mathbf{G}_{M1}}{s\mathbf{C} + \mathbf{G}_1 + \mathbf{G}_2} \]

This circuit has a rather large common-mode gain and will not reject common-mode signals.
Applications of Quarter-Circuit Concept to Op Amp Design

consider initially the basic single-ended amplifier
Single-stage single-input low-gain op amp

Basic Structure

Quarter Circuit

Counterpart Circuit

Practical Implementation
Small signal model of half-circuit

\[ G = G_1 + G_2 \]

\[ G_M = G_{M1} \]
Single-stage low-gain differential op amp

Quarter Circuit

Single-Ended Output : Differential Input Gain

\[
A(s) = \frac{-g_{m1}}{2} \frac{1}{sC_L + g_{o1} + g_{o3}}
\]

\[
A_0 = \frac{2}{g_{o1} + g_{o3}}
\]

\[
GB = \frac{g_{m1}}{2C_L}
\]

Need a CMFB circuit to establish \( V_{b1} \)
Single-stage low-gain differential op amp

\[
A(s) = \frac{-g_{m1}}{2} \frac{2}{sC_L + g_{o1} + g_{o3}}
\]

\[
A_o = \frac{g_{m1}}{2} \frac{2}{g_{o1} + g_{o3}}
\]

\[
GB = \frac{g_{m1}}{2C_L}
\]

What are the number of degrees of freedom?
(assume \(V_{DD}, C_L\) fixed)

Natural Parameters:
\[
\left\{ \frac{W_1}{L_1}, \frac{W_3}{L_3}, \frac{W_5}{L_5}, V_{B1}, V_{B3} \right\}
\]

Constraints: \(I_{D5} \approx 2I_{D3}\)
Net Degrees of Freedom: 4

Practical Parameters:
\[
\left\{ V_{EB1}, V_{EB3}, V_{EB5}, P \right\}
\]

Need a CMFB circuit to establish \(V_{b1}\)
Single-stage low-gain differential op amp

Quarter Circuit

Single-Ended Output : Differential Input Gain

\[
A(s) = \frac{-g_{m1}}{2} \frac{2}{sC_L + g_{o1} + g_{o3}}
\]

\[
A_o = \frac{2}{g_{o1} + g_{o3}} g_{m1}
\]

\[
GB = \frac{g_{m1}}{2c_L}
\]

Need a CMFB circuit to establish \(V_{b1}\)
Single-stage low-gain differential op amp

Quarter Circuit

\[ V_{OD} = V_o^+ - V_o^- \]

Differential Output : Differential Input Gain

\[ A(s) = \frac{g_{m1}}{sC_L + g_{o1} + g_{o3}} \]

\[ A_o = \frac{g_{m1}}{g_{o1} + g_{o3}} \]

\[ GB = \frac{g_{m1}}{C_L} \]

\[ A_0 = \left[ \frac{1}{\lambda_1 + \lambda_3} \right] \left( \frac{2}{V_{EB1}} \right) \]

\[ GB = \left( \frac{P}{V_{DD}C_L} \right) \bullet \left[ \frac{1}{V_{EB1}} \right] \]

Need a CMFB circuit to establish \( V_{B1} \) or \( V_{B2} \)
Operational Amplifier Small Signal Characteristics in Terms of Quarter Circuit Performance

Assumptions: Bias current in quarter circuits same as in Op Amps and $C_L$ is load capacitance on each side of op amp

Single-ended Output

$$A(s) = \frac{-g_{MN}}{2sC_L + g_{ON} + g_{OP}}$$

$$A_0 = \frac{1}{2} \frac{g_{MN}}{g_{ON} + g_{OP}}$$

$$GB = \frac{1}{2} \frac{g_{MN}}{C_L}$$

Differential Output

$$A(s) = \frac{-g_{MN}}{sC_L + g_{ON} + g_{OP}}$$

$$A_0 = \frac{g_{MN}}{g_{ON} + g_{OP}}$$

$$GB = \frac{g_{MN}}{C_L}$$

Expressions valid for both tail-current and tail-voltage op amp
Expressions valid for both tail-current and tail-voltage op amp

So which one should be used?

- Common-mode input range large for tail current bias
- Improved rejection of common-mode signals for tail current bias
- Extra design degree of freedom for tail current bias
- Improved output signal swing for tail voltage bias (will show later)
Slew Rate

Definition: The slew rate of an amplifier is the maximum rate of change that can occur at an output node.

SR is a nonlinear large-signal characteristic.
Input is over-driven hard (some devices in amplifier usually leave normal operating region).
Magnitude of $SR^+$ and $SR^-$ usually same and called SR (else $SR^+$ and $SR^-$ must be given).
Slew Rate

With step input on $V_{iN}^+$, all tail current ($I_T$) will go to $M_1$ thus turning off $M_2$ thus current through $M_4$ which is $\frac{1}{2}$ of $I_T$ will go to load capacitor $C_L$

The I-V characteristics of any capacitor is

$$I = C \frac{dV}{dt}$$

Substituting $I = I_T/2$, $V = V_{OUT}^+$ and $C = C_L$ obtain a voltage ramp at the output thus

$$SR^+ = \frac{dV_{OUT}^+}{dt} = \frac{I_T}{2C_L} = \frac{P}{V_{DD}2C_L}$$
Slew Rate

It can be similarly shown that putting a negative step on the input steer all current to $M_2$ thus the current to the capacitor $C_L$ will be $I_T$ minus the current from $M_2$ which is still $I_T/2$. This will cause a negative ramp voltage on $V_{OUT}^+$ of value

$$SR^- = \frac{dV_{OUT}^+}{dt} = -\frac{I_T}{2C_L} = -\frac{P}{V_{DD}2C_L}$$

Since the magnitude of $SR^+$ and $SR^-$ are the same, obtain a single SR for the amplifier of value

$$SR = \frac{P}{V_{DD}2C_L}$$
Single-stage low-gain differential op amp

Consider single-ended output performance:

Will term this the **reference op amp**
Will make performance comparisons of other op amps relative to this

$$A(s) = \frac{2}{sC_L + g_{o1} + g_{o3}}$$

mixed parameters

$$A_{VO} = \frac{1}{2} g_{m1} \frac{1}{g_{o1} + g_{o3}}$$

$$GB = \frac{g_{m1}}{2C_L}$$

$$SR = \frac{I_T}{2C_L}$$

practical parameters

$$A_{v0} = \left[ \frac{1}{\lambda_1 + \lambda_3} \right] \left( \frac{1}{V_{EB1}} \right)$$

$$GB = \left( \frac{P}{2V_{DD}C_L} \right) \cdot \left[ \frac{1}{V_{EB1}} \right]$$

$$SR = \frac{P}{2V_{DD}C_L}$$

Reference Op Amp

Need a CMFB circuit to establish $V_{b1}$
Reference Op Amp

single-ended output

\[ A(s) = \frac{g_{m1}}{2sC_L + g_{o1} + g_{o3}} \]

\[ A_{vo} = \frac{1}{2} \frac{g_{m1}}{g_{o1} + g_{o3}} \]

\[ GB = \frac{g_{m1}}{2C_L} \]

\[ SR = \frac{I_T}{2C_L} \]

\[ A_{vo} = \left[ \frac{1}{\lambda_1 + \lambda_3} \right] \left[ \frac{1}{V_{EB1}} \right] \]

\[ GB = \left[ \frac{P}{2V_{DD}C_L} \right] \cdot \left[ \frac{1}{V_{EB1}} \right] \]

\[ SR = \frac{P}{2V_{DD}C_L} \]

Need a CMFB circuit to establish \( V_{b1} \)
# Amplifier Structure Summary

## Small Signal Parameter Domain

<table>
<thead>
<tr>
<th>Common Source</th>
<th>$A_{vo} = \frac{g_m}{g_o}$</th>
<th>$GB = \frac{g_m}{C_L}$</th>
</tr>
</thead>
</table>

## Practical Parameter Domain

<table>
<thead>
<tr>
<th>Common Source</th>
<th>$A_{vo} = \left(\frac{2}{\lambda} \right) \frac{1}{V_{EB}}$</th>
<th>$GB = \left(\frac{2P}{V_{DD}C_L} \right) \frac{1}{V_{EB}}$</th>
</tr>
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## Small Signal Parameter Domain

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<tr>
<th>Reference Op Amp</th>
<th>$A_{vo} = \frac{1}{2} \frac{g_{m1}}{g_{o1} + g_{o3}}$</th>
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<th>$SR = \frac{I_T}{2C_L}$</th>
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## Practical Parameter Domain

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<th>Reference Op Amp</th>
<th>$A_{vo} = \left[ \frac{1}{\lambda_1 + \lambda_3} \right] \frac{1}{V_{EB1}}$</th>
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<th>$SR = \frac{P}{2V_{DD}C_L}$</th>
</tr>
</thead>
</table>
What basic type of amplifier is this op amp?

\[ A(s) = \frac{g_{m1}}{2 sC_L + g_{o1} + g_{o3}} \]
What basic type of amplifier is this op amp? Does it really matter?

\[ A(s) = \frac{g_{m1}}{2} \frac{1}{sC_L + g_{o1} + g_{o3}} \]
Single-stage low-gain differential op amp

Need a CMFB circuit to establish $V_{B1}$ or $V_{B2}$

CMFB amplifies difference between $V_{B1}$ and average of two signal inputs

Can apply to either $V_{B1}$ or $V_{B2}$ but not both
Single-stage low-gain differential op amp

- Can eliminate CMFB circuit if only single-ended output is needed by connecting counterpart circuits as a current mirror
- This will double the voltage gain and the GB as well
- Still uses counterpart circuits but terminated in different ways
- Although not symmetric, previous analysis results with specified modifications still nearly apply
Single-stage low-gain differential op amp

Current-Mirror Connected Counterpart Circuit

No CMFB Circuit Needed

\[
A(s) = \frac{g_{m1}}{sC_L + g_{o1} + g_{o3}}
\]

\[
A_o = \frac{g_{m1}}{g_{o1} + g_{o3}}
\]

\[
G_B = \frac{g_{m1}}{C_L} \quad S_R = \frac{I_T}{C_L}
\]

In terms of practical design space parameters

\[
A_o = \left[ \frac{1}{\lambda_1 + \lambda_3} \right] \left[ \frac{2}{V_{EB1}} \right] \quad G_B = \left[ \frac{P}{V_{DD}C_L} \right] \cdot \left[ \frac{1}{V_{EB1}} \right] \quad S_R = \frac{P}{V_{DD}C_L}
\]
End of Lecture 5
Signal Swing of Single-Stage Op Amp

Constraining Equations:

To keep $M_2$ in Saturation:

$$V_{OUT} > V_{ic} - V_{T2}$$

To keep $M_4$ in Saturation:

$$V_{OUT} < V_{DD} - |V_{EB4}|$$

To keep $M_1$ in Saturation:

$$V_{ic} < V_{DD} + V_{T1} - |V_{T3}| - |V_{EB3}|$$

To keep $M_5$ in Saturation:

$$V_{ic} > V_{T1} + V_{EB1} + V_{EB5} + V_{SS}$$
Signal Swing of Single-Stage Op Amp

Constraining Equations:

\[ V_{\text{OUT}} < V_{\text{DD}} - |V_{\text{EB4}}| \]
\[ V_{\text{OUT}} > V_{\text{ic}} - V_{T2} \]
\[ V_{\text{ic}} < V_{\text{DD}} + V_{T1} - |V_{T3}| - |V_{\text{EB3}}| \]
\[ V_{\text{ic}} > V_{T1} + V_{\text{EB1}} + V_{\text{EB5}} + V_{\text{SS}} \]
Signal Swing of Single-Stage Op Amp
**Signal Swing of Single-Stage Op Amp**

**Constraining Equations:**

\[
\begin{align*}
V_{\text{OUT}} &< V_{\text{DD}} - |V_{\text{EB4}}| \\
V_{\text{OUT}} &> V_{\text{ic}} - V_{T2} \\
V_{\text{ic}} &< V_{\text{DD}} + V_{T1} - |V_{T3}| - |V_{\text{EB3}}| \\
V_{\text{ic}} &> V_{T1} + V_{\text{EB1}} + V_{\text{EB5}} + V_{SS}
\end{align*}
\]

Signal swings are Important Performance Parameters !!
Design space for single-stage op amp

Performance Parameters in Practical Parameter Domain \{ V_{EB1}, V_{EB2}, V_{EB5}, P \}:

\[
A_0 = \frac{1}{\lambda_1 + \lambda_3} \left( \frac{2}{V_{EB1}} \right)
\]

\[
GB = \left( \frac{P}{V_{DD}C_L} \right) \left[ \frac{2}{V_{EB1}} \right]
\]

\[
SR = \frac{P}{V_{DD}C_L}
\]

\[
V_{OUT} < V_{DD} - |V_{EB3}|
\]

\[
V_{OUT} > V_{iC} - V_{T2}
\]

\[
V_{iC} < V_{DD} + V_{T1} - |V_{T3}| - |V_{EB3}|
\]

\[
V_{iC} > V_{T1} + V_{EB1} + V_{EB5} + V_{SS}
\]

Simple Expressions in Practical Parameter Domain