Other Gain Enhancement Strategies
- Cascaded Amplifiers
Review from Last Time

Current-Mirror Op Amps – Another Perspective!

Differential Half-Circuit

\[
A_v = -\frac{1}{2} \left( \frac{g_{m2}}{g_{m4}} \right) \left( \frac{g_{m6}}{g_{O6} + g_{O8}} \right)
\]

From Current Mirror Analysis:

\[
A_{vo} = -\frac{2}{g_{O6} + g_{O8}} = -\frac{g_{m6} g_{m1}}{g_{O6} + g_{O8}}
\]

Cascade of n-channel common source amplifier with p-channel common-source amplifier!
Current-Mirror Op Amps – Another Perspective!

Differential Half-Circuit

Are there stability issues or concerns?

\[ p_2 \approx -\left(\frac{g_{o6} + g_{o8}}{C_2}\right) \]

\[ p_1 \approx -\frac{g_{m4}}{C_1} \]

\[ |p_1| >> |p_2| \]

No stability problems provided \( C_2 \) is sufficiently large!
Current Mirror Op Amp Summary

- Current-mirror op amp offers no improvement in performance over the reference op amp
- Current-mirror op amp can be viewed as a cascade of two common-source amplifiers, one with a low gain and the other with a larger gain
- Current-mirror op amp is useful as an open-loop programmable transconductance amplifier (OTA)
Increasing Gain by Cascading

Provided the stages are non-interacting

\[
\frac{X_{\text{OUT}}}{X_{\text{IN}}} = A_1 A_2
\]

\[
\frac{X_{\text{OUT}}}{X_{\text{IN}}} = A_1 A_2 A_3
\]

\[
\frac{X_{\text{OUT}}}{X_{\text{IN}}} = \prod_{i=1}^{n} A_i
\]

Gain can be easily increased to almost any desired level!
Review of Basic Concepts

Theorem: A linear system is stable iff all poles lie in the open left half-plane.

- Stable with two negative real axis poles and two LHP CC poles
- Unstable with positive real axis pole
Consider Again the Frequency Response of a Feedback Amplifier

\[ A_{FB} = \frac{A^n_0}{(s/p+1)^n + \beta A^n_0} \]

Example: Assume \( n=3 \)

\[ A_{FB} = \frac{A}{1+\beta A} = \frac{A^3_0}{(s/p+1)^3 + \beta A^3_0} \]

The poles with feedback, \( p_F \), are given by

\[ p_F = \left( (-1)^{1/3} \beta^{1/3} A_0 - 1 \right) \tilde{p} \simeq (-1)^{1/3} \beta^{1/3} A_0 \tilde{p} \]

Note this amplifier is unstable !!!
Consider Again the Frequency Response of Feedback Amplifier

Example: If \( n=3 \) and stages are identical

\[
A_{FB} = \frac{A}{1 + A\beta} = \frac{A^3}{(s + \frac{1}{\beta})^3 + \beta A_0^3}
\]

Routh-Hurwitz Stability Criteria:

A third-order polynomial \( s^3 + a_2s^2 + a_1s + a_0 \) has all poles in the LHP iff all coefficients are positive and \( a_1a_2 > a_0 \)

Consider

\[
D_{FB}(s) = \left(\frac{s}{\bar{p}} + 1\right)^3 + \beta A_0^3 = s^3 \left(\frac{1}{\bar{p}^3}\right) + s^2 \frac{3}{\bar{p}^2} + s \frac{3}{\bar{p}} + (1 + \beta A_0^3)
\]

For stability

\[
(3\bar{p})(3\bar{p}^2) > \bar{p}^3(1 + \beta A_0^3) \quad 8 > \beta A_0^3
\]

Not only is the 3-stage amplifier unstable, it is far from being stable!
Routh-Hurwitz Stability Criteria:

A third-order polynomial $s^3+a_2s^2+a_1s+a_0$ has all poles in the LHP iff all coefficients are positive and $a_1a_2>a_0$

• Very useful in amplifier and filter design
• Can easily determine if poles in LHP without finding poles
• But tells little about how far in LHP poles may be
• RH exists for higher-order polynomials as well
Example:

Assume an amplifier has a transfer function that has a denominator polynomial that can be expressed as

\[ D(s) = s^3 + 2ks^2 + 4s + 16 \]

Determine the minimum value of \( k \) that will result in a stable amplifier.
Solution:

Assume an amplifier has a transfer function that has a denominator polynomial that can be expressed as

\[ D(s) = s^3 + 2ks^2 + 4s + 16 \]

Determine the minimum value of \( k \) that will result in a stable amplifier

Solution: Recall from the RH criteria that all roots of a third-order polynomial of the form \( s^3 + a_2s^2 + a_1s + a_0 \) will lie in the LHP provided all coefficients are positive and \( a_1a_2 > a_0 \)

Thus, for the current problem, must have

\[(2k)^4 > 16\]

or

\[ k > 2 \]
Consider Again the Frequency Response of the basic Feedback Amplifier

\[ A_{FB} = \frac{A}{1+ \beta A} = \frac{A_{01}A_{02}A_{03}}{\left( \frac{s}{\tilde{p}_1} + 1 \right) \left( \frac{s}{\tilde{p}_2} + 1 \right) \left( \frac{s}{\tilde{p}_3} + 1 \right) + \beta A_{02}A_{03}A_{03}} \]

\[ D_{FB}(s) = s^3 + s^2(\tilde{p}_1 + \tilde{p}_2 + \tilde{p}_3) + s(\tilde{p}_1 \tilde{p}_2 + \tilde{p}_1 \tilde{p}_3 + \tilde{p}_2 \tilde{p}_3) + \tilde{p}_1 \tilde{p}_2 \tilde{p}_3 (1 + \beta A_{0TOT}) \]

where \( A_{0TOT} = A_{01}A_{02}A_{03} \)
Consider Again the Frequency Response of Feedback Amplifier

Example: If n=3 and stages are not identical (cont)

\[ D_{FB}(s) = s^3 + s^2(\tilde{p}_1 + \tilde{p}_2 + \tilde{p}_3) + s(\tilde{p}_1 \tilde{p}_2 + \tilde{p}_1 \tilde{p}_3 + \tilde{p}_2 \tilde{p}_3) + \tilde{p}_1 \tilde{p}_2 \tilde{p}_3 (1 + \beta A_{TOT}) \]

Routh-Hurwitz Stability Criteria: (by assuming \(1 + \beta A_{TOT} \approx \beta A_{TOT}\))

\[ (\tilde{p}_1 + \tilde{p}_2 + \tilde{p}_3) (\tilde{p}_1 \tilde{p}_2 + \tilde{p}_1 \tilde{p}_3 + \tilde{p}_2 \tilde{p}_3) > \tilde{p}_1 \tilde{p}_2 \tilde{p}_3 \beta A_{TOT} \]

WOLG, assume \(\tilde{p}_1 < \tilde{p}_2 < \tilde{p}_3\) and define \(\tilde{p}_2 = k_2 \tilde{p}_1\) and \(\tilde{p}_3 = k_3 \tilde{p}_1\)

Thus the RH criteria can be expressed as

\[ (1 + k_2 + k_3)(k_2 + k_3 + k_2 k_3) > \beta A_{TOT} \]
Example: If \( n=3 \) and stages are not identical

RH criteria:

\[
(1 + k_2 + k_3)(k_2 + k_3 + k_2 k_3) > \beta A_{\text{TOT}}
\]

Since \( A_{\text{TOT}} \) will, in general, be very large for the cascade of 3 stages, a very large pole ratio is required just to maintain stability and an even larger ratio needed to avoid a close to becoming unstable situation

Practically it is difficult to obtain such a large spread in the bandwidth of the amplifiers

Problem can be viewed as one of accumulating too much phase shift before gain drops to an acceptable value

For many years there was limited commercial use of the cascade of three amplifiers (each with gain) in the design of op amps though some academic groups have worked on this approach with minimal practical success

In recent years, industry is looking at ways to “compensate” amplifiers to work with 3 (or more) high gain stages due to low headroom and shrinking \( g_m/g_o \) ratios
Similar implications on amplifier even if not a basic voltage feedback amplifier

\[ A_V = \frac{V_{OUT}}{V_{IN}} = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{1}{A_V} \left( 1 + \frac{R_2}{R_1} \right)} \]

\[ \beta = \frac{R_1}{R_2 + R_1} \]

\[ A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{A_V}{1 + \beta A_V} \]

\[ A_{VF} = \frac{V_{OUT}}{V_{IN}} = -\frac{R_2}{R_1} \frac{1}{1 + \frac{1}{A_V} \left( 1 + \frac{R_2}{R_1} \right)} \]

\[ \beta = \frac{R_1}{R_2 + R_1} \]

\[ A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{A_V}{1 + \beta A_V} \]
Similar implications on amplifier even if not a basic voltage feedback amplifier

\[ A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{1}{A_V} \left( 1 + \frac{R_2}{R_1} \right)} \]

These circuits have

- same \( \beta \)
- same dead network
- same characteristic polynomial
- same poles
- different zeros

\[ \beta = \frac{R_1}{R_2 + R_1} \]

\[ D(s) = 1 + A\beta \quad \text{(expressed as polynomial)} \]
Example: Determine the dc open-loop gain, dc closed-loop gain, the open-loop poles, the open-loop zeros, the closed-loop poles, the closed-loop zeros, and the characteristic polynomial if

\[ A(s) = 10^7 \frac{s+1}{(s+10)(s+1000)} \]
Example: Determine the dc open-loop gain, dc closed-loop gain, the open-loop poles, the open-loop zeros, the closed-loop poles, the closed-loop zeros, and the characteristic polynomial if

\[ A(s) = 10^7 \frac{s+1}{(s+10)(s+1000)} \]

\[ A_{OL} = \]

Open-loop zeros =

Open-loop poles =
Example: Determine the dc open-loop gain, dc closed-loop gain, the open-loop poles, the open-loop zeros, the closed-loop poles, the closed-loop zeros, and the characteristic polynomial if

\[
A(s) = \frac{10^7}{(s+10)(s+1000)}
\]

\[
\beta = \frac{R_1}{R_2 + R_1}
\]

\[
A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{-\frac{R_2}{R_1}}{1 + \frac{1}{A_V} \left(1 + \frac{R_2}{R_1}\right)}
\]

\[
A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{-\frac{R_2}{R_1} \left(\frac{10^7}{s+10} \frac{1}{s+1000}\right)}{1 + \frac{\beta}{10^7}(s+1)}
\]

\[
A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{-\frac{R_2}{R_1} \left(10^7 \beta(s+1)\right)}{(s+1)(10^7 \beta + (s+10)(s+1000))}
\]

\[
A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{-\frac{R_2}{R_1} \left(10^7 \beta(s+1)\right)}{(s+1)(10^7 \beta + (s+10)(s+1000))}
\]
Example: Determine the dc open-loop gain, dc closed-loop gain, the open-loop poles, the open-loop zeros, the closed-loop poles, the closed-loop zeros, and the characteristic polynomial if

\[
A(s) = \frac{s+1}{(s+10)(s+1000)}
\]

\[
A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{-\frac{R_2}{R_1}10^7\beta(s+1)}{(s+1)10^7\beta + (s+10)(s+1000)}
\]

\[
D_{FB}(s) = (s+1)10^7\beta + (s+10)(s+1000)
\]

In integer-monic form:

\[
D_{FB}(s) = s^2 + s(10+1000+10^7\beta) + 10^7\beta
\]

\[
A_{OF} =
\]

Closed-loop zeros =

Closed-loop poles =
Cascaded Amplifier Issues

- Three amplifier cascades - for ideally identical stages $8 > \beta A_0^3$
  -- seldom used in industry though some recent products use this method!

- Four or more amplifier cascades - problems even larger than for three stages
  -- seldom used in industry!
Consider the Frequency Response of Feedback Amplifier

Consider cascade of two stages, i.e. \( n=2 \)

\[
A_{FB} = \frac{A}{1 + A\beta} = \frac{A_0 A_0}{\left(\frac{s}{\tilde{\rho}_1} + 1\right)\left(\frac{s}{\tilde{\rho}_2} + 1\right) + \beta A_0 A_0}
\]

If we assume \( \tilde{\rho}_2 \geq \tilde{\rho}_1 \) and thus express \( \tilde{\rho}_2 = k\tilde{\rho}_1 \)

The characteristic polynomial can be expressed as

\[
D_{FB}(s) = s^2 + s\tilde{\rho}_1(1 + k) + k\tilde{\rho}_1^2(1 + \beta A_{0\text{TOT}})
\]

Note this amplifier is stable !!!!
(at least based upon this analysis)
Two-stage Cascade (continued)

\[ D_{FB}(s) = s^2 + s\tilde{p}_1(1+k) + k\tilde{p}_1^2(1+\beta A_{TOT}) \]

Consider special case of identical stages (i.e. \( k=1 \))

\[ D_{FB}(s) = s^2 + s\tilde{p}_1(2) + \tilde{p}_1^2(1+\beta A_{TOT}) \approx s^2 + s\tilde{p}_1(2) + \tilde{p}_1^2(\beta A_{TOT}) \]

thus the poles of the feedback amplifier are located at

\[ p_{1,2} = -\tilde{p}_1 \pm \sqrt{\tilde{p}_1^2(1-\beta A_{TOT})} \approx -\tilde{p}_1(1 \pm j\sqrt{\beta A_{TOT}}) \]

- FB poles are very close to the imaginary axis
- Very highly under damped
- Not useful as an amplifier (excessive ringing)
- Other poles will make it unstable
Two-stage Cascade (continued)

\[ D_{FB}(s) = s^2 + s\tilde{p}_1(1+k) + k\tilde{p}_1^2(1+\beta A_{0\text{TOT}}) \]

Thus, must make \( k \gg 1 \) if there is any potential for the two-stage cascade

\[ D_{FB}(s) \simeq s^2 + s\tilde{p}_1(k) + k\tilde{p}_1^2(\beta A_{0\text{TOT}}) \]

thus the poles of the feedback amplifier are located at

\[ p_{1,2} \simeq \frac{\tilde{p}_1}{2} \left( -k \pm j\sqrt{4A_{0\text{TOT}} k\beta - k^2} \right) \]

Case 1: No complex conjugate poles; must make discriminate 0, thus

\[ k \simeq 4\beta A_{0\text{TOT}} \]

Two equal FB poles will provide maximally fast time-domain response w/o ringing
Two-stage Cascade (continued)

\[ p_{1,2} \approx \frac{p_1}{2} \left( -k \pm j\sqrt{4A_{0\text{TOT}} k\beta - k^2} \right) \]

Case 2: Maximally flat magnitude response; must make real and imaginary parts equal

\[ k = \sqrt{4A_{0\text{TOT}} k\beta - k^2} \]

\[ k \approx 2\beta A_{0\text{TOT}} \]

- Small ringing in step response
- Factor of 2 reduction in pole spread
Two-stage Cascade (continued)

\[ p_{1,2} \approx \frac{p_1}{2} \left( -k \pm j\sqrt{4A_{_{0TOT}} k\beta - k^2} \right) \]

- The pole spread for maximal frequency domain flatness or fast non-ringing time domain response is quite large for the two-stage amplifier but can be achieved.

- Usually will make angle of feedback poles with imaginary axis between 45° and 90°

- This results in an open loop pole spread that satisfies the relationship
  \[ 4\beta A_{_{0TOT}} > k > 2\beta A_{_{0TOT}} \]

- “Compensation” is the modification of the pole locations of an amplifier to achieve a desired closed-loop pole angle.
Cascaded Amplifier Issues

- Single-stage amplifiers
  -- widely used in industry, little or no concern about compensation

- Two amplifier cascades
  \[ 4\beta A_{\text{TOT}} > k > 2\beta A_{\text{TOT}} \]
  -- widely used in industry but compensation is essential!

- Three amplifier cascades - for ideally identical stages
  \[ 8 > \beta A_0^3 \]
  -- seldom used in industry but starting to appear!

- Four or more amplifier cascades - problems even larger than for three stages
  -- seldom used in industry!

Note: Some amplifiers that are termed single-stage amplifiers in many books and papers are actually two-stage amplifiers and some require modest compensation. Some that are termed two-stage amplifiers are actually three-stage amplifiers. These invariable have a very small gain on the first stage and a very large bandwidth. The nomenclature on this summary refers to the number of stages that have reasonably large gain.
Summary of Cascaded Amplifier Characteristics

A cascade of amplifiers can result in a very high dc gain!

Characteristics of feedback amplifier (where the op amp is applied) are of ultimate concern

Some critical and fundamental issues came up with even the most basic cascades when they are used in a feedback configuration

Must understand how open-loop and closed-loop amplifier performance relate before proceeding to design amplifiers by cascading
Summary of Amplifier Characteristics

An amplifier is stable iff all poles lie in the open LHP

Routh-Hurwitz Criteria is often a practical way to determine if an amplifier is stable.

Although stability of an amplifier is critical, a good amplifier must not only be stable but generally must satisfy magnitude peaking and/or settling requirements thus poles need to be moved a reasonable distance from the imaginary axis.

The cascade of three identical high-gain amplifiers will result in a pole-pair far in the right half plane when feedback is applied so FB amplifier will be unstable.

\[ A_{FB} = \frac{A}{1 + A\beta} = \frac{A_0^3}{\left( \frac{s}{\beta} + 1 \right)^3 + \beta A_0^3} \]

For stability

\[ 8 > \beta A_0^3 \]
End of Lecture 12