EE 435

Lecture 10

Current Mirror Op Amps (wrap up)
Other Gain Enhancement Strategies
- Cascaded Amplifiers
OTA Applications

Review from Last Time

Noninverting Voltage Controlled Amplifier

\[ V_{\text{OUT}} = \frac{g_{m1}}{g_{m2}} V_{\text{in}} \]

Inverting Voltage Controlled Amplifier

\[ V_{\text{OUT}} = -\frac{g_{m1}}{g_{m2}} V_{\text{in}} \]

Extremely large gain adjustment is possible

Voltage Controlled Resistorless Amplifiers
OTA Applications

Review from Last Time

Noninverting Voltage Controlled Integrator

\[ V_{OUT} = \frac{g_m}{sC} V_{in} \]

Inverting Voltage Controlled Integrator

\[ V_{OUT} = -\frac{g_m}{sC} V_{in} \]

Voltage Controlled Integrators
Fully Differential Current Mirror Op Amp with Improved Slew Rate

SR \( = \frac{M I_T}{C_L} \)

\( SR_{CMOpAmp} = \frac{M I_T}{2C_L} \)

Improved a factor of 2!

but …

\( P_{CMOpAmp} = V_{DD} I_T (1 + M) \)

\( P = V_{DD} I_T (1 + 2M) \)

SR actually about the same for “improved SR circuit” and basic OTA
Comparison of Current-Mirror Op Amps with Previous Structures

How does the GB power efficiency compare with previous amplifiers?

\[
\text{GB} = \frac{g_{mEQ}}{C_L} = \frac{Mg_{m1}}{2} = \frac{MI_T}{2V_{EB1}C_L}
\]

\[
P = V_{DD} I_T \left(1 + M\right)
\]

GB for Telescopic Cascode and Ref Op Amp!

GB efficiency decreased for small M!!
Current-Mirror Op Amps – Another Perspective!

Differential Half-Circuit
Current-Mirror Op Amps – Another Perspective!

Differential Half-Circuit

Cascade of n-channel common source amplifier with p-channel common-source amplifier!
Current-Mirror Op Amps – Another Perspective!

Differential Half-Circuit

From Current Mirror Analysis:

\[ A_v = -\frac{1}{2} \left( \frac{g_{m2}}{g_{m4}} \right) \left( \frac{g_{m6}}{g_{o6} + g_{o8}} \right) \]

\[ A_{vo} = -\frac{M \cdot g_{m1}}{2} = -\frac{g_{m6} \cdot g_{m1}}{g_{o6} + g_{o8}} \]

Cascade of n-channel common source amplifier with p-channel common-source amplifier!
Stability

- Sometimes circuits that have been designed to operate as amplifiers do not amplify a signal but rather oscillate when no input signal is present ($V_{in}=0V$ or $I_{in}=0A$) or “latch up”
- Circuits that are designed to operate as amplifiers that oscillate or “latch up” are said to be unstable
- The stability of any circuit is determined by the location of the poles
- We will discuss stability with more rigor later
- It will be shown that if the poles of an open-loop amplifier are widely separated on the negative real axis, then the feedback amplifier built using the open-loop amplifier will be stable
Poles of an Amplifier

• The poles of an amplifier are the roots of the denominator of the transfer function.

• Each energy storage element (capacitor or inductor) introduces an additional pole (except when capacitor or inductor loops exist).

• The poles of an amplifier can often be approximated by independently considering the impedance facing each capacitor and assuming all other capacitors are either open circuits or short circuits.
Current-Mirror Op Amps – Another Perspective!

Differential Half-Circuit

Are there stability issues or concerns?

\[
p_2 \approx -\frac{(g_{o6} + g_{o8})}{C_2}
\]

\[
p_1 \approx -\frac{g_{m4}}{C_1}
\]

\[|p_1| >> |p_2|\]

No stability problems provided \(C_2\) is sufficiently large!
Current Mirror Op Amp Summary

• Current-mirror op amp offers no improvement in performance over the reference op amp
• Current-mirror op amp can be viewed as a cascade of two common-source amplifiers, one with a low gain and the other with a larger gain
• Current-mirror op amp is useful as an open-loop programmable transconductance amplifier (OTA)
Other Methods of Gain Enhancement

The current mirror op amp is actually a cascade of two amplifiers but did not give a real improvement in gain.

Provided the stages are non-interacting

\[
\frac{X_{\text{OUT}}}{X_{\text{IN}}} = A = A_1A_2
\]

For the current mirror op amp

\[
A_1 \ll A_2
\]

Could the gain be increased by cascading two or more amplifiers if the amplifiers had a higher gain?
Increasing Gain by Cascading

Provided the stages are non-interacting

\[ \frac{X_{\text{OUT}}}{X_{\text{IN}}} = A_1A_2 \]

\[ \frac{X_{\text{OUT}}}{X_{\text{IN}}} = A_1A_2A_3 \]

\[ \frac{X_{\text{OUT}}}{X_{\text{IN}}} = \prod_{i=1}^{n} A_i \]

Gain can be easily increased to almost any desired level!
Increasing Gain by Cascading

\[ \frac{X_{\text{OUT}}}{X_{\text{IN}}} = \prod_{i=1}^{n} A_i \]

But each of the gains will roll off with frequency so can be modeled as

\[ A_k(s) = \frac{A_{0k}}{s + \tilde{p}_k} \]

A\(_{0k}\) is the dc gain of stage \( k \)
\( \tilde{p}_k \) is the negative of the pole of stage \( k \)

Thus

\[ \frac{X_{\text{OUT}}}{X_{\text{IN}}} = A = \frac{\prod_{i=1}^{n} A_{0i}}{\prod_{k=1}^{n} \left( \frac{s}{\tilde{p}_k} + 1 \right)} \]
Increasing Gain by Cascading

\[ \frac{X_{\text{OUT}}}{X_{\text{IN}}} = A = \prod_{i=1}^{n} \frac{A_{0i}}{s + \tilde{p}_k} + 1 \]

Assume for case of an example that all stages are identical with \( A_{0k} = A_0 \) and \( \tilde{p}_k = \tilde{p} = -p \)

- Much larger gain
- Much steeper gain transition
- Much more phase shift
Increasing Gain by Cascading

\[ \frac{X_{\text{OUT}}}{X_{\text{IN}}} = A = \frac{\prod_{i=1}^{n} A_{0i}}{\prod_{k=1}^{n} \left( \frac{s}{p_k} + 1 \right)} \]

Dramatic improvement in performance for the open-loop amplifier!!

But – op amps seldom used open loop

How does the cascaded amplifier perform in a feedback application?

\[ A_{\text{FB}} = \frac{A}{1 + A \beta} \]
Feedback Amplifier Representation

\[
\frac{V_{\text{OUT}}}{V_{\text{IN}}} = \frac{1 + \frac{R_2}{R_1}}{1 + \left(1 + \frac{R_2}{R_1}\right)A}
\]

\[
\beta = \frac{R_1}{R_1 + R_2}
\]

\[
A_{\text{FB}} = \frac{A}{1 + A\beta} = \frac{\beta^{-1}}{1 + \frac{\beta^{-1}}{A}} = \frac{1 + \frac{R_2}{R_1}}{1 + \left(1 + \frac{R_2}{R_1}\right)A}
\]
Frequency Response of Feedback Amplifier

Consider the special case where $A$ is the cascade of $n$ identical stages

\[
\frac{X_{\text{OUT}}}{X_1} = A = \left( \prod_{k=1}^{n} \frac{s}{\beta_k} + 1 \right) = \left( \frac{s}{\beta + 1} \right)^n
\]

\[
A_{\text{FB}} = \frac{A}{1 + A \beta} = \frac{A_0^n}{\left( \frac{s}{\beta} + 1 \right)^n + \beta A_0^n}
\]

How do we determine how the amplifier is performing from $A_{\text{FB}}$?
Review of Basic Concepts

If \( T(s) = \frac{N(s)}{D(s)} \) is the transfer function of a linear system,

Roots of \( N(s) \) are termed the zeros,

Roots of \( D(s) \) are termed the poles.

\( X \) denotes poles
\( O \) denotes zeros
Review of Basic Concepts

If \( T(s) \) is the transfer function of a linear system

\[
T(s) = \frac{N(s)}{D(s)}
\]
is the transfer function of a linear system

Roots of \( N(s) \) are termed the zeros

Roots of \( D(s) \) are termed the poles

Theorem: A linear system is stable iff all poles lie in the open left half-plane

Claim: A circuit that is not stable is not a useful amplifier
Claim: A circuit that is close to becoming unstable is not a useful amplifier
Theorem: A linear system is stable iff all poles lie in the open left half-plane.

Stable with two negative real axis poles and two LHP CC poles.

Unstable with positive real axis pole.
Theorem: A linear system is stable iff all poles lie in the open left half-plane.

Stable with negative real axis poles

Unstable with cc RHP poles
Review of Basic Concepts

Theorem: A linear system is stable iff all poles lie in the open left half-plane.

Stable with negative real-axis poles and RHP zero.
System zero locations do not have any impact on stability.
Review of Basic Concepts

Theorem: A linear system is stable iff all poles lie in the open left half-plane

Close to becoming unstable since poles are close to the RHP
Review of Basic Concepts

\[
\begin{align*}
X_{\text{IN}} & \quad T(s) \quad X_{\text{OUT}} \\
\end{align*}
\]

\[
T(s) = \frac{N(s)}{D(s)}
\]

Theorem: A linear system is stable iff all poles lie in the open left half-plane.

What are the practical implications of stability and “close to becoming unstable”?

For any input to a linear system, the response can be written as

\[
X_{\text{OUT}}(s) = X_{\text{IN}}(s)T(s) = \sum_{k=1}^{n} \frac{a_k}{s + \tilde{p}_k} + \sum_{k=1}^{h} \frac{b_k}{s + \tilde{x}_k}
\]

where the terms \( \tilde{p}_k \) are the negative of the poles of \( T(s) \), the terms \( \tilde{x}_k \) are the negative of the roots of the denominator of the excitation and the terms \( a_k \) and \( b_k \) are the partial fraction expansion coefficients.

If \( \tilde{p}_k \) is the negative of any pole, then \( \tilde{p}_k \) can be expressed as

\[
\tilde{p}_k = -\alpha_k - j\beta_k
\]

where \( \alpha_k \) is the real part of the pole and \( \beta_k \) is the imaginary part of the pole.

\[
p_k = -\tilde{p}_k
\]
Review of Basic Concepts

\[
T(s) = \frac{N(s)}{D(s)}
\]

\[X_{IN} \quad T(s) \quad X_{OUT}\]

**Theorem:** A linear system is stable iff all poles lie in the open left half-plane.

What are the practical implications of stability and “close to becoming unstable”?

It thus follows that

\[
X_{OUT}(t) = \mathcal{L}^{-1}(X_{IN}(s)T(s)) = \sum_{k=1}^{n} a_k e^{a_k t} e^{j \beta_k t} + \sum_{k=1}^{h} b_k e^{-j \alpha_k t}
\]

Thus, for the output to be bounded for ANY input, must have ALL \( \alpha_k < 0 \)

That is equivalent to saying all poles must lie in the left half-plane.

If a pole is in the RHP, output for any input (even very small noise) will grow to infinity. If the corresponding \( \beta_k = 0 \), output will latch up. If corresponding \( \beta_k \neq 0 \), output will be a growing sinusoid.
Consider Again the Frequency Response of a Feedback Amplifier

\[ A_{FB} = \frac{A_n}{(s + \frac{1}{\beta}) + \beta A_n} \]

Example: Assume \( n=3 \)

\[ A_{FB} = \frac{A}{1 + A\beta} = \frac{A_0^3}{(s + \frac{1}{\beta})^3 + \beta A_0^3} \]

The poles with feedback, \( p_F \), are given by

\[ p_F = \left( (-1)^{\frac{1}{3}} \beta^{\frac{1}{3}} A_0 - 1 \right) \tilde{p} \approx (-1)^{\frac{1}{3}} \beta^{\frac{1}{3}} A_0 \tilde{p} \]

Note this amplifier is unstable !!!
End of Lecture 10