Problem 1

$g_{o eq}$ can be found by shorting the input, applying a test current to the output, and finding $\frac{I_{test}}{V_{out}}$

@ $V_i$

$V_i g_{m2} + V_i (g_{m1} + g_{ds1}) + (V_i - V_{out}) g_{ds2} = 0$

$V_i = \frac{V_o g_{ds2}}{g_{m1} + g_{m2} + g_{ds1} + g_{ds2}}$

@ $V_{out}$

$I_{in} - (-V_i) g_{m} = (V_o - V_i) g_{ds2}$

Substitute $V_i$

$I_{in} + (g_{m} + g_{ds2}) g_{ds2} V_o = V_o g_{ds2}$

$\frac{I_{in}}{V_o} = \frac{g_{ds2} (g_{m1} + g_{m2} + g_{ds1} + g_{ds2}) - (g_{m2} + g_{ds2}) g_{ds2}}{g_{m1} + g_{m2} + g_{ds1} + g_{ds2}} = \frac{g_{ds2} (g_{m} + g_{ds1})}{g_{m1} + g_{m2} + g_{ds1} + g_{ds2}} \approx \frac{g_{ds2}}{2}$
\[ g_{meq} = \frac{I_0}{V_{in}} = \frac{I_0}{V_{o}} \cdot \frac{V_{o}}{V_{in}} = g_{o eq} \cdot \text{Gain} \]

\[ V_{in} \]

\[ Vgs \]

\[ V_1 \]

\[ g_{m2} \]

\[ g_{ds2} \]

\[ G \]

\[ g_{m2} (V_{in} - V_1) + (V_1 - V_1) g_{ds2} = 0 \]

\[ V_1 \]

\[ g_{m2} (V_{in} - V_1) + (V_0 + V_1) g_{ds2} = V_1 (g_{m1} + g_{ds1}) \]

\[ V_1 \]

\[ g_{m2} \]

\[ g_{ds2} \]

\[ g_{meq} = \frac{g_{ds2}}{2}, \quad \frac{g_{m2}}{g_{ds2}} = \frac{g_{m2}}{2} \]

Sub in \( V_1 = 0 \) into equation at \( V_{out} \)

\[ \frac{V_0}{V_{in}} = \frac{g_{m2}}{g_{ds2}} \]
b)

\[ \text{Gain} = \frac{g_{mea}}{1 \left( g_{oeq} + g_{oteq} \right)} \]

\[ BW = \frac{g_{mea} + g_{oeq}}{c_L} \]

\[ GB = \frac{g_{mea}}{c_L} \]

\[ \text{Gain} = \frac{g_m}{2 \cdot g_{ds}} \]

\[ BW = \frac{g_{ds}}{c_L} \]

\[ GB = \frac{g_m}{2 \cdot c_L} \]
(c)

\[ \text{Gain} = \frac{g_m}{2g_{ds}} = \frac{2I_{DA}}{V_{EB}} \]

\[ \text{BW} = \frac{g_{ds}}{C_L} = \frac{\lambda I_{Da}}{C_L} = \frac{\lambda I_{tail}}{2C_L} = \frac{\lambda P}{2C_L(V_{DD} - V_{SS})} \]

\[ GB = \frac{g_m}{2C_L} = \frac{2I_{Da}}{2C_L} = \frac{I_{tail}}{V_{EB}C_L} = \frac{P}{V_{EB}C_L(V_{DD} - V_{SS})} \]

**Problem 2**

(a) For SumsWrong's circuit

\[ \text{Gain} = \frac{1}{V_{EB}\lambda} \]

\[ GB = \frac{P}{2V_{EB}C_L(V_{DD} - V_{SS})} \]

\[ V_{EB} = .15\text{ V} \]

For Reference Amplifier

\[ \text{Gain} = \frac{1}{V_{EB}\lambda} \]

\[ GB = \frac{P}{V_{EB}C_L(V_{DD} - V_{SS})} \]

\[ V_{EB} = .5\text{ V} \]

From these equations, the gain is the same for the same \( V_{EB} \), however the equations for gain bandwidth show the SumsWrong circuit is \( \frac{1}{2} \) of the gain bandwidth of the reference amplifier. The difference between the \( V_{EB} \)'s is more than a factor of 3, so the "gains" of SumsWrong's \( GB \) are not from the circuit topography.
b) This is not a fair comparison of these circuits as the "gains" come from uneven designs of VEB so the results are favorable towards Sam's wrong's circuit.

c) A fair comparison would use the same VEB for both circuits. Doing so allows the superiority of the reference op amp to be seen.

To find gain, find equivalent gm, g01 at the input and g02 of the modified Wilson current mirror.
To find $g_{os2}$ set input to 0 and find
\[
\frac{I_{\text{test}}}{V_{\text{out}}} = g_{os2}.
\]

At $V_2$, it is essentially a common source amplifier where $V_{\text{in}} = V_I$ and $V_{\text{out}} = V_2$.

\[
V_2 = -g_{m1} \frac{V_I}{g_{ds1}}
\]

At $V_1$, $I_{\text{test}}$ splits through $g_{m2}$ and $g_{ds2}$ and recombines at $V_1$, so

\[
V_1 = \frac{I_{\text{test}}}{g_{m3} + g_{ds3}}
\]

At $V_{\text{out}}$,

\[
I_{\text{test}} = g_{m2} (V_2 - V_I) + g_{ds2} (V_{\text{out}} - V_I)
\]

\[
I_{\text{test}} = g_{m2} \left(- \frac{g_{m1} I_{\text{test}}}{g_{ds1} (g_{m3} + g_{ds3})} \right) - \left(\frac{I_{\text{test}}}{g_{m3} + g_{ds3}}\right) + g_{ds2} V_{\text{out}} - \frac{g_{ds2} I_{\text{test}}}{g_{m3} + g_{ds3}}
\]

\[
I_{\text{test}} \left(1 + \frac{g_{m1} g_{m2}}{g_{ds1} (g_{m3} + g_{ds3})} + \frac{g_{m2}}{g_{m3} + g_{ds3}} + \frac{g_{ds2}}{g_{m3} + g_{ds3}}\right) = g_{ds2} V_{\text{out}}
\]
\[ I_{\text{test}} \left( \frac{g_{ds1} (g_{m3} + g_{d3}) + g_{m2} g_{ds1} + g_{ds2} g_{ds5} + g_{m1} + g_{m2}}{g_{ds1} (g_{m3} + g_{d3})} \right) = g_{ds2} \text{ Vout} \]

\[ I_{\text{test}} = \frac{g_{ds2} g_{ds1} (g_{m3} + g_{d3})}{g_{m1} g_{m2} g_{ds1} g_{ds5} + g_{m3} g_{ds2} g_{ds5} + g_{m2} g_{ds1} g_{ds5} + g_{m1} g_{d5} g_{ds2}} \approx \frac{g_{m3} g_{ds2} g_{ds5}}{g_{m1} g_{m2} g_{ds5}} \text{ Veq2} \]

\( g_{m1eq} \) and \( g_{d5eq} \) are \( g_{m1eq} \) and \( g_{d5eq} \) of the cascade amplifier.

\[ g_{m1eq} = g_{m2} \quad g_{d5eq} = \frac{g_{ds7} g_{ds5}}{g_{m5}} \]

\[ A_0(s) = \frac{g_{m1eq}}{s C_L + g_{m1eq} + g_{d5eq}} \]

\[ \text{DC Gain} = \frac{g_{m7}}{s C_L + \frac{g_{ds7} g_{ds5}}{g_{m5}} + \frac{g_{m3} g_{ds2} g_{ds5}}{g_{m1} g_{m2}}} \]

\[ g_B = \frac{g_{m1eq}}{C_L} = \frac{g_{m7}}{C_L} \]

Notice that both DC gain and \( g_B \) are fairly close to the DC gain and \( g_B \) of the telescopic cascade.
Problem 4

b) First, currents in each branch must be found

\[ I_{\text{tot}} = \frac{10 \text{mW}}{3 \text{V}} = 3.33 \text{ mA} \]
\[ I_{\text{tail}} = \frac{I_{\text{tot}}}{3} = 1.11 \text{ mA} \]

\[ I_{\text{da}} = \frac{I_{\text{tail}}}{2} = 555 \text{ mA} \]
\[ I_7 = \frac{I_{\text{tot}}}{2} - I_{\text{da}} = 1.11 \text{ mA} \]

\[ V_{\text{EB}} = 0.2 \text{ V} \]
\[ I_{\text{mcox}} = 100 \frac{\text{mA}}{V^2} \]
\[ I_{\text{mcox}} = 33 \frac{\text{mA}}{V^2} \]

Using these parameters and the currents, the square law model can be used to size all transistors.
1. \[ \frac{2}{W_{1}} = \frac{I_{N1}}{L_{1}} = 555 \]

2. \[ \frac{2}{W_{2,3}} = \frac{I_{N2}}{L_{2,3}} = 277.5 \]

3. \[ \frac{2}{W_{4,5}} = \frac{I_{N4}}{L_{4,5}} = 2522.7 \]

4. \[ \frac{2}{W_{6,7}} = \frac{I_{N6}}{L_{6,7}} = 1681.8 \]

5. \[ \frac{2}{W_{8,9,10,11}} = \frac{I_{N8}}{L_{8,9,10,11}} = 555 \]

C) DC gain can be found by finding equivalent \( g_{m} \) and \( g_{0} \) of folded cascode and current mirror cascode. Only difference between regular cascode and folded cascode is the \( g_{ds3} \) is added to \( g_{ds3} \) to account for conductance of the current bias, so

\[ g_{meq} = g_{m3} \quad g_{0eq} = \frac{g_{ds7}(g_{ds5} + g_{ds3})}{g_{m7}} \quad g_{0eq} = \frac{g_{ds5} + g_{ds11}}{g_{m9}} \]
\[ A_0 = \frac{g_{m3} g_{m7}}{g_{ds7}(g_{ds5} + g_{ds3})} = \frac{g_{m3}}{g_{m7} + \frac{g_{dsq} g_{ds11}}{g_{mq}}} \]
\[ \approx \frac{g_{m3} g_{m7}}{g_{ds7}(g_{ds5} + g_{ds3}) + g_{m7} g_{dsq} g_{ds11}} \]

\[ g_{m3} = \frac{2I_{DQ}}{V_{EB}} \]
\[ I_{tail} = \frac{\Theta I_{tot}}{V_{EB}} \]
\[ I_{DQ3} = \frac{\Theta I_{tot}}{2} \]

\[ g_{m6} = \frac{2I_{DQ}}{V_{EB}} = \frac{2(\frac{I_{tot}}{2} - \frac{\Theta I_{tot}}{2})}{V_{EB}} = \frac{I_{tot}(1-\Theta)}{V_{EB}} \]
\[ I_{DS6} = \frac{I_{tot}(1-\Theta)}{2} \]

\[ A_0 = \frac{g_{m3} g_{m7}}{g_{ds7}(g_{ds5} + g_{ds3})} = \frac{\Theta (1-\Theta) I_{tot}^2}{V_{EB}^2} \]
\[ = \frac{4\Theta I_{tot}}{V_{EB}^2 \lambda_5 \lambda_7 (\lambda_5 + \Theta \lambda_7)} \]

\[ A_0 = \frac{4\Theta}{V_{EB}^2 \lambda_7 (\lambda_5 + \Theta \lambda_7)} \cdot \frac{\lambda_5}{0.1(0.1 + 0.1)} = 2.50 \times 10^3 = 108 \text{ dB} \]
Gain Band Width is the gain multiplied by the band width

\[ GB = BW \cdot A_0 = \frac{g_{1e} + g_{0z}}{C_L}, \quad \text{Imeq} = \frac{g_{meq}}{C_L} \]

\[ g_{meq} = g_{m3} = \frac{\Theta \cdot I_{total}}{V_{EB}} = \frac{\Theta \cdot P}{V_{EB} (V_{DD} - V_{IS})} \]

\[ GB = \frac{\Theta \cdot P}{V_{EB} \cdot V_{DD} \cdot C_L} = \frac{10 \text{mW}}{3} = 0.0033 \quad \frac{\text{V}}{\text{V} \cdot \text{C}_L} = \frac{0.0055}{C_L} \]

If \( C_L = 1 \text{pF} \) for example \( GB = 5.56 \text{Erad} = 884 \text{MHz} \)

Problem 5

\[ V_{T1} = \frac{V_{DD}}{5}, \quad V_{T2} = \frac{V_{DD}}{10} \]

\[ V_{EB2} = V_{T1} - \frac{V_{DD}}{10} \]

\[ V_{EB1} = V_{EB2} - \frac{V_{DD}}{10} \]

\[ V_{EB1} = V_{EB2} = \frac{V_{DD}}{10} \]

practical parameters are \( P, V_{EB2} \)

\[ \text{or} V_{EB2} = V_{EB1} + \Delta V_{th} \quad V_{T1} - V_{T2} \]

\[ g_{meq} = g_{m1} + g_{m2} \]

\[ g_{m1} = g_{01} + g_{02} \]
DC gain = \frac{g_{mca}}{g_{oeq} + g_{o2eq}}

GB = \frac{g_{meq}}{g_{oeq}}

DC gain = \frac{g_{m1} + g_{m2}}{g_{ds1} + g_{ds2} + g_{ds3} + g_{ds6}}

= \frac{2 I_{DA1}}{V_{EB1}} + \frac{2 I_{DA2}}{V_{EB2}}

= \lambda \left( I_{DA1} + I_{DA2} + I_{DA3} + I_{DA4} \right)

= 2 \left( I_{DA1} (V_{EB1} + \Delta V_{th}) + \left( \frac{I_{tail}}{2} - I_{DA1} \right) V_{EB1} \right)

= \lambda \left( \frac{I_{tail}}{2} \right)

= I_{DA1} \Delta V_{th} + \frac{I_{tail} V_{EB1}}{2}

= \frac{I_{DA1}}{\lambda} \Delta V_{th} + \frac{I_{tail} V_{EB1}}{2 \lambda}

F_{D0} = \Theta \frac{I_{tail}}{2}

DC gain = \frac{\Theta \Delta V_{th} + V_{EB1}}{\lambda}
\[ G_B = \frac{2 \left( I_{DQ1} \Delta V_{th} + \frac{I_{th1} V_{EB}}{2} \right)}{C_L} \]

\[ = \frac{P ( \Theta V_{th} + V_{EB})}{(V_{DD} - V_{SS}) C_L} \]