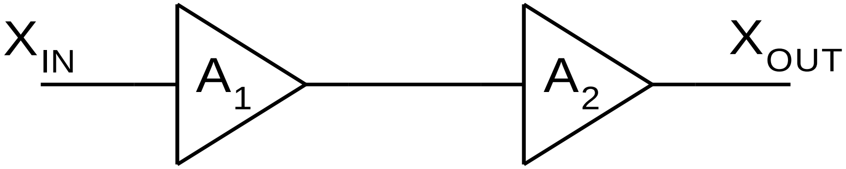
EE 435
Lecture 13

Cascaded Amplifiers

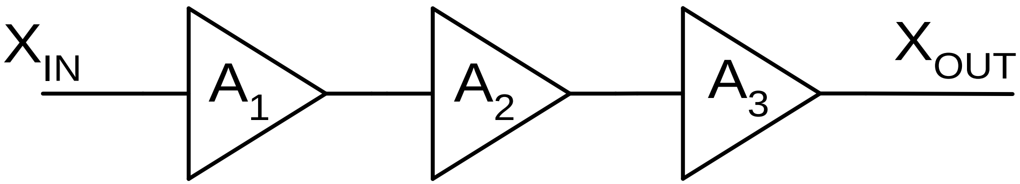
-- Two-Stage Op Amp Design

Increasing Gain by Cascading

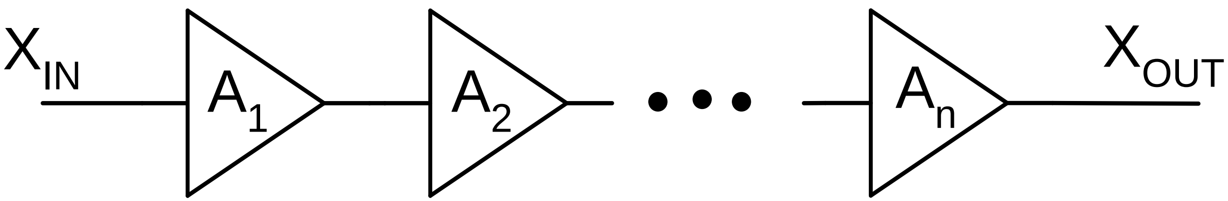
Provided the stages are non-interacting



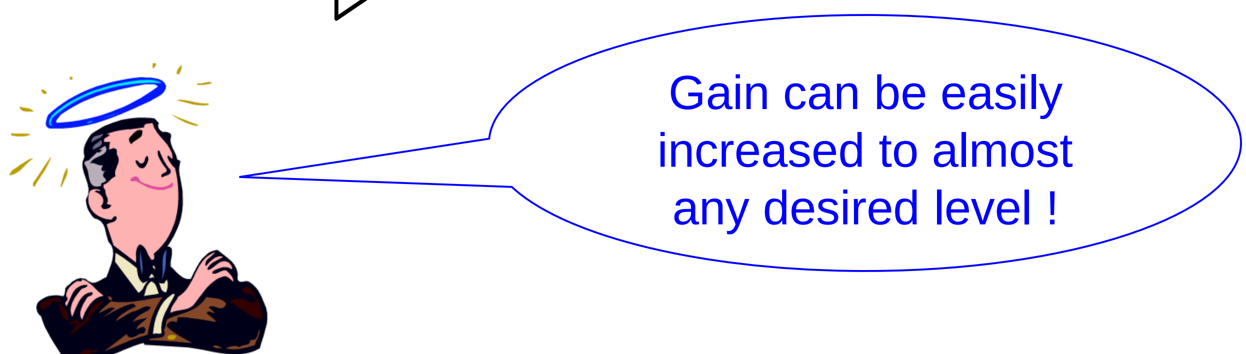
$$\frac{X_{OUT}}{X_{IN}} = A_1 A_2$$



$$\frac{X_{OUT}}{X_{IN}} = A_1 A_2 A_3$$

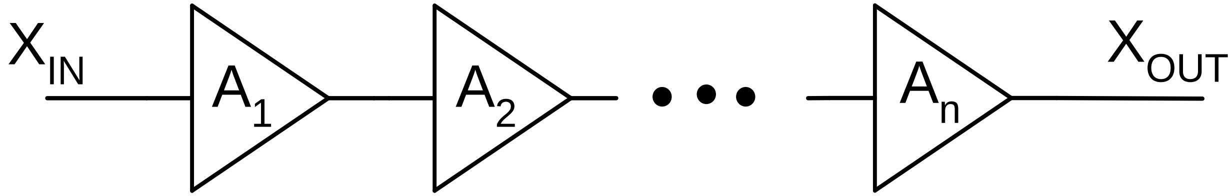


$$\frac{X_{OUT}}{X_{IN}} = \prod_{i=1}^{n} A_i$$



Review from Last Time

Increasing Gain by Cascading



$$\frac{X_{OUT}}{X_{IN}} = \prod_{i=1}^{n} A_i$$

But each of the gains will roll off with frequency so can be modeled as

$$A_k(s) = \frac{A_{0k}}{\frac{s}{\tilde{p}_k} + 1}$$

A_{0k} is the dc gain of stage k

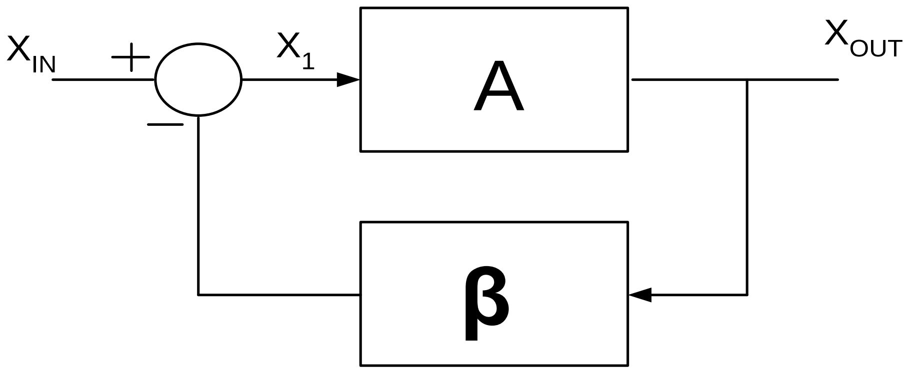
$\tilde{p}_k$ is the negative of the pole of stage k

Thus

$$\frac{X_{OUT}}{X_{IN}} = A = \frac{\prod_{i=1}^{n} A_{0i}}{\prod_{k=1}^{n}\left(\frac{s}{\tilde{p}_k} + 1\right)}$$

Review from Last Time

Frequency Response of Feedback Amplifier



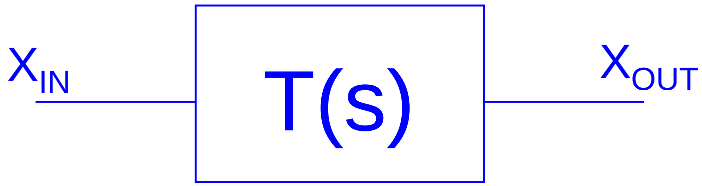
Consider the special case where A is the cascade of n identical stages

$$\frac{X_{OUT}}{X_1} = A = \frac{\prod_{k=1}^{n} A_{0k}}{\prod_{k=1}^{n}\left(\frac{s}{\tilde{p}_k}+1\right)} = \frac{A_0^n}{\left(\frac{s}{\tilde{p}}+1\right)^n}$$

$$A_{FB} = \frac{A}{1+A\beta} = \frac{A_0^n}{\left(\frac{s}{\tilde{p}}+1\right)^n + \beta A_0^n}$$

How do we determine how the amplifier is performing from A_{FB}?

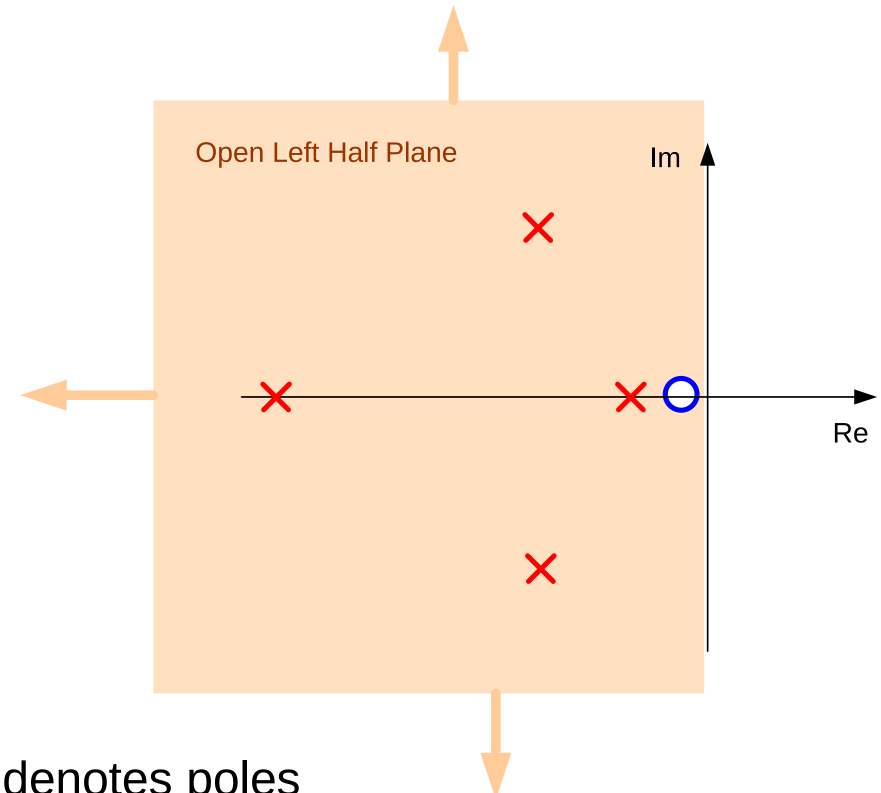
Review of Basic Concepts



If $\mathbf{T(s)} = \frac{\mathbf{N(s)}}{\mathbf{D(s)}}$ is the transfer function of a linear system

Roots of N(s) are termed the zeros

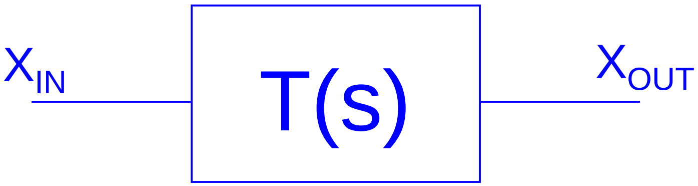
Roots of D(s) are termed the poles



X denotes poles

O denotes zeros

Review of Basic Concepts



If $\mathbf{T(s)} = \dfrac{\mathbf{N(s)}}{\mathbf{D(s)}}$ is the transfer function of a linear system

Roots of N(s) are termed the zeros

Roots of D(s) are termed the poles

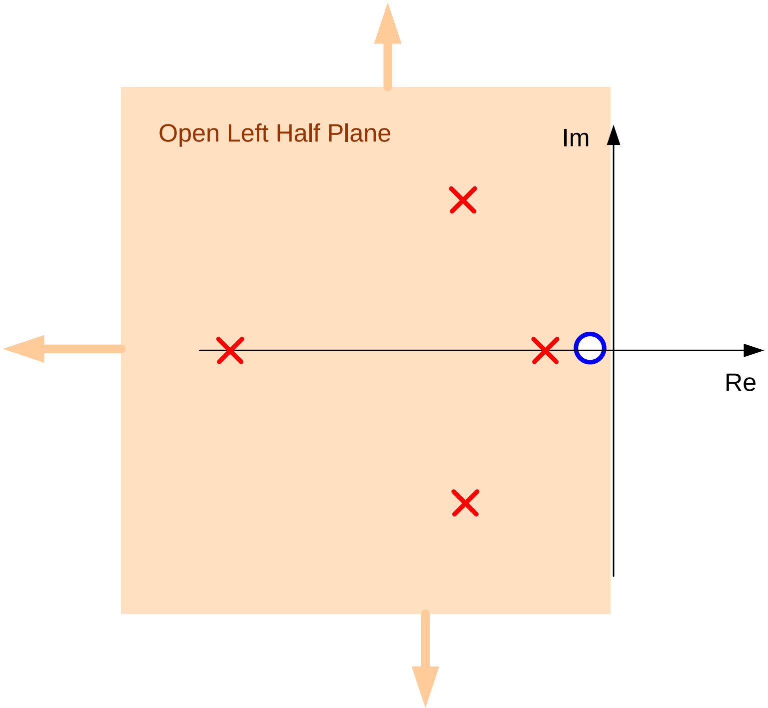
Theorem: A linear system is stable iff all poles lie in the open left half-plane

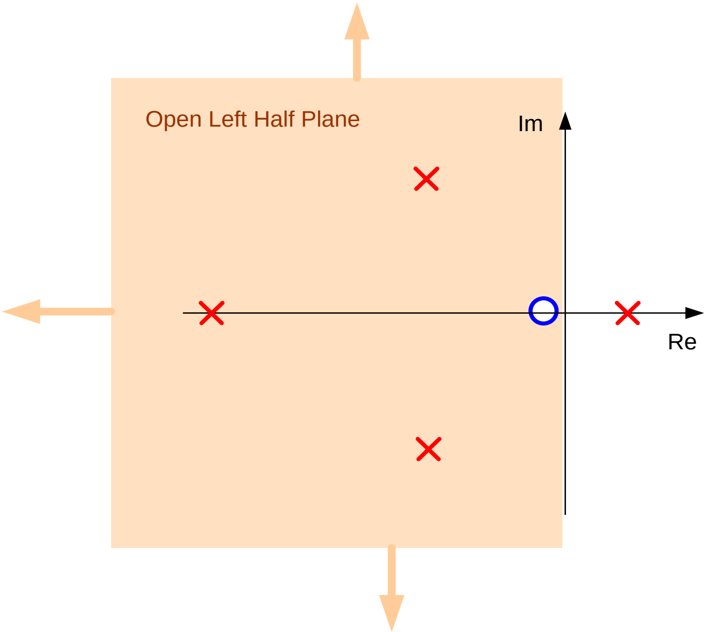
Claim: If a circuit is unstable, the output will either diverge to infinity or oscillate even if the input is set to 0

Claim: A circuit that is not stable is not a useful amplifier

Claim: A circuit that is “close” to becoming unstable is not a useful amplifier

Theorem: A linear system is stable iff all poles lie in the open left half-plane

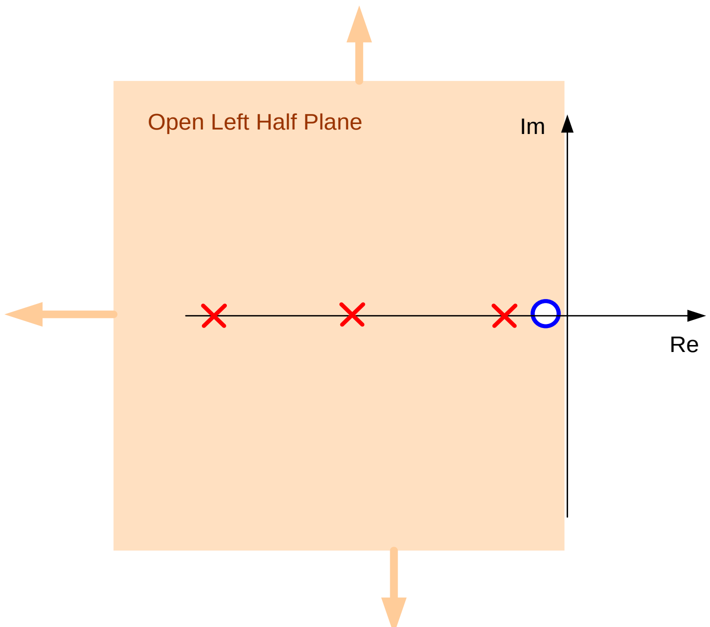




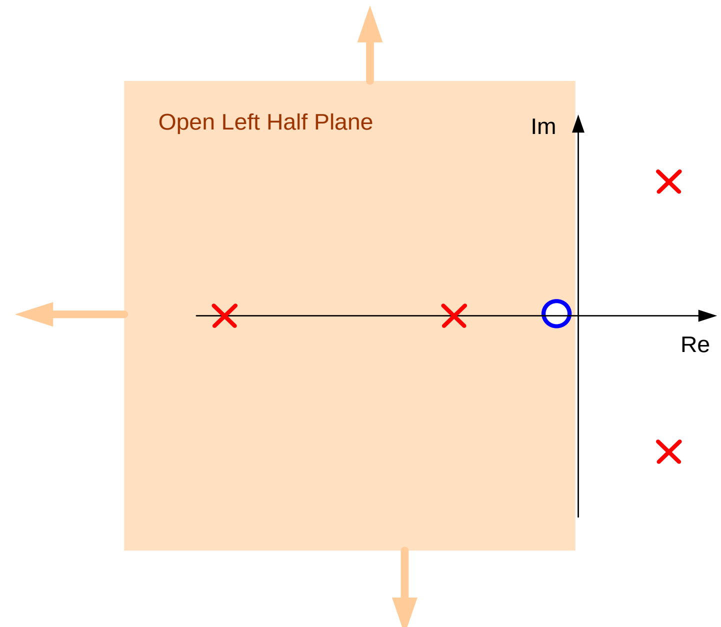
Stable with two negative
real axis poles and two LHP CC poles

Unstable with positive real
axis pole

Theorem: A linear system is stable iff all poles lie in the open left half-plane

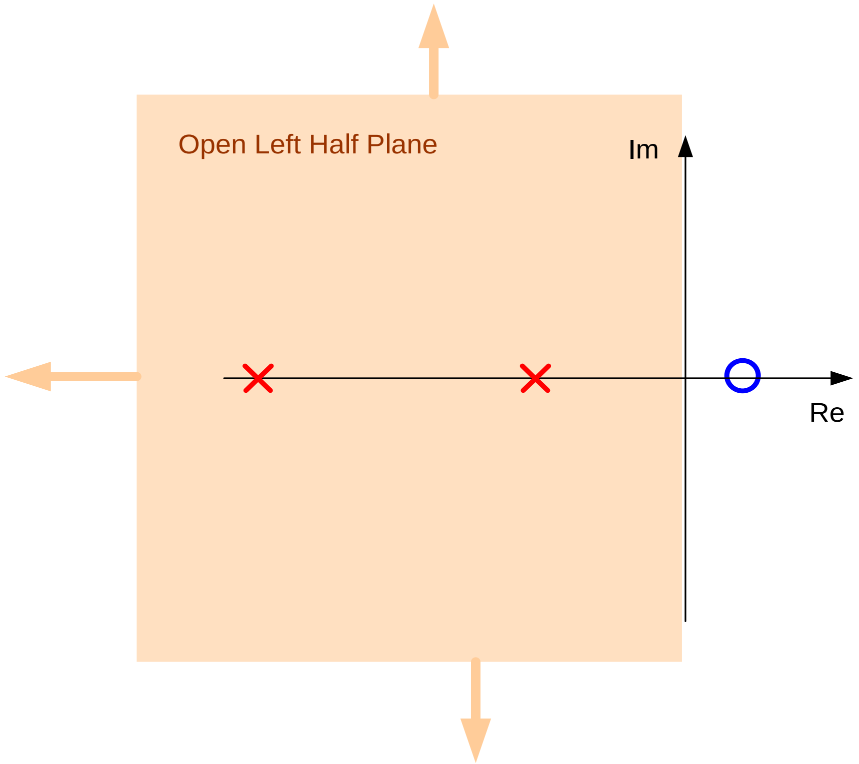


Stable with negative real axis poles



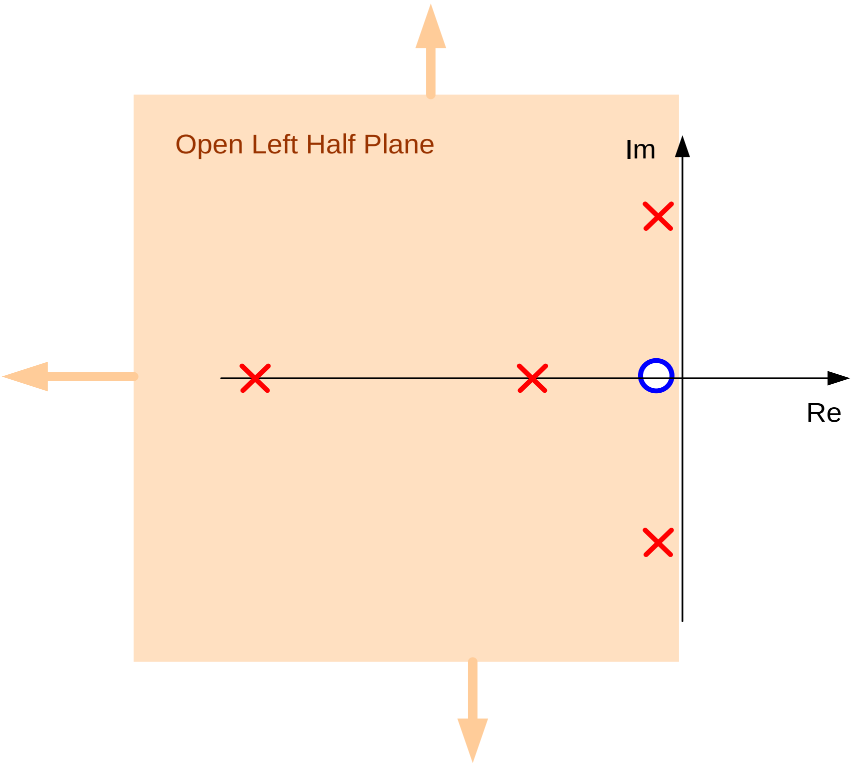
Unstable with cc RHP poles

Theorem: A linear system is stable iff all poles lie in the open left half-plane



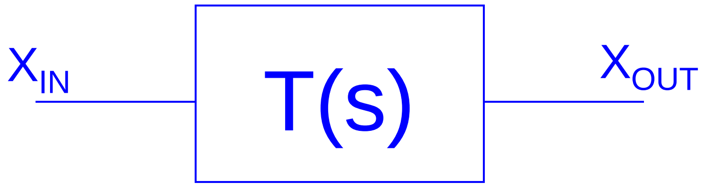
Stable with negative real-axis poles and RHP zero
System zero locations of have no impact on stability

Theorem: A linear system is stable iff all poles lie in the open left half-plane



Close to becoming unstable since poles are close to the RHP

Review of Basic Concepts



$$T(s) = \frac{N(s)}{D(s)}$$

Theorem: A linear system is stable iff all poles lie in the open left half-plane

What are the practical implications of stability and “close to becoming unstable” ?

For any input to a linear system, the response can be written as

$$X_{OUT}(s) = X_{IN}(s)T(s) = \sum_{k=1}^{n} \frac{a_k}{s+\tilde{p}_k} + \sum_{k=1}^{h} \frac{b_k}{s+\tilde{x}_k}$$

where the terms $\tilde{p}_k$ are the <u>negative</u> of the poles of T(s), the terms $\tilde{x}_k$ are the negative of the roots of the denominator of the excitation and the terms a_k and b_k are the partial fraction expansion coefficients

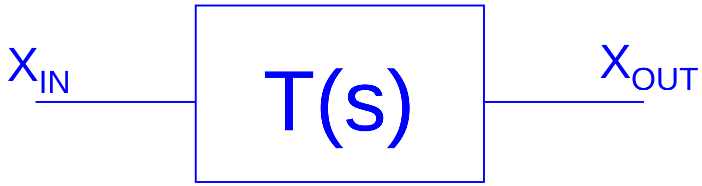
If $\tilde{p}_k$ is the negative of any pole, then $\tilde{p}_k$ can be expressed as

$$\tilde{p}_k = -\alpha_k - j\beta_k$$

where α_k is the real part of the pole and β_k is the imaginary part of the pole

$$p_k = -\tilde{p}_k = \alpha_k + j\beta_k$$

Review of Basic Concepts



$$T(s)=\frac{N(s)}{D(s)}$$

Theorem: A linear system is stable iff all poles lie in the open left half-plane

What are the practical implications of stability and “close to becoming unstable” ?

It thus follows that

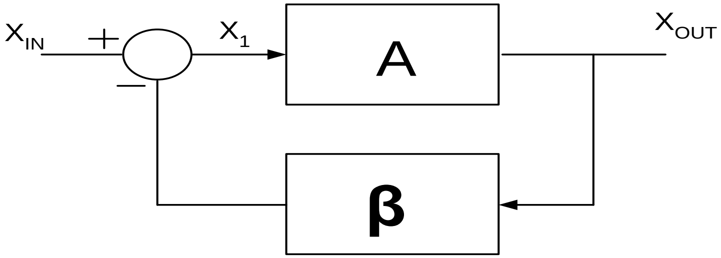
$$X_{OUT}(t)=\mathcal{L}^{-1}\left(X_{IN}(s)T(s)\right)=\sum_{k=1}^{n}a_k e^{\alpha_k t}e^{j\beta_k t}+\sum_{k=1}^{h}b_k e^{-j\tilde{x}_k t}$$

Thus, for the output to be bounded for ANY input, must have ALL $\alpha_k<0$

That is equivalent to saying all poles must lie in the left half-plane

If a pole is in the RHP, output for any input (even very small noise) will grow to infinity. If the corresponding β_k=0, output will latch up. If corresponding $\beta_k \neq 0$, output will be a growing sinusoid

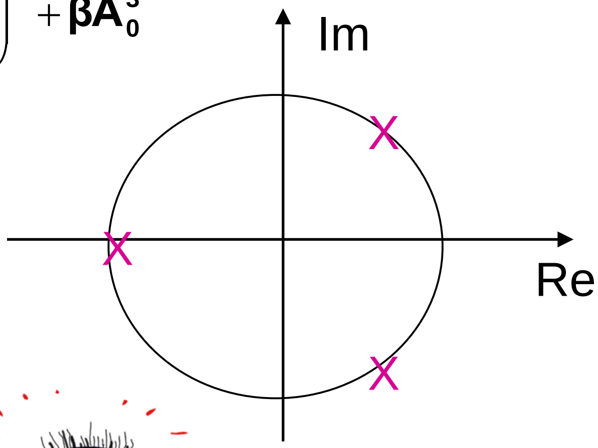
Consider Again the Frequency Response of a Feedback Amplifier with identical gain stages



$$A_{FB} = \frac{A_0^n}{\left(\frac{s}{\tilde{p}}+1\right)^n + \beta A_0^n}$$

Example: Assume n=3

$$A_{FB} = \frac{A}{1+A\beta} = \frac{A_0^3}{\left(\frac{s}{\tilde{p}}+1\right)^3 + \beta A_0^3}$$

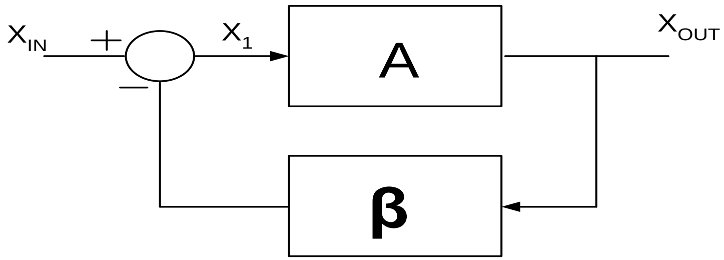


The poles with feedback, p_F, are given by

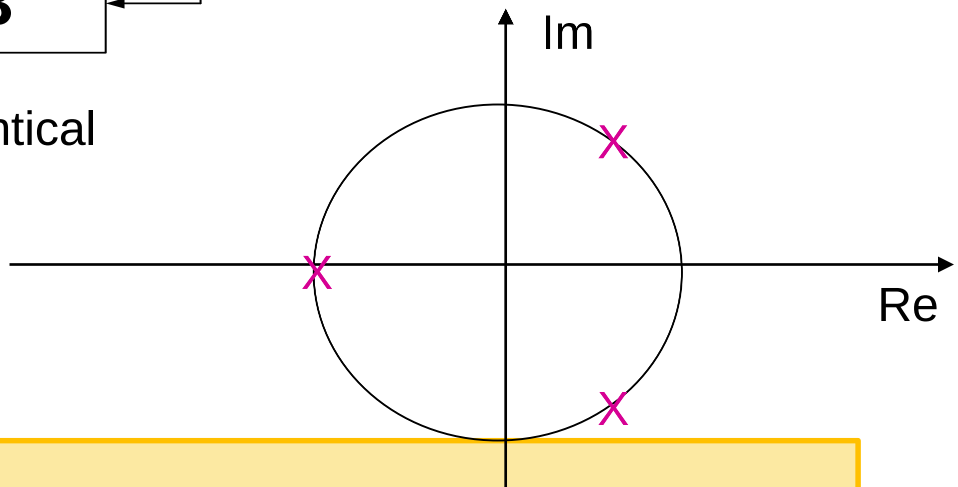
$$p_F = \left((-1)^{1/3}\beta^{1/3}A_0 - 1\right)\tilde{p} \cong (-1)^{1/3}\beta^{1/3}A_0\tilde{p}$$

Note this amplifier is unstable !!!

Consider Again the Frequency Response of Feedback Amplifier



$$A=\frac{A_0\,\tilde{p}}{s+\tilde{p}}$$



Example: If n=3 and stages are identical

$$A_{FB}=\frac{A}{1+A\beta}=\frac{A_0^3}{\left(\frac{s}{\tilde{p}}+1\right)^3+\beta A_0^3}$$

Routh-Hurwitz Stability Criteria:

A third-order polynomial $s^3+a_2s^2+a_1s+a_0$ has all poles in the LHP iff all coefficients are positive and $a_1a_2>a_0$

Consider
$$D_{FB}(s)=\left(\frac{s}{\tilde{p}}+1\right)^3+\beta A_0^3=s^3\left(\frac{1}{\tilde{p}^3}\right)+s^2\frac{3}{\tilde{p}^2}+s\frac{3}{\tilde{p}}+\left(1+\beta A_0^3\right)$$

For stability

$$\left(3\tilde{p}\right)\left(3\tilde{p}^2\right)>\tilde{p}^3\left(1+\beta A_0^3\right)\qquad\qquad 8>\beta A_0^3$$

Not only is the 3-stage amplifier unstable, it is far from being stable!

Routh-Hurwitz Stability Criteria:

A third-order polynomial $s^3+a_2s^2+a_1s+a_0$ has all poles in the LHP iff all coefficients are positive and $a_1a_2>a_0$

- Very useful in amplifier and filter design
- Can easily determine if poles in LHP without finding poles
- But tells little about how far in LHP poles may be
- RH exists for higher-order polynomials as well

Example:

Assume an amplifier has a transfer function that has a denominator polynomial that can be expressed as

$$D(s)=s^3+2ks^2+4s+16$$

Determine the minimum value of k that will result in a stable amplifier

Solution:

Assume an amplifier has a transfer function that has a denominator polynomial that can be expressed as

$$D(s)=s^3+2ks^2+4s+16$$

Determine the minimum value of k that will result in a stable amplifier

Solution: Recall from the RH criteria that all roots of a third-order polynomial of the form $s^3+a_2s^2+a_1s+a_0$ will lie in the LHP provided all coefficients are positive and $a_1a_2 > a_0$

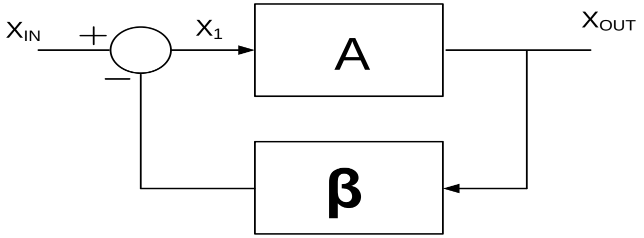
Thus, for the current problem, must have

$$(2k)4 > 16$$

or

$$k>2$$

Consider Again the Frequency Response of the basic Feedback Amplifier



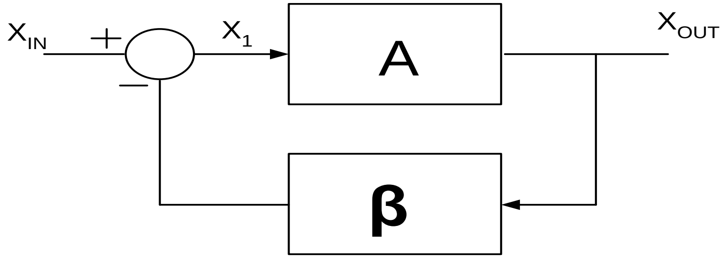
Example: If n=3 and stages are not identical

$$A_{FB}=\frac{A}{1+A\beta}=\frac{A_{01}A_{02}A_{03}}{\left(\frac{s}{\tilde{p}_1}+1\right)\left(\frac{s}{\tilde{p}_2}+1\right)\left(\frac{s}{\tilde{p}_3}+1\right)+\beta A_{02}A_{03}A_{03}}$$

$$D_{FB}(s)=s^3+s^2\left(\tilde{p}_1+\tilde{p}_2+\tilde{p}_3\right)+s\left(\tilde{p}_1\tilde{p}_2+\tilde{p}_1\tilde{p}_3+\tilde{p}_2\tilde{p}_3\right)+\tilde{p}_1\tilde{p}_2\tilde{p}_3\left(1+\beta A_{0TOT}\right)$$

where $A_{0TOT}=A_{01}A_{02}A_{03}$

Consider Again the Frequency Response of Feedback Amplifier



Example: If n=3 and stages are not identical (cont)

$$D_{FB}(s) = s^3 + s^2\left(\tilde{p}_1 + \tilde{p}_2 + \tilde{p}_3\right) + s\left(\tilde{p}_1\tilde{p}_2 + \tilde{p}_1\tilde{p}_3 + \tilde{p}_2\tilde{p}_3\right) + \tilde{p}_1\tilde{p}_2\tilde{p}_3\left(1 + \beta A_{0TOT}\right)$$

Routh-Hurwitz Stability Criteria: (by assuming $1+\beta A_{0TOT} \cong \beta A_{0TOT}$)

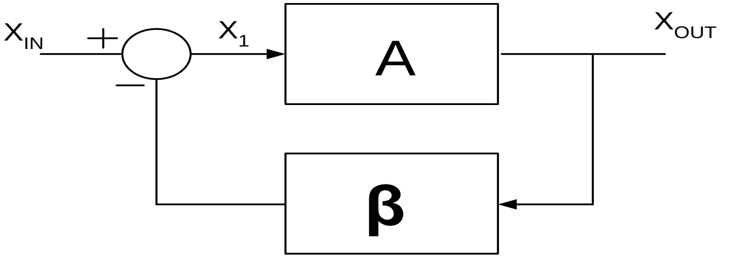
$$\left(\tilde{\mathbf{p}}_1 + \tilde{\mathbf{p}}_2 + \tilde{\mathbf{p}}_3\right)\left(\tilde{\mathbf{p}}_1\tilde{\mathbf{p}}_2 + \tilde{\mathbf{p}}_1\tilde{\mathbf{p}}_3 + \tilde{\mathbf{p}}_2\tilde{\mathbf{p}}_3\right) > \tilde{\mathbf{p}}_1\tilde{\mathbf{p}}_2\tilde{\mathbf{p}}_3\,\boldsymbol{\beta}\,\mathbf{A}_{0TOT}$$

WOLG, assume $\tilde{\mathbf{p}}_1 < \tilde{\mathbf{p}}_2 < \tilde{\mathbf{p}}_3$ and define $\tilde{\mathbf{p}}_2 = \mathbf{k}_2\tilde{\mathbf{p}}_1$ **and** $\tilde{\mathbf{p}}_3 = \mathbf{k}_3\tilde{\mathbf{p}}_1$

Thus the RH criteria can be expressed as

$$\left(\mathbf{1} + \mathbf{k}_2 + \mathbf{k}_3\right)\left(\mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_2\mathbf{k}_3\right) > \boldsymbol{\beta}\mathbf{A}_{0TOT}$$

Consider Again the Frequency Response of Feedback Amplifier (cont)



Example: If n=3 and stages are not identical

RH criteria:

$$\left(1+k_2+k_3\right)\left(k_2+k_3+k_2k_3\right)>\beta A_{0TOT}$$

Since A_{0TOT} will, in general, be very large for the cascade of 3 stages, a very large pole ratio is required just to maintain stability and an even larger ratio needed to avoid a close to becoming unstable situation

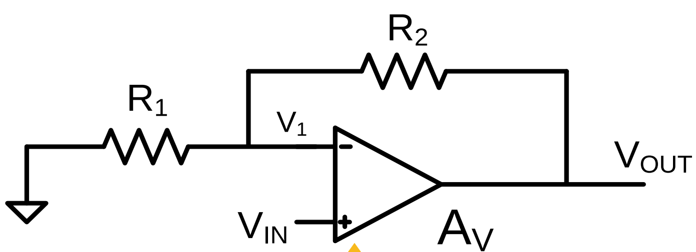
Practically it is difficult to obtain such a large spread in the bandwidth of the amplifiers

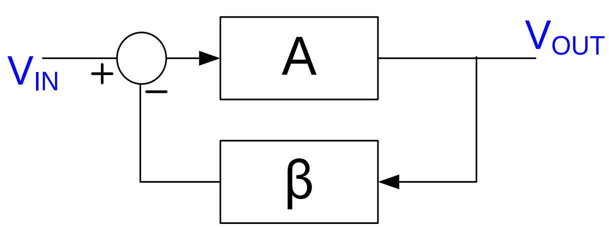
Problem can be viewed as one of accumulating too much phase shift before gain drops to an acceptable value

For many years there was limited commercial use of the cascade of three amplifiers (each with gain) in the design of op amps though some academic groups have worked on this approach with minimal practical success

In recent years, industry is looking at ways to “compensate” amplifiers to work with 3 (or more) high gain stages due to low headroom and shrinking g_m/g_o ratios

Similar implications on inverting amplifier even if not a basic voltage feedback amplifier

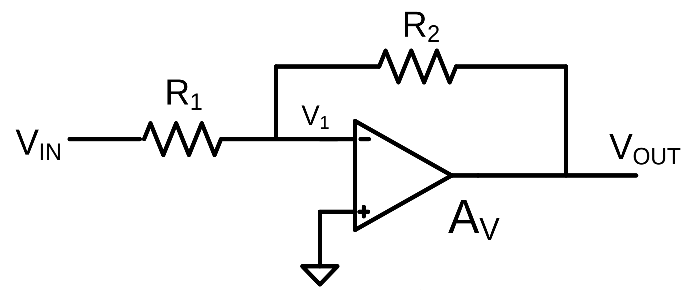


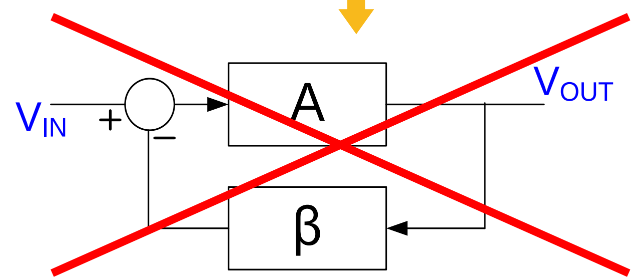


$$A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{1+\frac{R_2}{R_1}}{1+\frac{1}{A_V}\left(1+\frac{R_2}{R_1}\right)}$$

$$\beta = \frac{R_1}{R_2+R_1}$$

$$A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{A_V}{1+\beta A_V}$$



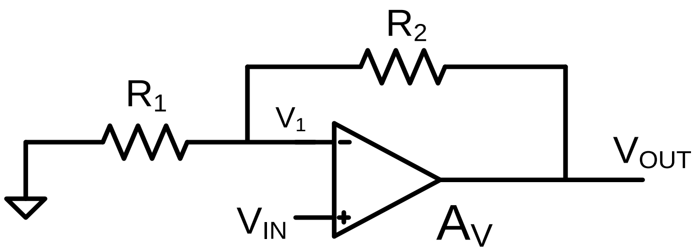


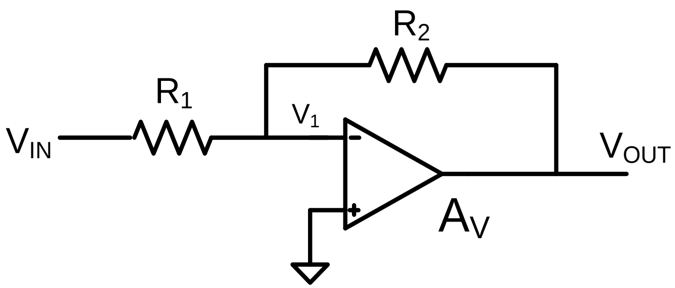
$$A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{-\frac{R_2}{R_1}}{1+\frac{1}{A_V}\left(1+\frac{R_2}{R_1}\right)}$$

$$\beta = \frac{R_1}{R_2+R_1}$$

$$A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{A_V}{1+\beta A_V}$$

Similar implications on inverting amplifier even if not a basic voltage feedback amplifier





$$A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{1+\frac{R_2}{R_1}}{1+\frac{1}{A_V}\left(1+\frac{R_2}{R_1}\right)}$$

$$A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{A_V}{1+A_V\left(\frac{R_1}{R_2+R_1}\right)}$$

$$A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{-\frac{R_2}{R_1}}{1+\frac{1}{A_V}\left(1+\frac{R_2}{R_1}\right)}$$

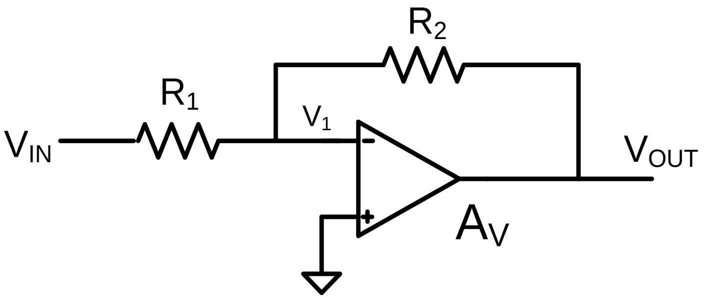
$$A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{A_V\left(\frac{-R_2}{R_1}\right)}{1+A_V\left(\frac{R_1}{R_2+R_1}\right)}$$

These circuits have

- same β $\quad \beta = \frac{R_1}{R_2+R_1}$
- same dead network
- same characteristic polynomial $\quad D(s)=1+A\beta$ (expressed as polynomial)
- same poles
- different zeros

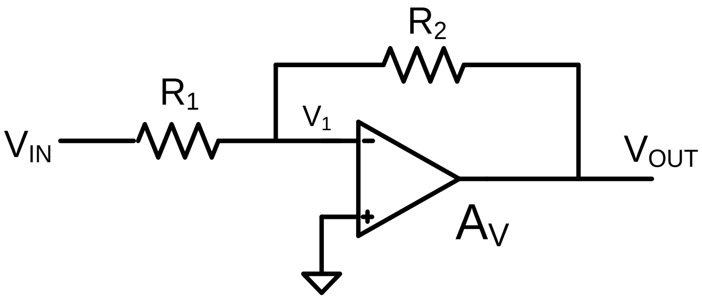
Example: Determine the dc open-loop gain, dc closed-loop gain, the open-loop poles, the open-loop zeros, the closed-loop poles, the closed-loop zeros, and the characteristic polynomial if

$$A(s)=10^7 \frac{s+1}{(s+10)(s+1000)}$$



Example: Determine the dc open-loop gain, dc closed-loop gain, the open-loop poles, the open-loop zeros, the closed-loop poles, the closed-loop zeros, and the characteristic polynomial if

$$A(s)=10^7\frac{s+1}{(s+10)(s+1000)}$$

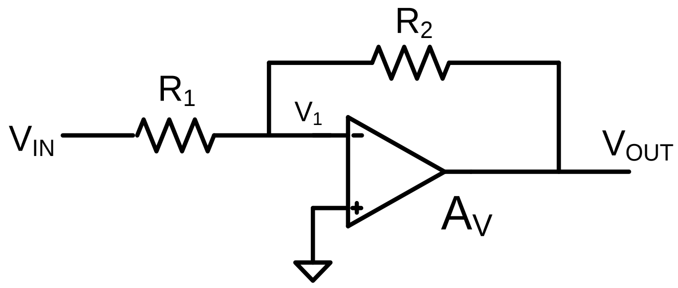


$A_{OL}=$

Open-loop zeros =

Open-loop poles =

Example: Determine the dc open-loop gain, dc closed-loop gain, the open-loop poles, the open-loop zeros, the closed-loop poles, the closed-loop zeros, and the characteristic polynomial if $A(s)=10^7 \frac{s+1}{(s+10)(s+1000)}$



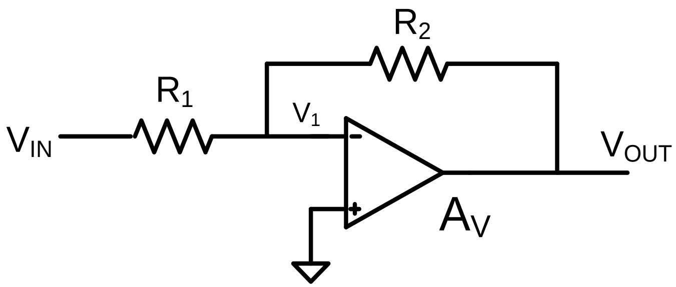
$$A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{-\frac{R_2}{R_1}}{1+\frac{1}{A_V}\left(1+\frac{R_2}{R_1}\right)}$$

$$\beta = \frac{R_1}{R_2+R_1}$$

$$A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{-\frac{R_2}{R_1}}{1+\frac{(s+10)(s+1000)}{10^7\beta(s+1)}}$$

$$A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{-\frac{R_2}{R_1}10^7\beta(s+1)}{(s+1)10^7\beta + (s+10)(s+1000)}$$

Example: Determine the dc open-loop gain, dc closed-loop gain, the open-loop poles, the open-loop zeros, the closed-loop poles, the closed-loop zeros, and the characteristic polynomial if $A(s)=10^7\dfrac{s+1}{(s+10)(s+1000)}$



$$A_{VF} = \frac{V_{OUT}}{V_{IN}} = \frac{-\frac{R_2}{R_1}10^7\beta(s+1)}{(s+1)10^7\beta + (s+10)(s+1000)}$$

$$D_{FB}(s) = (s+1)10^7\beta + (s+10)(s+1000)$$

In integer-monic form:

$$D_{FB}(s) = s^2 + s\left(10+1000+10^7\beta\right)+10^7\beta$$

A_{OF}=

Closed-loop zeros =

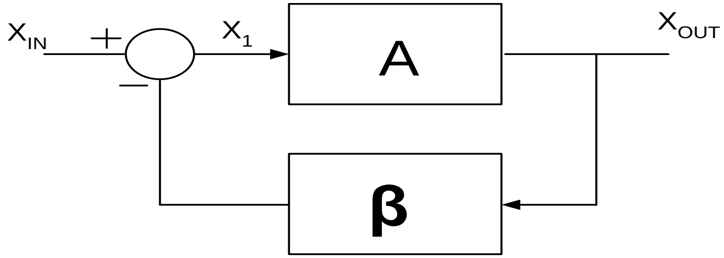
Closed-loop poles =

Cascaded Amplifier Issues

For first-order lowpass stage gains $$A = \frac{A_0 \tilde{p}}{s + \tilde{p}}$$

- Three amplifier cascades - for ideally identical stages $\mathbf{8 > \beta A_0^3}$
  - -- seldom used in industry though some recent products use this method !
  - -- invariably modify A
- Four or more amplifier cascades - problems even larger than for three stages
  - -- seldom used in industry !

Consider Again the Frequency Response of Feedback Amplifier



$$A = \frac{A_0\ \tilde{p}}{s+\tilde{p}}$$

Consider cascade of two stages, i.e. n=2

$$\mathbf{A_{FB}} = \frac{\mathbf{A}}{\mathbf{1}+\mathbf{A\beta}} = \frac{\mathbf{A_{01}A_{02}}}{\left(\frac{\mathbf{s}}{\tilde{\mathbf{p}}_1}+\mathbf{1}\right)\left(\frac{\mathbf{s}}{\tilde{\mathbf{p}}_2}+\mathbf{1}\right)+\mathbf{\beta A_{01}A_{02}}}$$

If we assume $\tilde{\mathbf{p}}_2 \geq \tilde{\mathbf{p}}_1$ **and thus express** $\tilde{\mathbf{p}}_2 = \mathbf{k}\tilde{\mathbf{p}}_1$

The characteristic polynomial can be expressed as

$$\mathbf{D_{FB}(s)} = \mathbf{s^2} + \mathbf{s}\tilde{\mathbf{p}}_1(\mathbf{1}+\mathbf{k}) + \mathbf{k}\tilde{\mathbf{p}}_1^2(\mathbf{1}+\mathbf{\beta A_{0TOT}})$$

Note this amplifier is stable !!!!
(at least based upon this analysis)

Two-stage Cascade (continued)

$$A = \frac{A_0\ \tilde{p}}{s+\tilde{p}}$$

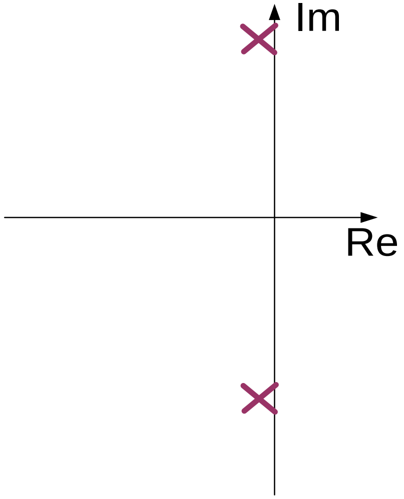
$$\mathbf{D_{FB}(s) = s^2 + s\tilde{p}_1(1+k) + k\tilde{p}_1^2(1+\beta A_{0TOT})}$$

Consider special case of identical stages (i.e. k=1)

$$\mathbf{D_{FB}(s) = s^2 + s\tilde{p}_1(2) + \tilde{p}_1^2(1+\beta A_{0TOT}) \cong s^2 + s\tilde{p}_1(2) + \tilde{p}_1^2(\beta A_{0TOT})}$$

thus the poles of the feedback amplifier are located at

$$\mathbf{p_{1,2} = -\tilde{p}_1 \pm \sqrt{\tilde{p}_1^2(1-\beta A_{0TOT})} \cong -\tilde{p}_1\left(1 \pm j\sqrt{\beta A_{0TOT}}\right)}$$



- FB poles are very close to the imaginary axis
- Very highly under damped
- Not useful as an amplifier (excessive ringing)
- Other poles will make it unstable

Two-stage Cascade (continued)

$$A = \frac{A_0\,\tilde{p}}{s+\tilde{p}}$$

$$\mathbf{D_{FB}(s) = s^2 + s\tilde{p}_1(1+k) + k\tilde{p}_1^2(1+\beta A_{0TOT})}$$

Thus, must make k >> 1 if there is any potential for the two-stage cascade

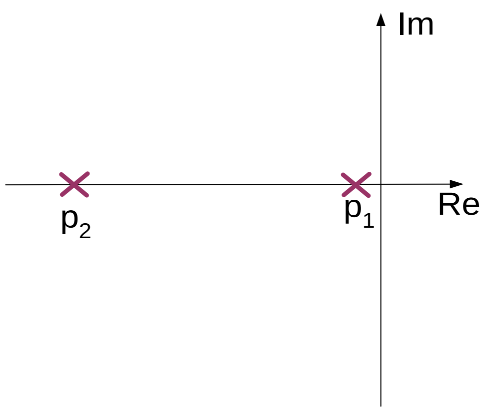
$$\mathbf{D_{FB}(s) \cong s^2 + s\tilde{p}_1(k) + k\tilde{p}_1^2(\beta A_{0TOT})}$$

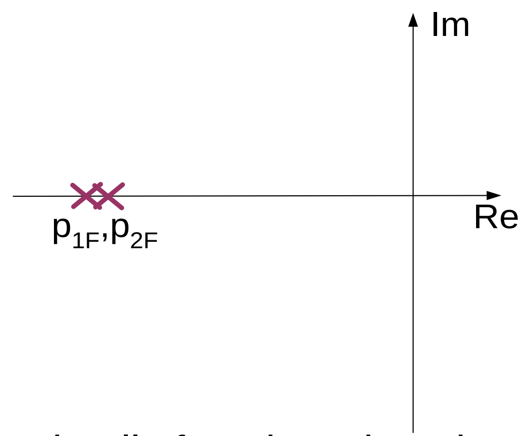
thus the poles of the feedback amplifier are located at

$$\mathbf{p_{1,2} \cong \frac{\tilde{p}_1}{2}\left(-k \pm j\sqrt{4A_{0TOT}\,k\beta - k^2}\right)}$$

Case 1: No complex conjugate poles; must make discriminate 0, thus

$$\mathbf{k \cong 4\beta\ A_{0TOT}}$$





Two equal FB poles on real axis will provide maximally fast time-domain response w/o ringing

Two-stage Cascade (continued)

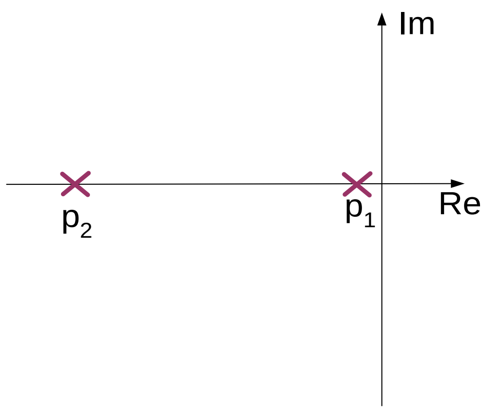
$$\mathbf{p_{1,2} \cong \frac{\tilde{p}_1}{2}\left(-k \pm j\sqrt{4A_{0TOT}\,k\beta - k^2}\right)}$$

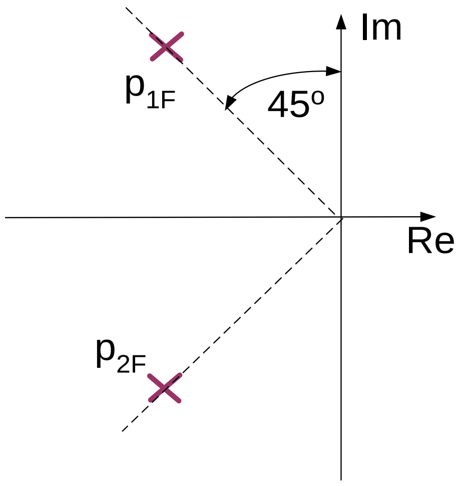
$$A = \frac{A_0\ \tilde{p}}{s+\tilde{p}}$$

Case 2: Maximally flat magnitude response; must make real and imaginary parts equal

$$\mathbf{k = \sqrt{4A_{0TOT}\,k\beta - k^2}}$$

$$\mathbf{k \cong 2\beta\ A_{0TOT}}$$





- Small ringing in step response
- Factor of 2 reduction in pole spread

Two-stage Cascade (continued)

$$\mathbf{p_{1,2} \cong \frac{\tilde{p}_1}{2}\left(-k \pm j\sqrt{4A_{0TOT}\,k\beta - k^2}\right)}$$

$$A = \frac{A_0\ \tilde{p}}{s+\tilde{p}}$$

- The pole spread for maximal frequency domain flatness or fast non-ringing time domain response is quite large for the two-stage amplifier but can be achieved
- Usually will make angle of feedback poles with imaginary axis between 45^o and 90^o
- This results (for all-pole cascade) in an open loop pole spread that satisfies the relationship $\mathbf{4\beta\ A_{0TOT} > k > 2\beta\ A_{0TOT}}$
- "Compensation" is the modification of the pole locations of an amplifier to achieve a desired closed-loop pole angle

Cascaded Amplifier Issues

$$A = \frac{A_0 \tilde{p}}{s + \tilde{p}}$$

- Single-stage amplifiers

  -- widely used in industry, little or no concern about compensation

- Two amplifier cascades $\quad 4\beta A_{0TOT} > k > 2\beta A_{0TOT}$

  -- widely used in industry but compensation is essential !

- Three amplifier cascades - for ideally identical stages $\quad 8 > \beta A_0^3$

  -- seldom used in industry but starting to appear !

- Four or more amplifier cascades - problems even larger than for three stages

  -- seldom used in industry !

Note: Some amplifiers that are termed single-stage amplifiers in many books and papers are actually two-stage amplifiers and some require modest compensation. Some that are termed two-stage amplifiers are actually three-stage amplifiers. These invariable have a very small gain on the first stage and a very large bandwidth. The nomenclature on this summary refers to the number of stages that have reasonably large gain. Results given above vary somewhat if a zero is present in the amplifier.

Summary of Cascaded Amplifier Characteristics

A cascade of amplifiers can result in a very high dc gain !

Characteristics of feedback amplifier (where the op amp is applied) are of ultimate concern

Some critical and fundamental issues came up with even the most basic cascades when they are used in a feedback configuration

Must understand how open-loop and closed-loop amplifier performance relate before proceeding to design amplifiers by cascading

Summary of Amplifier Characteristics

An amplifier is stable iff all poles lie in the open LHP

Routh-Hurwitz Criteria is often a practical way to determine if an amplifier is stable

Although stability of an amplifier is critical, a good amplifier must not only be stable but generally must satisfy magnitude peaking and/or settling requirements thus poles need to be moved a reasonable distance from the imaginary axis

The cascade of three identical high-gain amplifiers will result in a pole-pair far in the right half plane when feedback is applied so FB amplifier will be unstable

$$\mathbf{A_{FB}} = \frac{\mathbf{A}}{\mathbf{1} + \mathbf{A\beta}} = \frac{\mathbf{A_0^3}}{\left(\frac{\mathbf{s}}{\tilde{\mathbf{p}}} + \mathbf{1}\right)^3 + \boldsymbol{\beta}\mathbf{A_0^3}}$$

$$A = \frac{A_0\,\tilde{p}}{s + \tilde{p}}$$

For stability

$$\mathbf{8} > \boldsymbol{\beta}\mathbf{A_0^3}$$

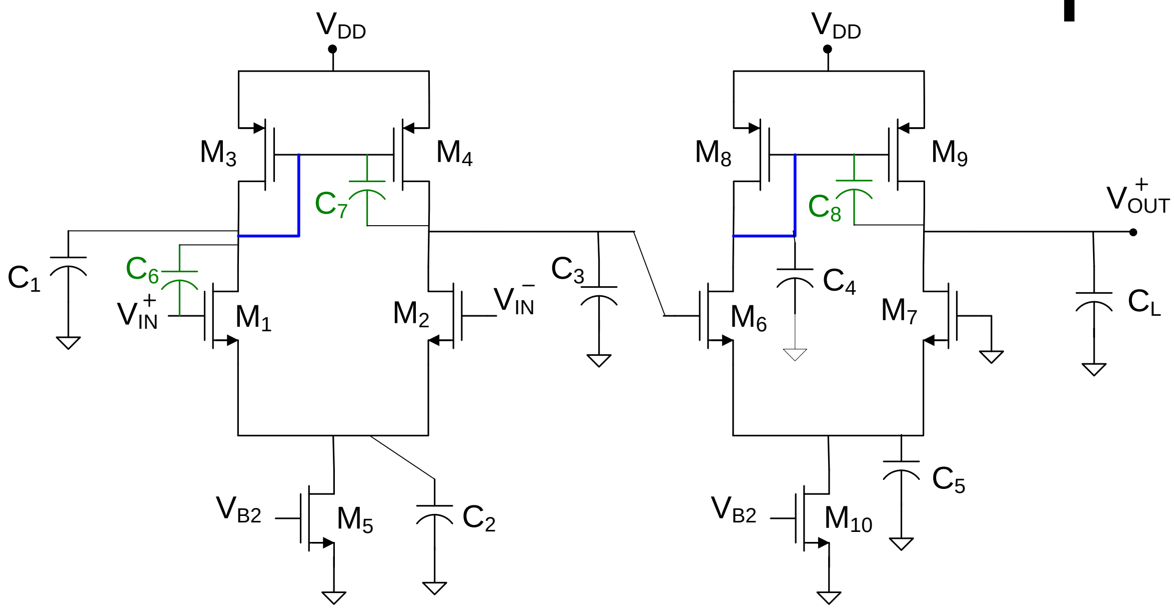
End of Lecture 13

- Fundamental Amplifier Design Issues
- Single-Stage Low Gain Op Amps
- Single-Stage High Gain Op Amps
- Two-Stage Op Amp
  - Compensation
  - Breaking the Loop
- Other Basic Gain Enhancement Approaches
- Other Issues in Amplifier Design
- Summary Remarks

Two-stage op amp design

It is essential to know where the poles of the op amp are located since there are some rather strict requirements about the relative location of the open-loop poles when the op amp is used in a feedback configuration.

Poles and Zeros of Amplifiers



Cascaded Amplifier showing <u>some</u> of the capacitors

- There are a large number of parasitic capacitors in an amplifier (appprox 5 for each transistor)
- Many will appear in parallel but the number of equivalent capacitors can still be large
- Order of transfer function is equal to the number of non-degenerate energy storage elements
- Obtaining the transfer function of a high-order network is a lot of work !
- Essentially every node in an amplifier has a capacitor to ground and these often dominate the frequency response of the amplifier (but not always)

Pole approximation methods

1. Consider all shunt capacitors

2. Decompose these into two sets, those that create low frequency poles and those that create high frequency poles (large capacitors create low frequency poles and small capacitors create high frequency poles)
   $\{C_{L1}, \ldots C_{Lk}\}$ and $\{C_{H1}, \ldots C_{Hm}\}$

3. To find the k low frequency poles, replace all independent voltage sources with ss shorts and all independent current sources with ss opens, all high-frequency capacitors with ss open circuits and, one at a time, select C_{Lh} and determine the impedance facing it, say R_{Lh} if all other low-frequency capacitors are replaced with ss open circuits. Then an approximation for the pole corresponding to C_{Lh} is

$$p_{Lh}=-1/(R_{Lh}C_{Lh})$$

4. To find the m high-frequency poles, replace all independent voltage sources with ss shorts and all independent current sources with ss opens, replace all low-frequency capacitors with ss short circuits and, one at a time, select C_{Hh} and determine the impedance facing it, say R_{Hh} if all other high-frequency capacitors are replaced with ss open circuits. Then the approximation for the pole corresponding to C_{Hh} is

$$p_{Hh}=-1/(R_{Hh}C_{Hh})$$

Pole approximation methods

These are just pole approximations but are often quite good

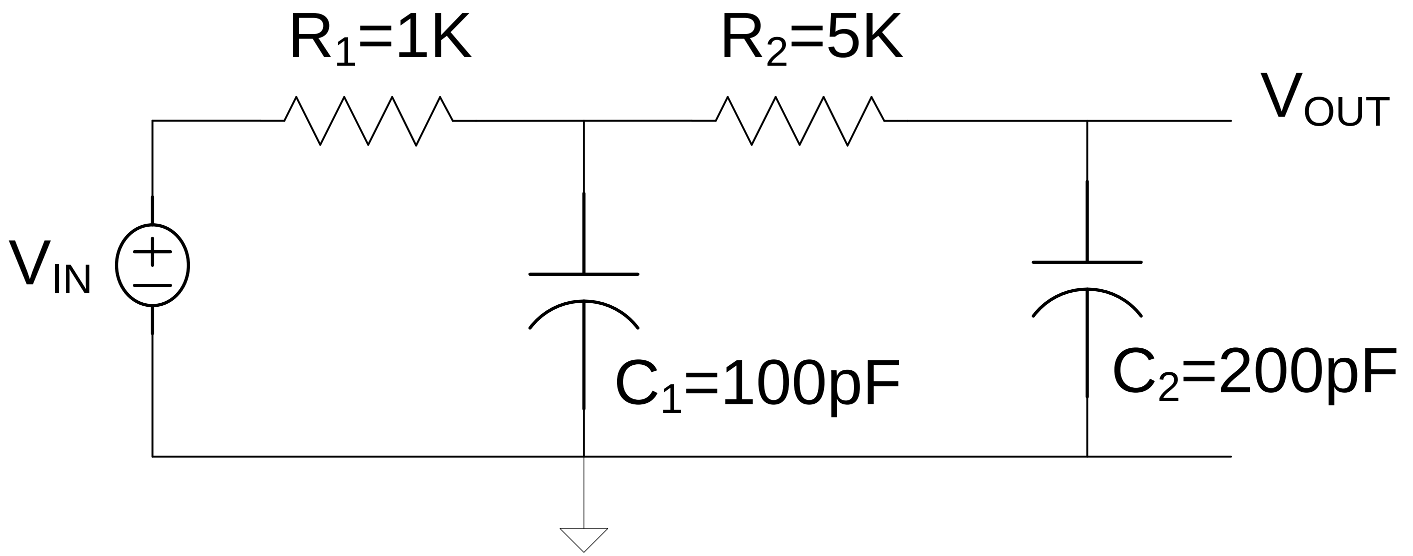
Provides closed-form analytical expressions for poles in terms of components of the network that can be managed during design

Provides considerable insight into what is affecting the frequency response of the amplifier

Pole approximation methods give no information about zero locations

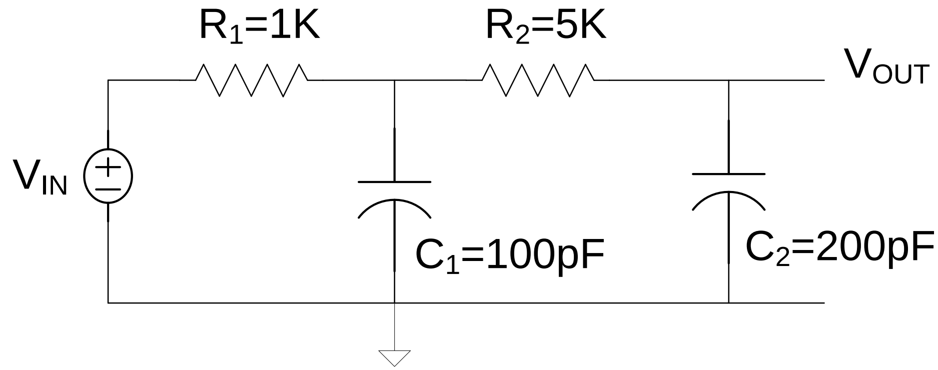
Many authors refer to the “pole on a node” and this notation comes from the pole approximation method discussed on previous slide

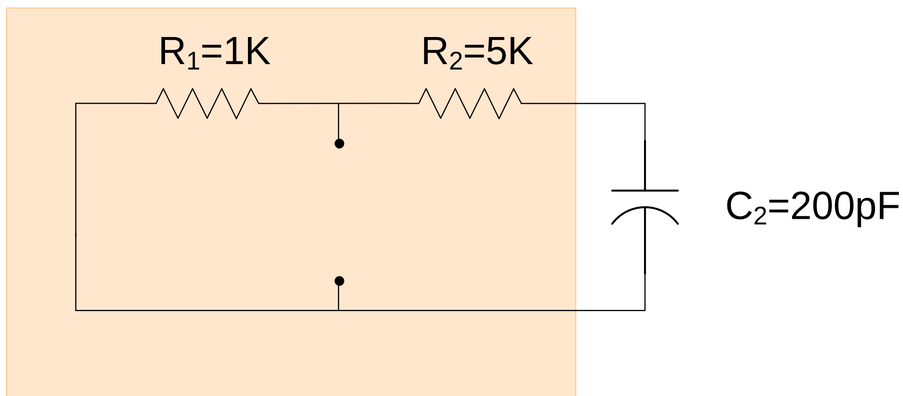
Example: Obtain the approximations to the poles of the following circuit



Since C_1 and C_2 and small, have two high-frequency poles

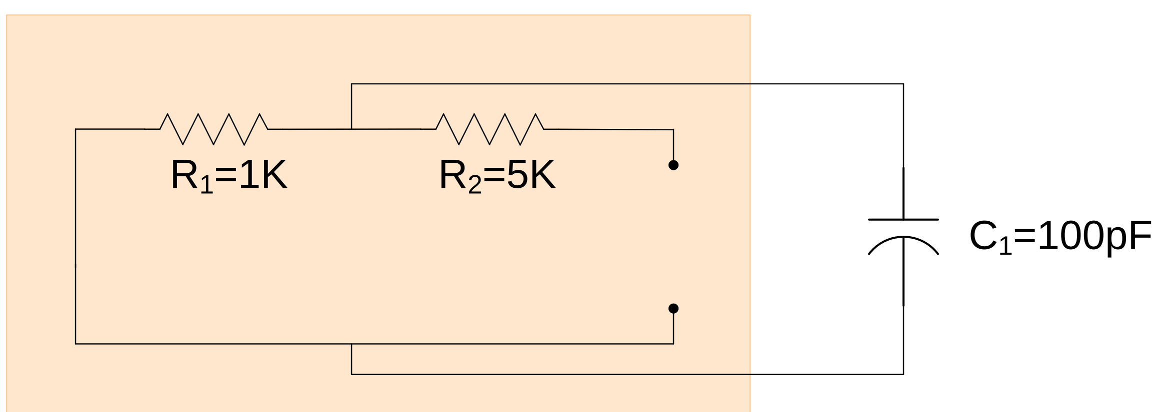
$\{C_1, C_2\}$





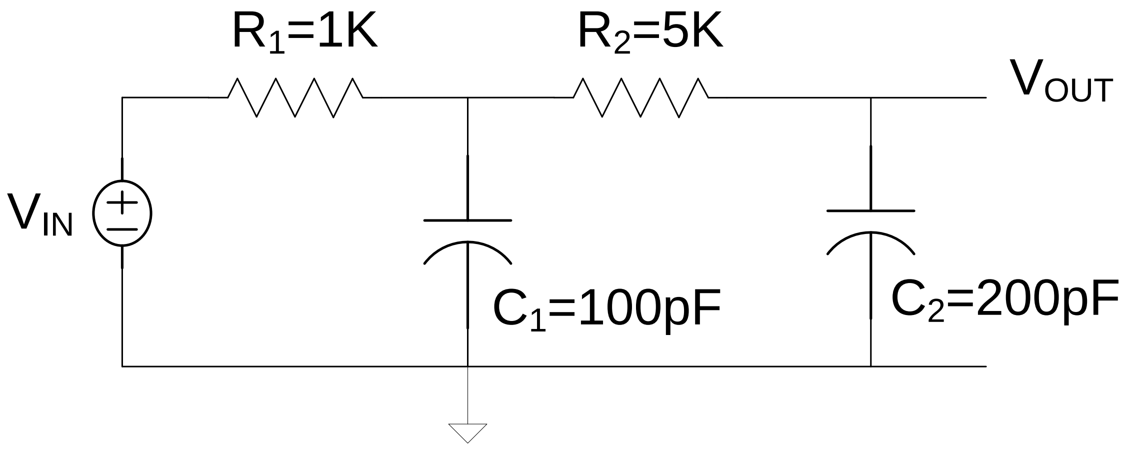
$$p_{H2} = -\frac{1}{C_2\left(R_1+R_2\right)}$$

$$p_{H2} = -\ 833\text{Krad/sec}$$



$$p_{H1} = -\frac{1}{C_1\ R_1}$$

$$p_{H1} = -10\text{M rad/sec}$$



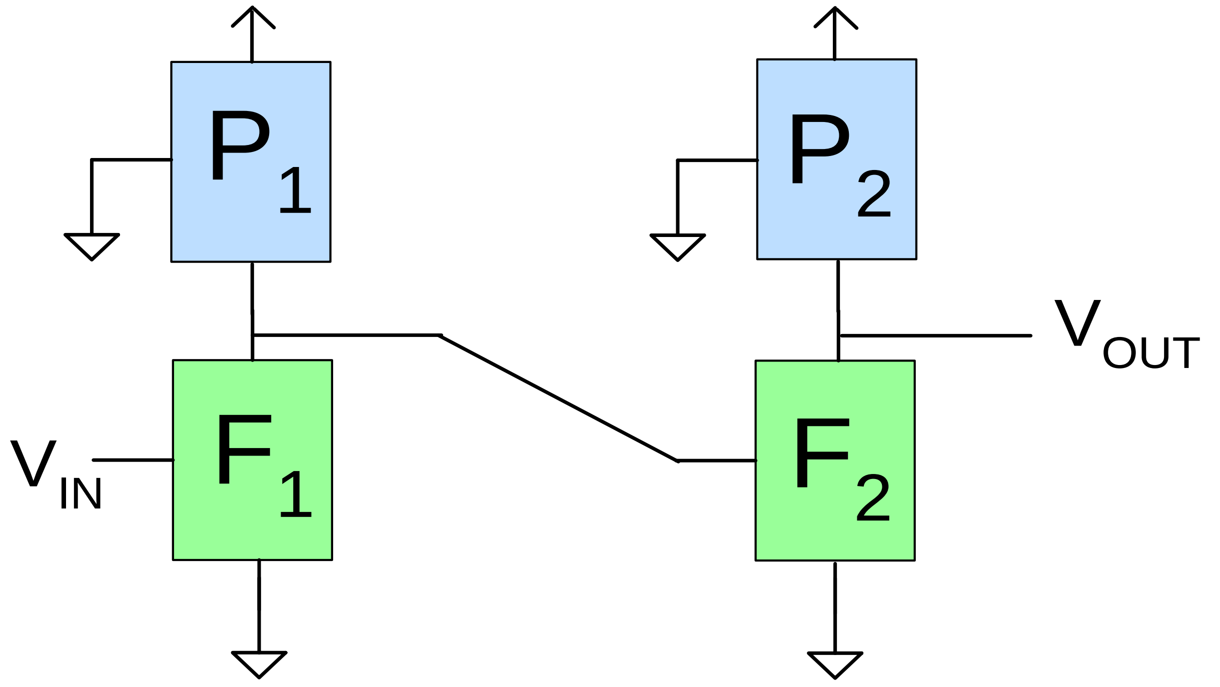
In this case, an exact solution is possible

$$T(s) = \frac{\frac{1}{R_1R_2C_1C_2}}{s^2 + \left[\frac{1}{R_1C_1} + \frac{1}{R_2C_2} + \frac{1}{R_2C_1}\right]s + \frac{1}{R_1R_2C_1C_2}}$$

p_{H1} = -12.2M rad/sec (18% error)

p_{H2} = - 821Krad/sec (1.4% error)

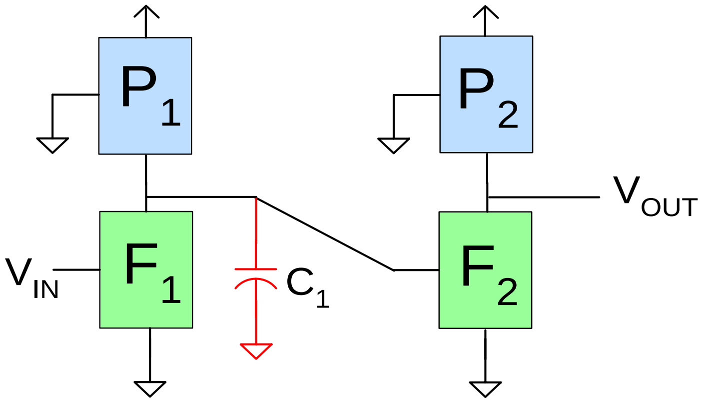
Basic Two-Stage Cascade



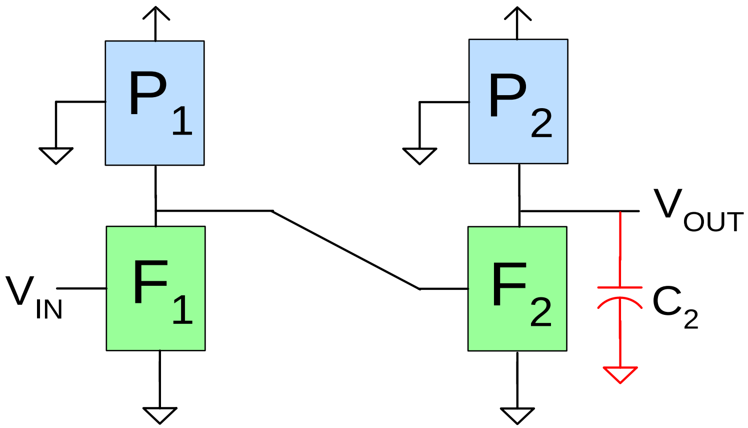
Can be extended to fully differential on first or second stage

- Simple Concept
- Must decide what to use for the two quarter circuits

Compensation of Basic Two-Stage Cascade



Internally Compensated

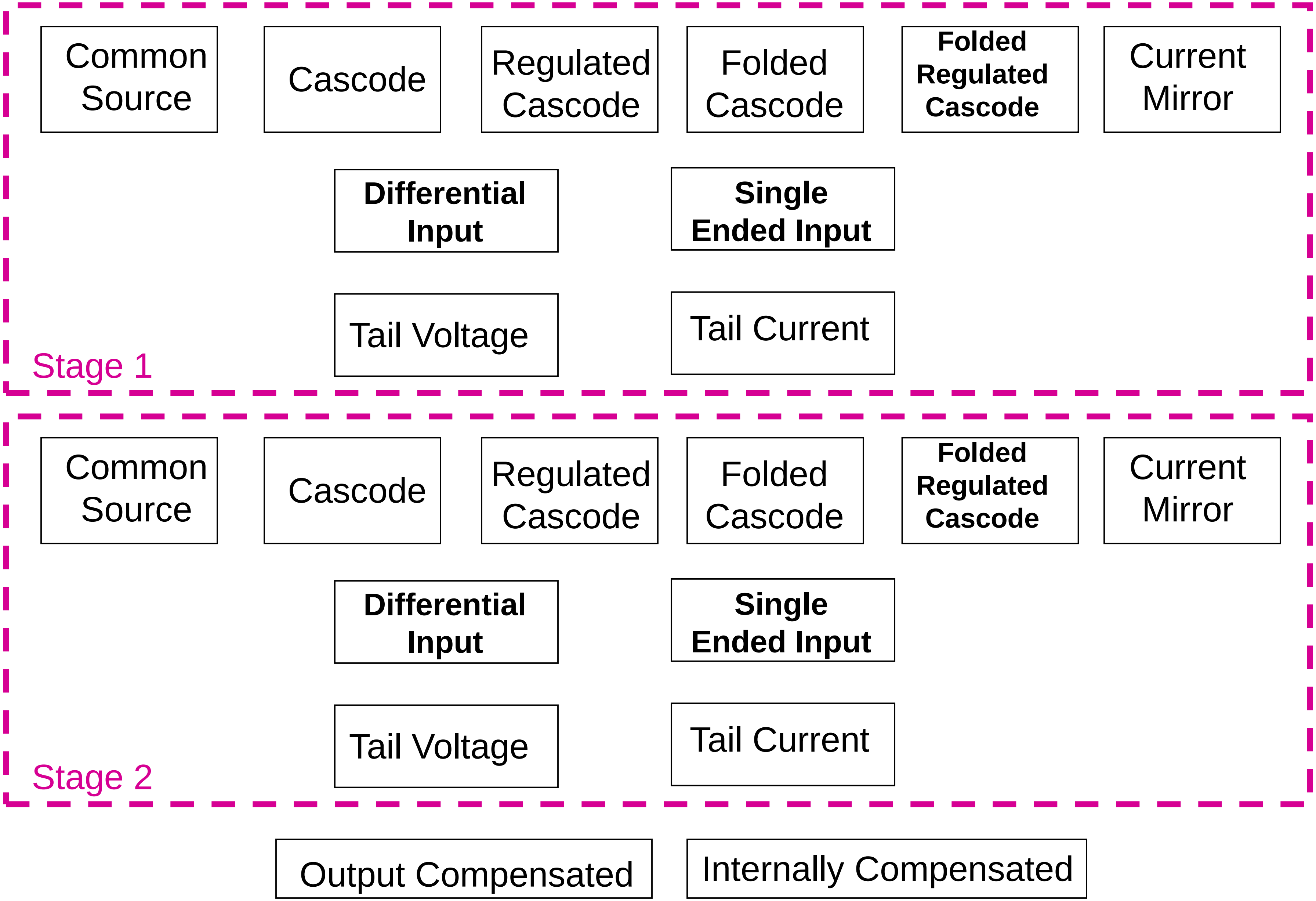


Output Compensated

- Modest variants of the compensation principle are often used
- Internally compensated creates the dominant pole on the internal node
- Output compensated created the dominant pole on the external node
- Output compensated often termed “self-compensated”

Everything else is just details !!

Two-stage Architectural Choices



Two-stage Architectural Choices

Stage 1

Common Source / Cascode / Regulated Cascode / Folded Cascode / **Folded Regulated Cascode** / Current Mirror	6
Differential Input / **Single Ended Input**	2
Tail Voltage / Tail Current	2

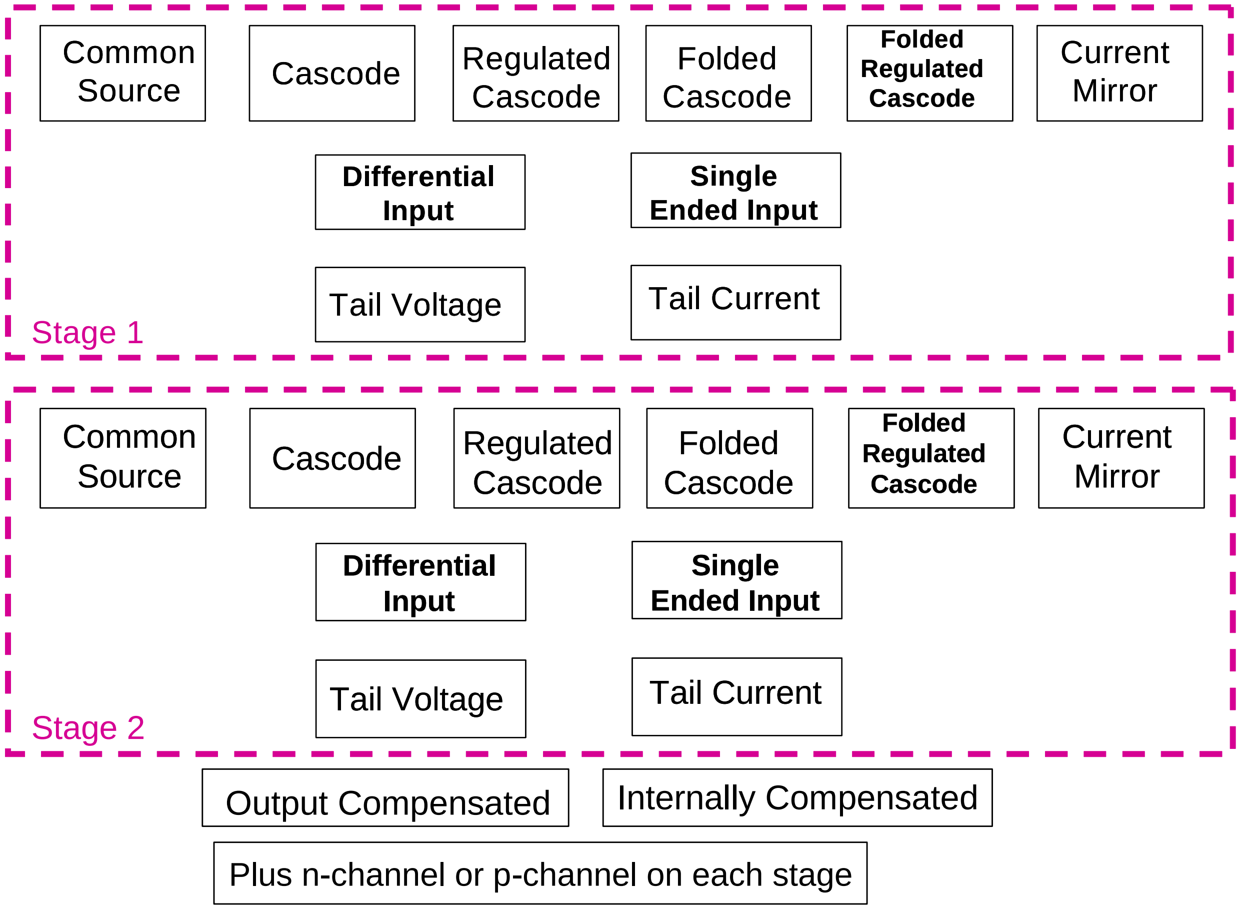
Stage 2

Common Source / Cascode / Regulated Cascode / Folded Cascode / **Folded Regulated Cascode** / Current Mirror	6
Differential Input / **Single Ended Input**	2
Tail Voltage / Tail Current	2

Output Compensated / Internally Compensated	2
Plus n-channel or p-channel on each stage	4

2304 Choices !!!

Two-stage Architectural Choices



Which of these 2304 choices can be used to build a good op amp?

All of them !!

Two-stage Architectural Choices

There are actually a few additional variants so the number of choices is larger

Basic analysis of all is about the same and can be obtained from the quarter circuit of each stage

A very small number of these are actually used

Some rules can be established that provide guidance as to which structure may be most useful in a given application

Two-stage Architectural Choices

Guidelines for Architectural Choices

Tail current source usually used in first stage, tail voltage source in second stage

Large gain usually used in first stage, smaller gain in second stage

First and second stage usually use quarter circuits of opposite types (n-p or p-n)

Input common mode input range of concern on first stage but output swing of first stage of reduced concern. Output range on second stage of concern.

CMRR of first stage of concern but not of second stage

Noise on first stage of concern but not of much concern on second stage

Two-stage Architectural Choices

Common Source | Cascode | Regulated Cascode | Folded Cascode | **Folded Regulated Cascode** | Current Mirror

Differential Input | **Single Ended Input**

Tail Voltage | Tail Current

Stage 1

Common Source | Cascode | Regulated Cascode | Folded Cascode | **Folded Regulated Cascode** | Current Mirror

Differential Input | **Single Ended Input**

Tail Voltage | Tail Current

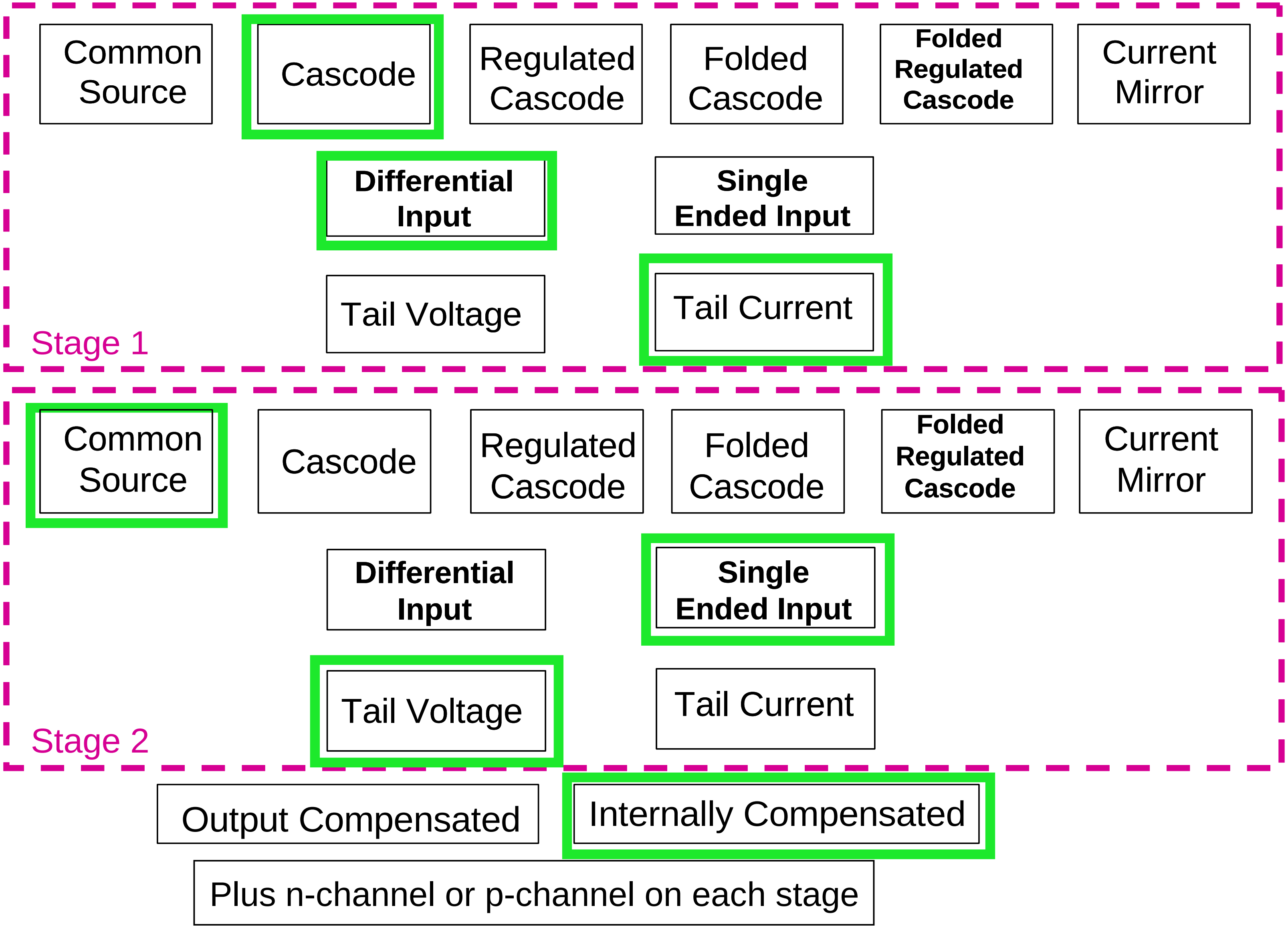
Stage 2

Output Compensated | Internally Compensated

Plus n-channel or p-channel on each stage

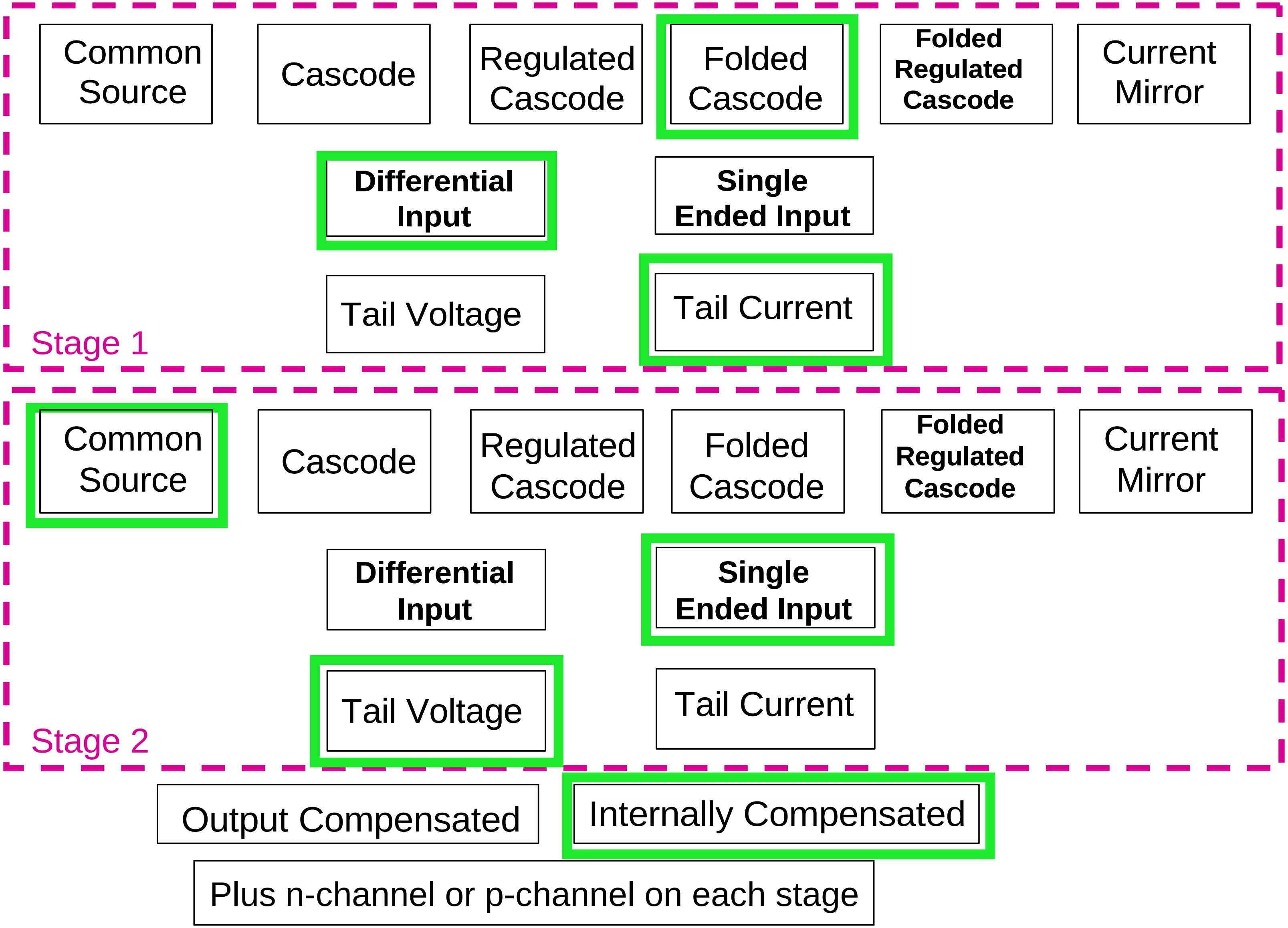
Basic Two-Stage Op Amp

Two-stage Architectural Choices



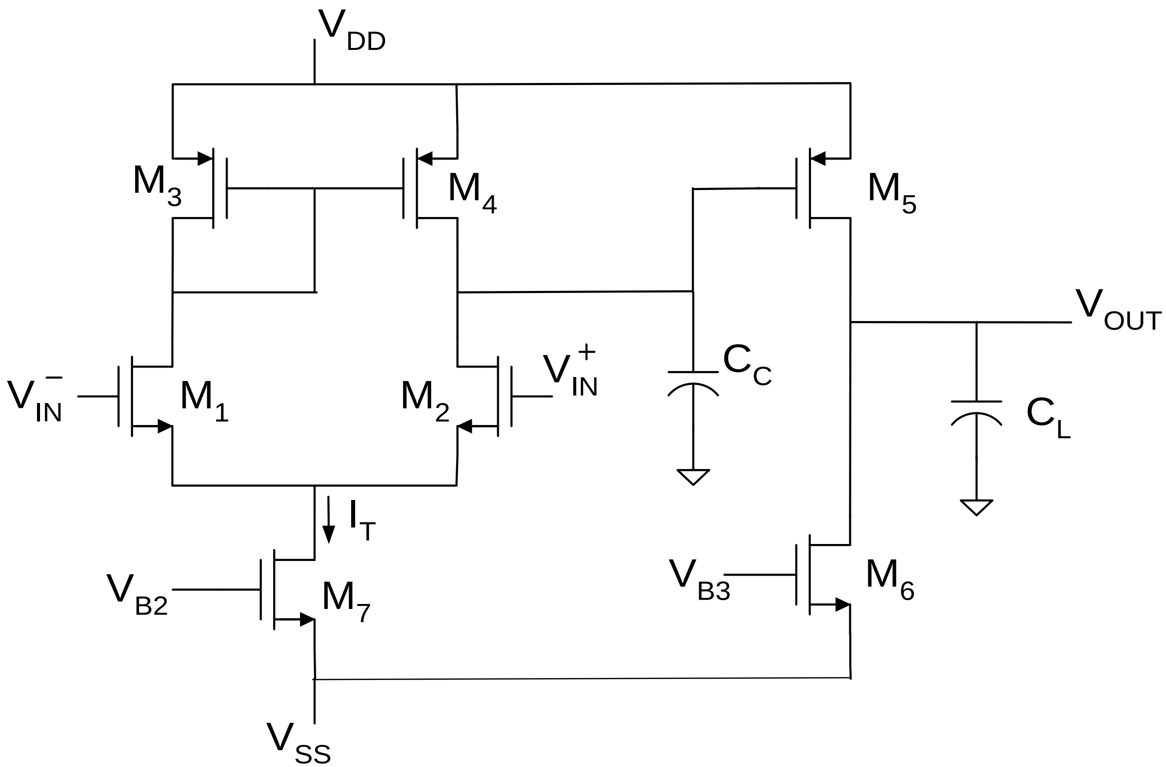
Cascode-Cascade Two-Stage Op Amp

Two-stage Architectural Choices



Folded Cascode-Cascade Two-Stage Op Amp

Basic Two-Stage Op Amp



- o One of the most widely used op amp architectures
- o Essentially just a cascade of two common-source stages
- o Compensation Capacitor C_C used to get wide pole separation
- o Two poles in amplifier
- o No universally accepted strategy for designing this seemingly simple amplifier

Pole spread $\propto \beta A_{01} A_{02}$ makes C_C unacceptably large

Example:

Sketch the circuit of a two-stage internally compensated op amp with a telescopic cascode first stage, single-ended output, tail current bias first stage, tail voltage bias second stage, p-channel inputs and n-channel inputs on the second stage.

Two-stage Architectural Choices

Stage 1

Common Source | Cascode | Regulated Cascode | Folded Cascode | Folded Regulated Cascode | Current Mirror

Differential Input | Single-Ended Input

Differential Output | Single-Ended Output

Tail Voltage Bias | Tail Current Bias

Stage 2

Common Source | Cascode | Regulated Cascode | Folded Cascode | Folded Regulated Cascode | Current Mirror

Differential Input | Single-Ended Input

Differential Output | Single-Ended Output

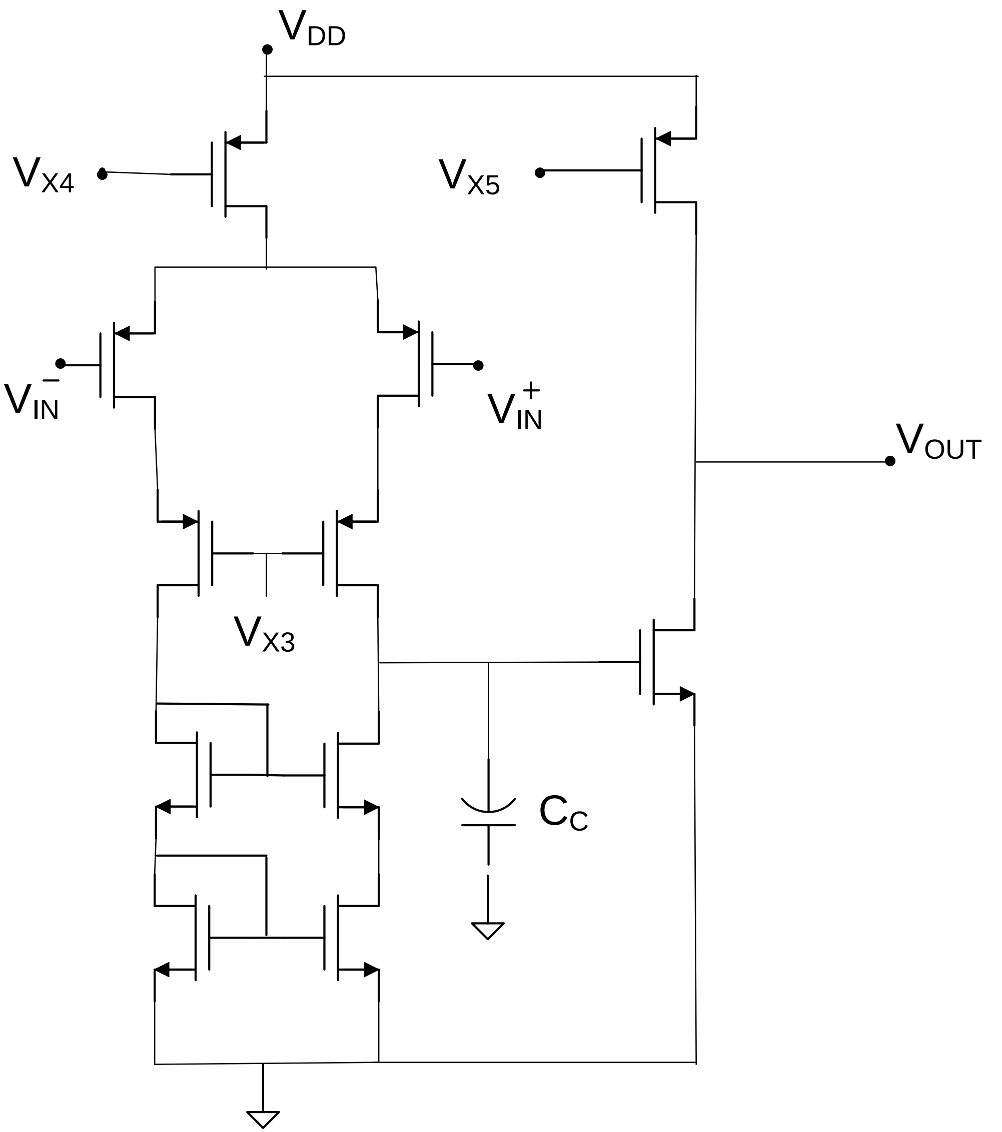
Tail Voltage Bias | Tail Current Bias

Internally Compensated | Output Compensated

p-channel Input | n-channel Input

Cascode-Cascade Two-Stage Op Amp

Example Solution



First Commercial Operational Amplifier

K2-W Op Amp by Philbrickk, 1952-1971

Inventor of the Two-Stage Op Amp

Robert Widlar

Many say he started the field of analog IC design, considered a brilliant engineer

"Widlar began his career at Fairchild semiconductor, where he designed a couple of pioneering op amps. By 1966, the commercial success of his designs became apparent, and Widlar asked for a raise. He was turned down, and jumped ship to the fledgling National Semiconductor. At National he continued to turn out amazing designs, and was able to retire just before his 30th birthday in 1970." (from posted www site)

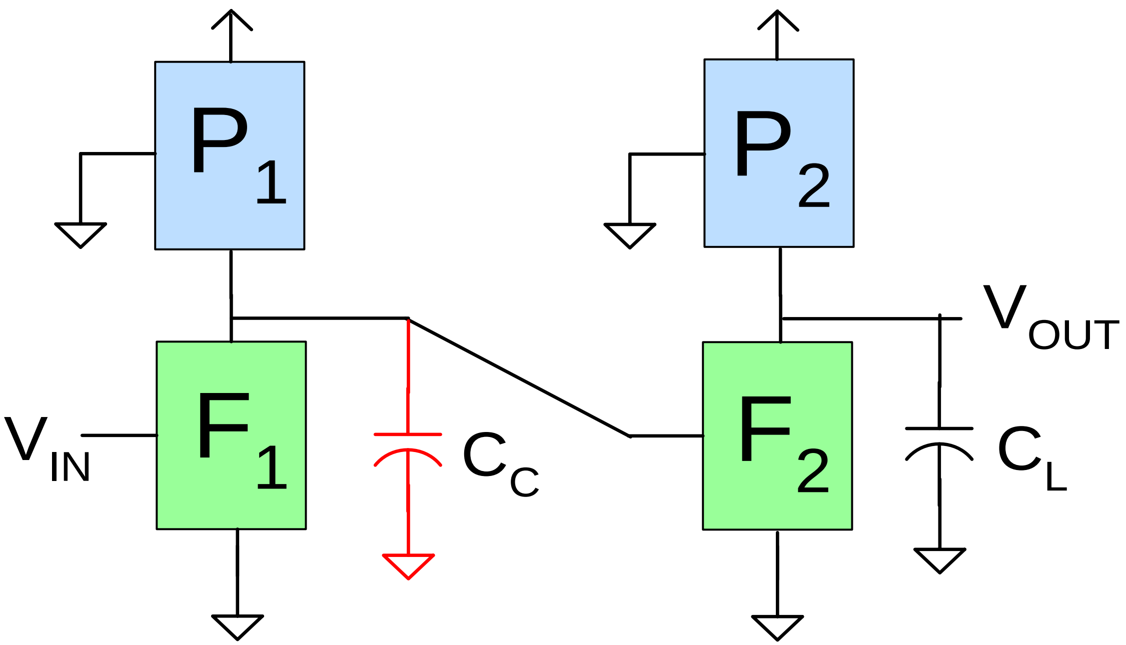
Inventor of the internally-compensated Op Amp

Dave Fullagar

(from posted www site)

- Designed the first internally-compensate op amp, the 741
- Fullagar was 26 years old when this was designed (introduced?)
- Introduced in 1968
- Largest selling integrated circuit ever
- Still in high-volume production even though over 40 years old
- Fullagar later started the linear design activities at Intersil
- Cofounder (catalyst) of Maxim

Analysis of Internally Compensated Two-Stage Op Amps



Consider single-ended input-output (differential analysis only slightly different)

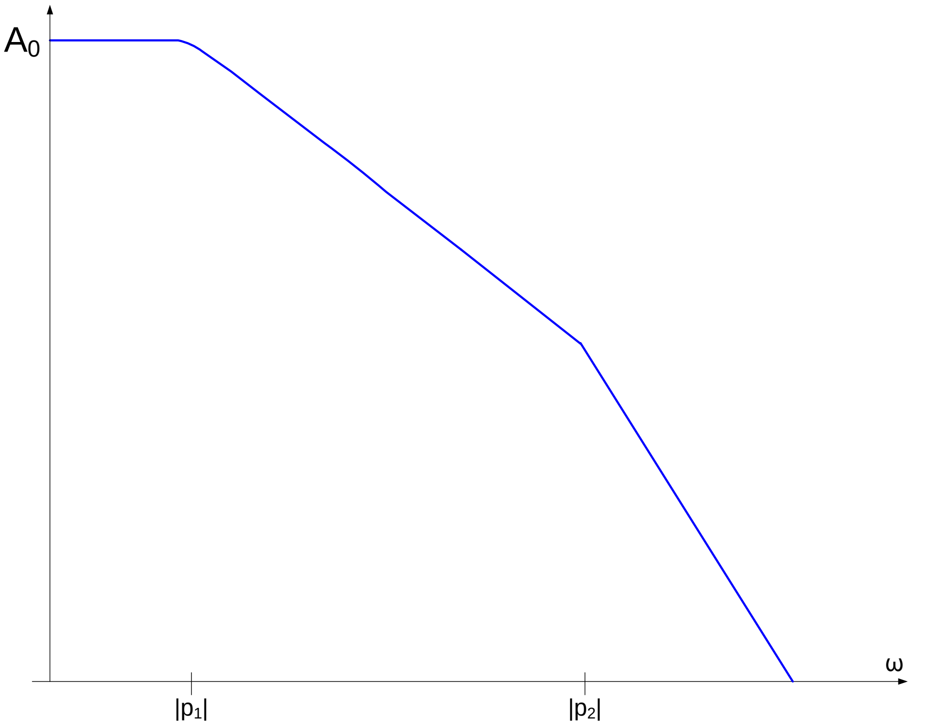
Can't get everything but can get most of the small-signal results

Since internally compensated, must have $p_1 << p_2$

Analysis of Internally Compensated Two-Stage Op Amps

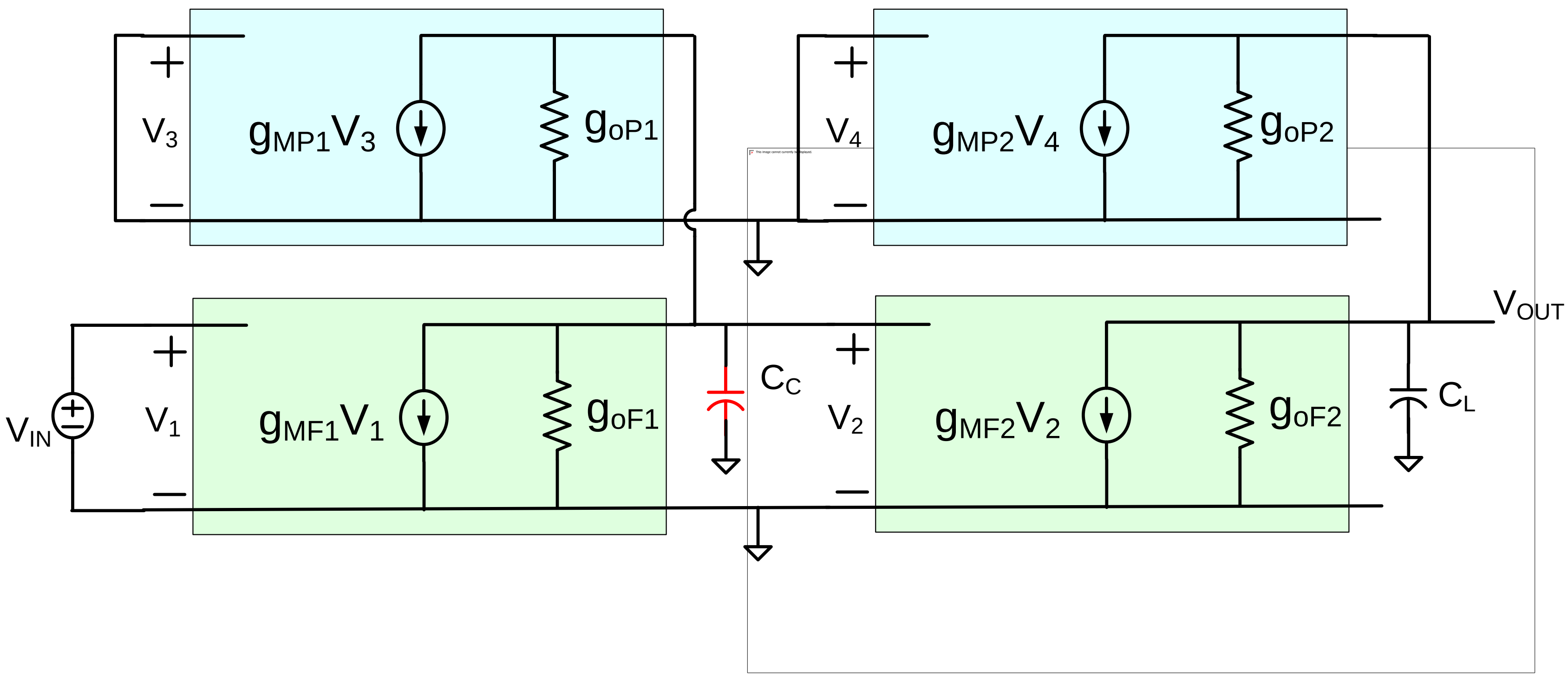
$$A(s) = \frac{A_0}{\left(\frac{s}{|p_1|}+1\right)\left(\frac{s}{|p_2|}+1\right)}$$

For $|p_1| << |p_2|$

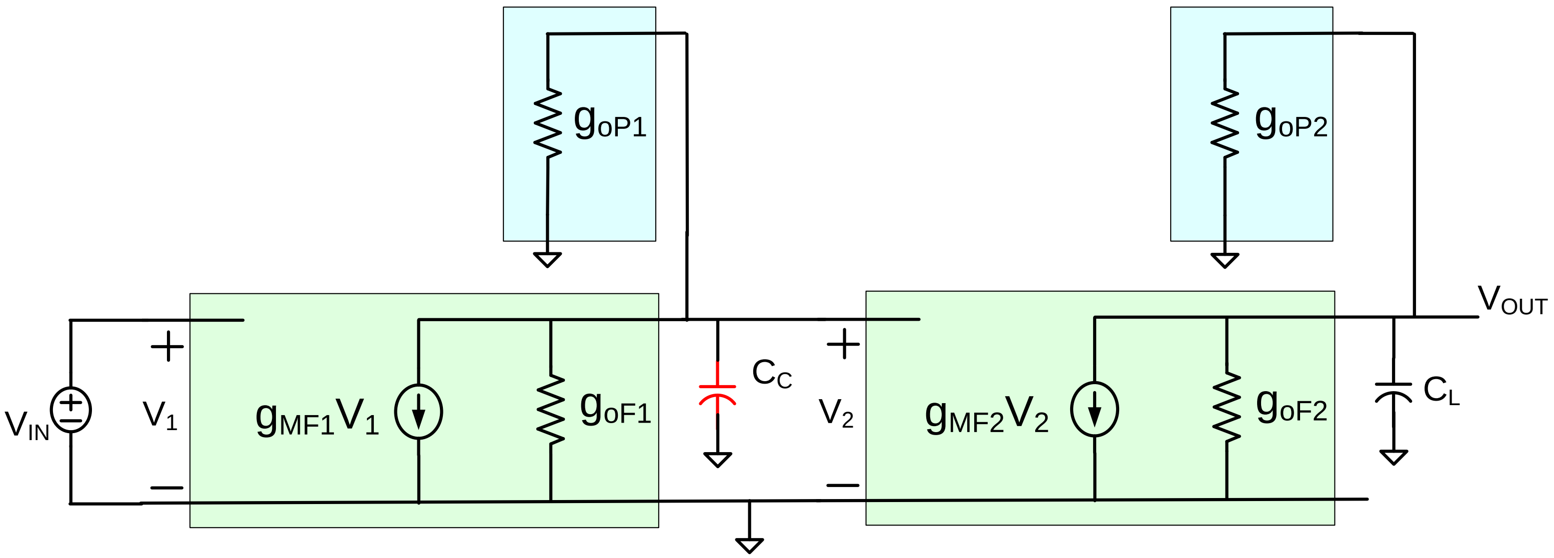


$BW \approx |p_1|$

Analysis of Internally Compensated Two-Stage Op Amps



Analysis of Internally Compensated Two-Stage Op Amps



$$A_{V0} = \left(\frac{g_{mF1}}{g_{oF1} + g_{oP1}}\right)\left(\frac{g_{mF2}}{g_{oF2} + g_{oP2}}\right)$$

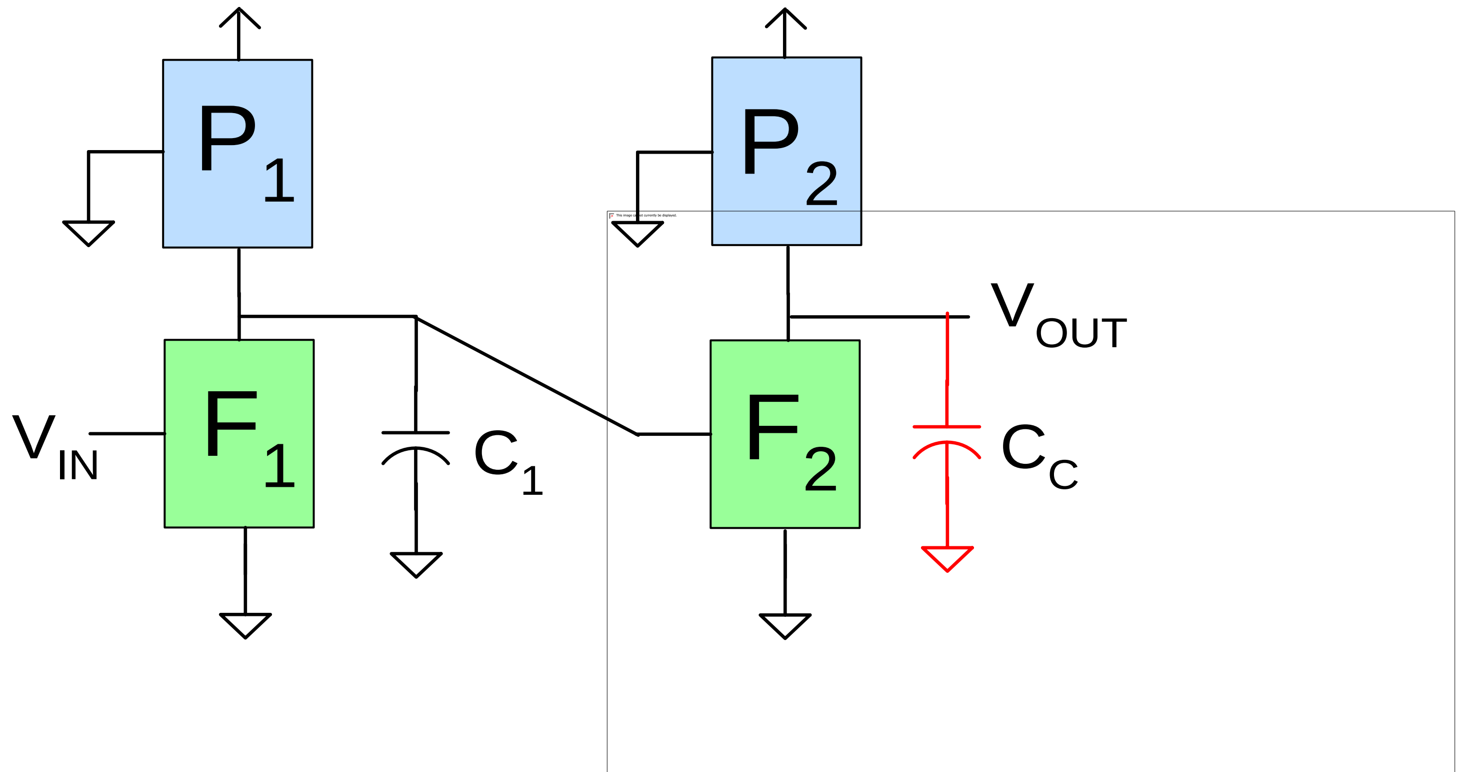
$$|p_2| = \frac{(g_{oF2} + g_{oP2})}{C_L}$$

$$BW = |p_1|$$

$$|p_1| = \frac{(g_{oF1} + g_{oP1})}{C_C}$$

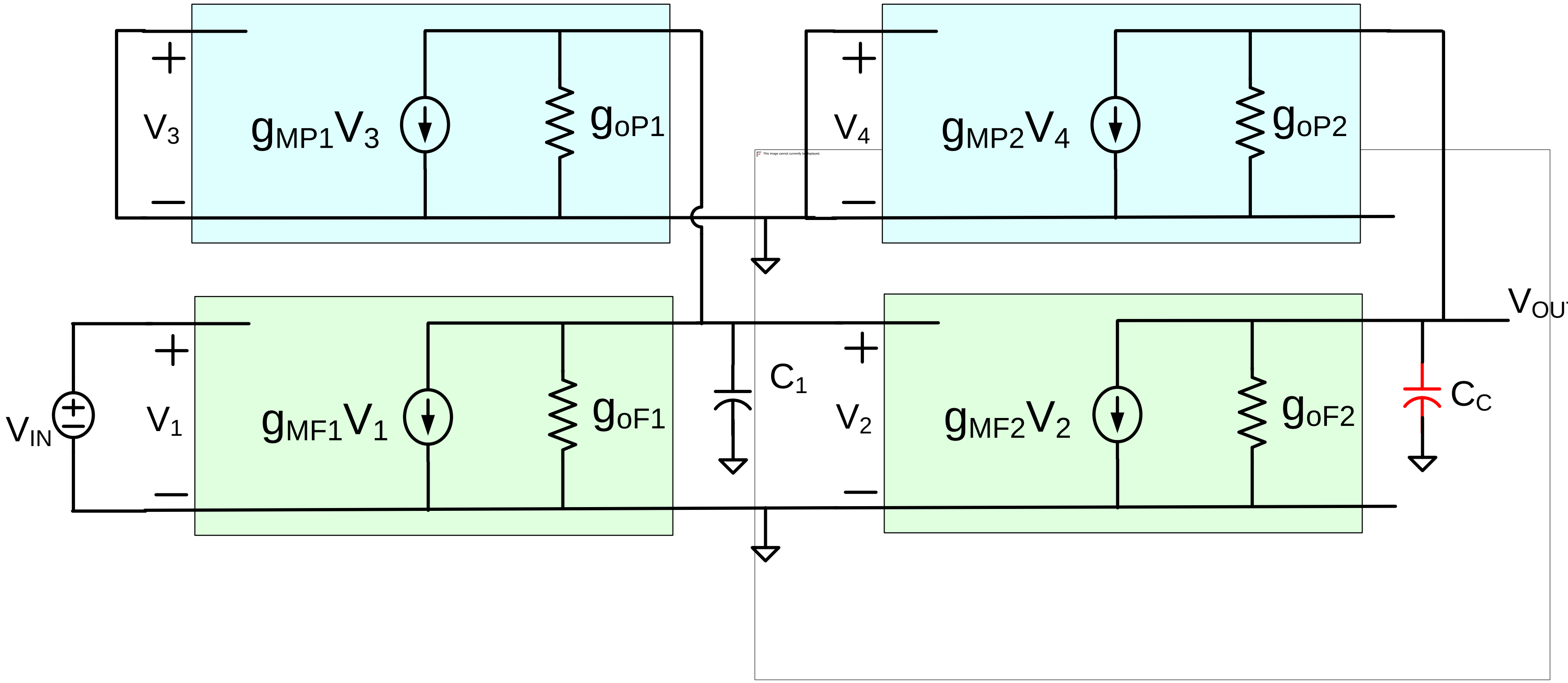
$$GB = \frac{g_{mF1} g_{mF2}}{(g_{oF2} + g_{oP2})C_C}$$

Analysis of Externally Compensated Two-Stage Op Amps

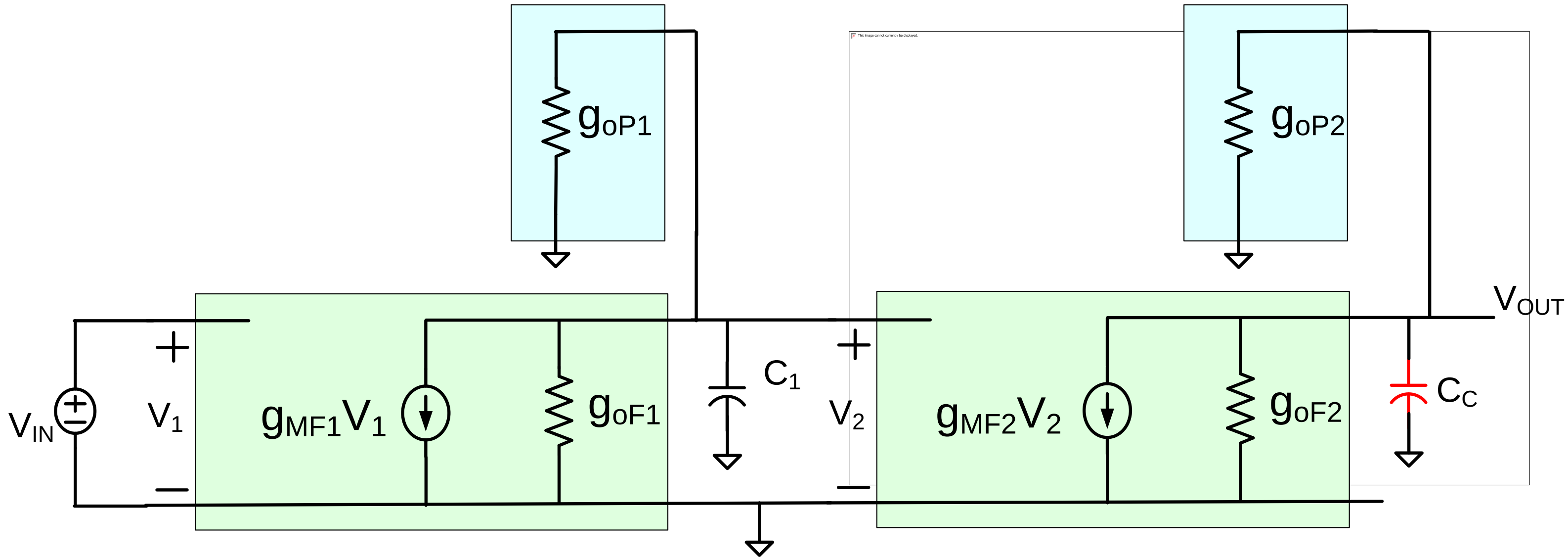


Can’t get everything but can get most of the small-signal results

Analysis of Externally Compensated Two-Stage Op Amps



Analysis of Externally Compensated Two-Stage Op Amps



$$A_{V0} = \left(\frac{g_{mF1}}{g_{oF1}+g_{oP1}}\right)\left(\frac{g_{mF2}}{g_{oF2}+g_{oP2}}\right)$$

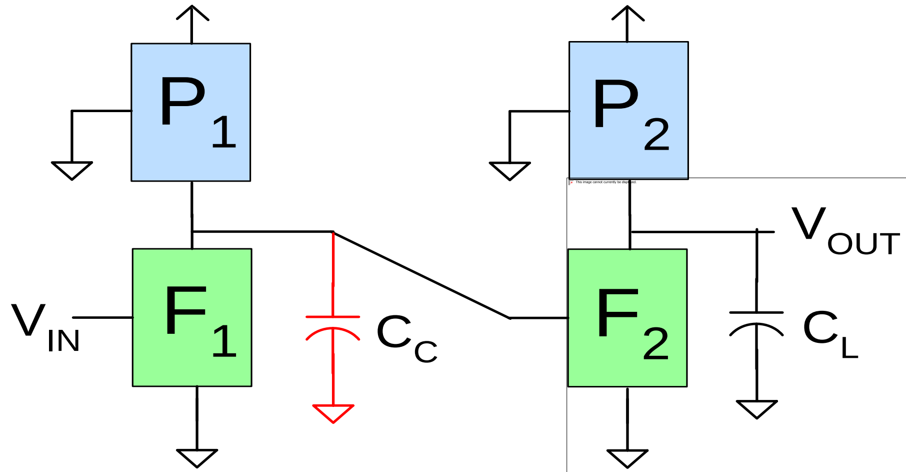
$$|p_2| = \frac{(g_{oF2}+g_{oP2})}{C_C}$$

$$BW = |p_2|$$

$$|p_1| = \frac{(g_{oF1}+g_{oP1})}{C_1}$$

$$GB = \frac{g_{mF1}g_{mF2}}{(g_{oF1}+g_{oP1})C_C}$$

Consider Again the Internally Compensated Two-Stage Op Amp



Recall very crude compensation requirements:

$$4\beta A_{0TOT} > k > 2\beta A_{0TOT}$$

where $|p_2| = k|p_1|$

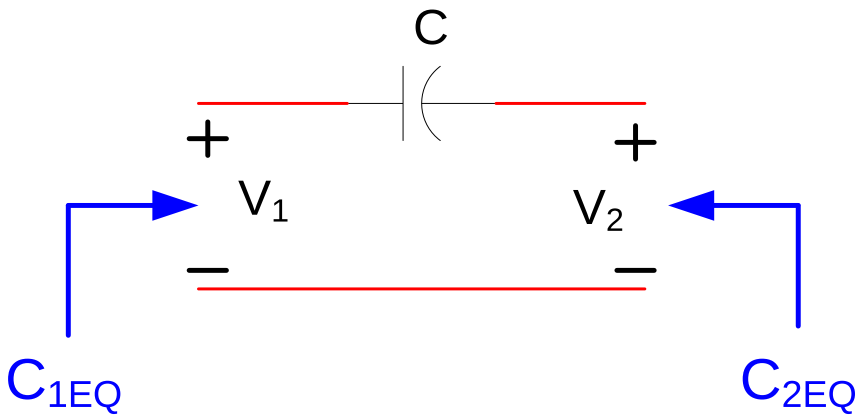
Thus, very approximately, $3\beta A_{0TOT} = \frac{|p_2|}{|p_1|}$

$$3\beta\left(\frac{g_{mF1}}{g_{oF1}+g_{oP1}}\right)\left(\frac{g_{mF2}}{g_{oF2}+g_{oP2}}\right) \approx \left(\frac{g_{oF2}+g_{oP2}}{C_L}\right)\left(\frac{C_C}{g_{oF1}+g_{oP1}}\right)$$

$$C_C \approx \left(3\beta\frac{g_{mF1}g_{mF2}}{\left(g_{oF2}+g_{oP2}\right)^2}\right)C_L$$

Since the pole ratio needs to be very large, C_C gets very large !

Miller Capacitance - Review

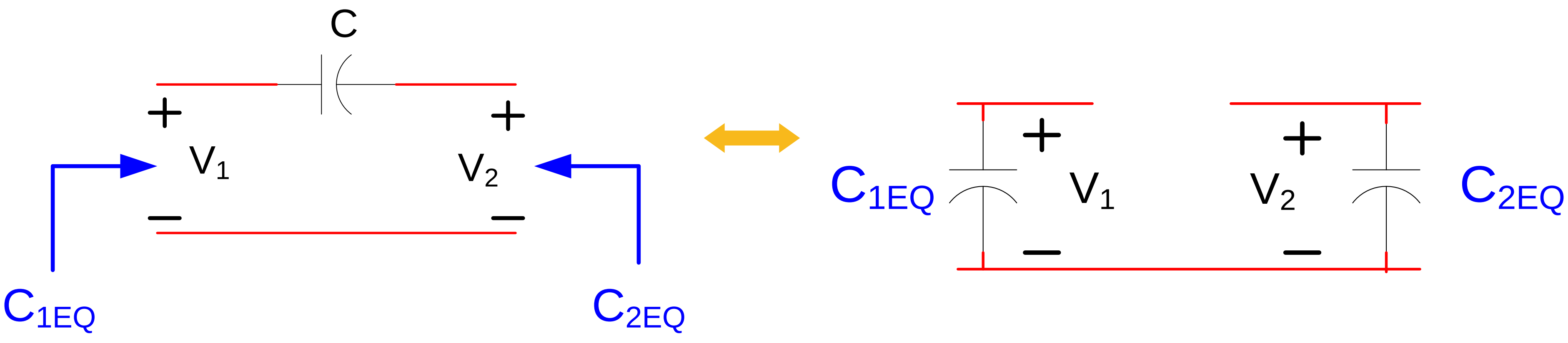


If $V_2 = -AV_1$ then for A large

$$\mathbf{C_{1EQ}} = \mathbf{C}(\mathbf{1} + \mathbf{A}) \approx \mathbf{CA} \qquad \mathbf{C_{2EQ}} = \mathbf{C}\left(\mathbf{1} + \frac{\mathbf{1}}{\mathbf{A}}\right) \approx \mathbf{C}$$

Thus, a large effective capacitance can be created with a much smaller capacitor if a capacitor bridges two nodes with a large inverting gain !!

Miller Capacitance - Review



If $V_2 = -AV_1$ then for A large

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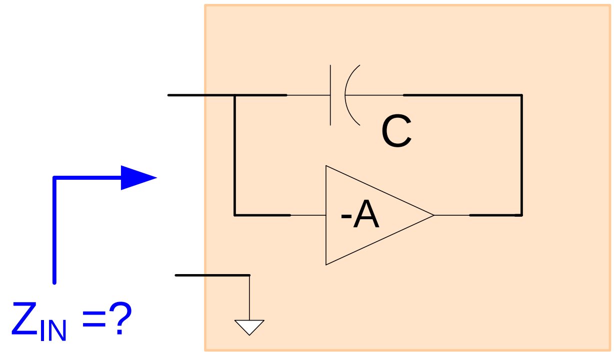
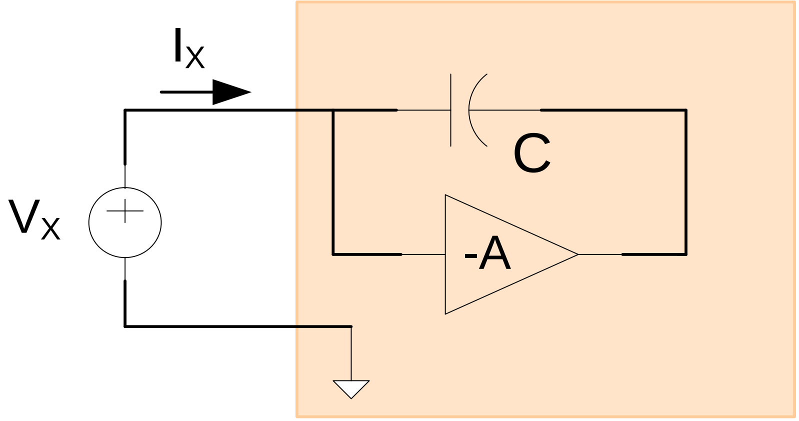
- If A changes with frequency, C_{1EQ} and C_{2EQ} are no longer pure capacitors
- More useful for giving a concept than for accurate actual analysis because of frequency dependence of A

Miller Capacitance - Review

The Basic Concept – from capacitance multiplication





$$I_X = [V_X - (-AV_X)]sC = V_X s[C(1+A)]$$

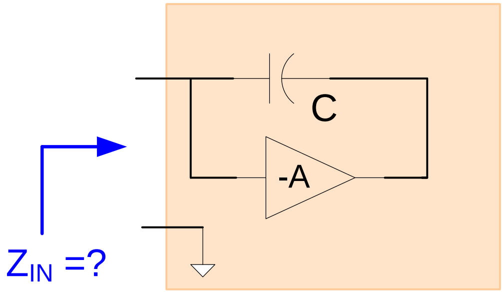
thus

$$Z_{IN} = \frac{V_X}{I_X} = \frac{1}{s[C(1+A)]}$$

So, if A is constant, input looks like a capacitor of value

$$C_{EQ} = C(1+A)$$

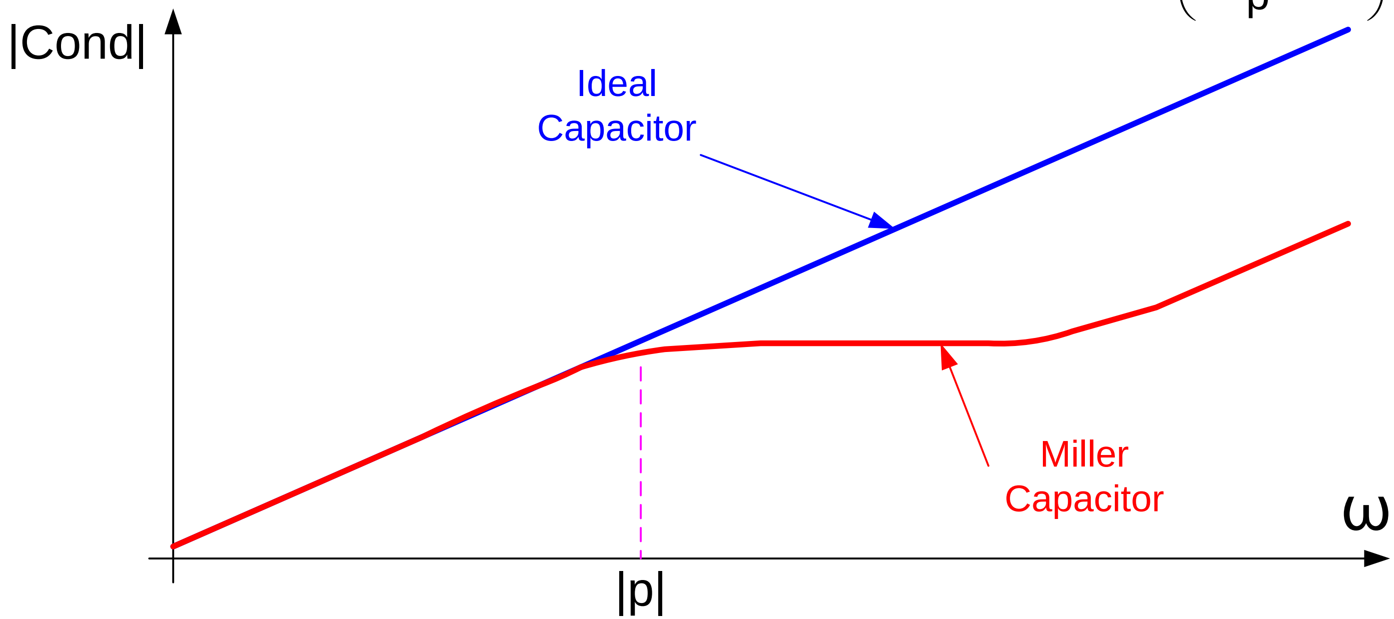
Miller Capacitance - Review



$$Z_{IN} = \frac{V_X}{I_X} = \frac{1}{s\left[C(1+A)\right]}$$

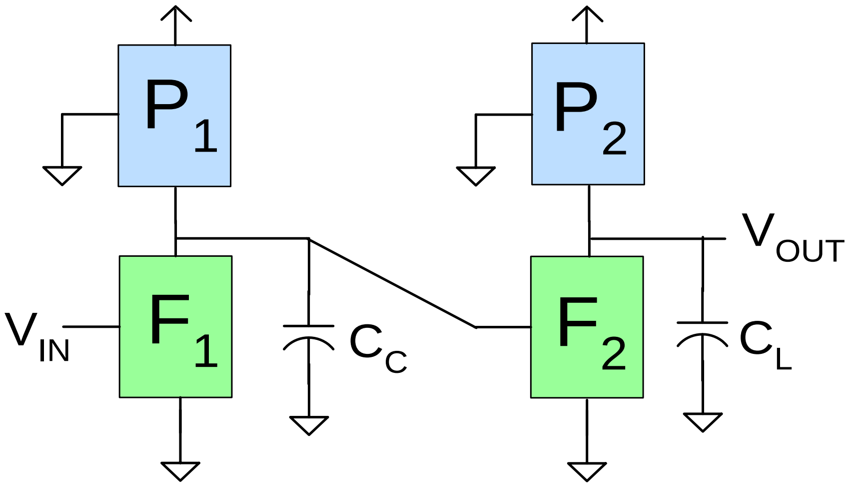
If $$A(s) = \frac{A_0}{\frac{s}{p}+1}$$

$$G_{IN} = s\left[C(1+A)\right] = sC\left(\frac{\frac{s}{p}+1+A_0}{\frac{s}{p}+1}\right)$$

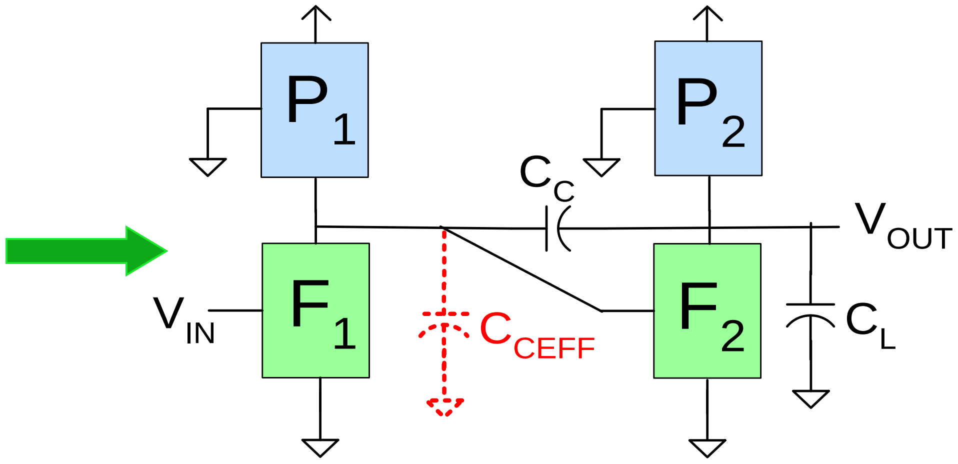


Does not behave as a capacitor for ω > p

Internal Miller-Compensated Two-Stage Op Amp



Standard Compensation

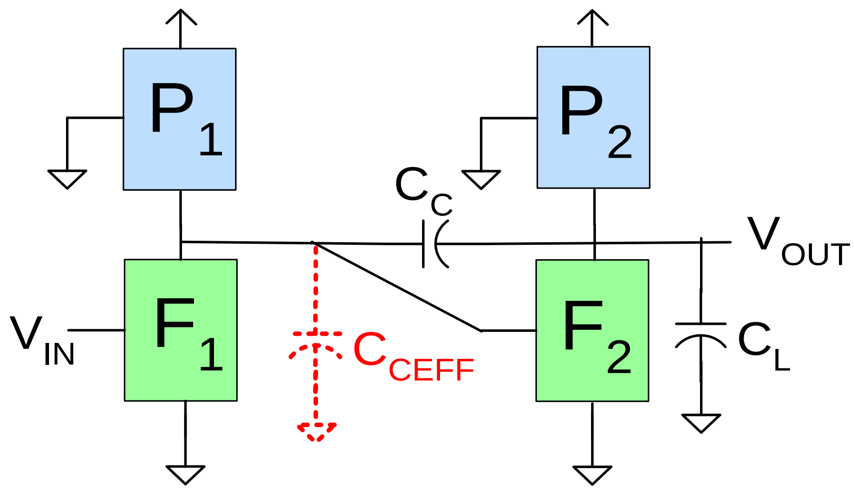


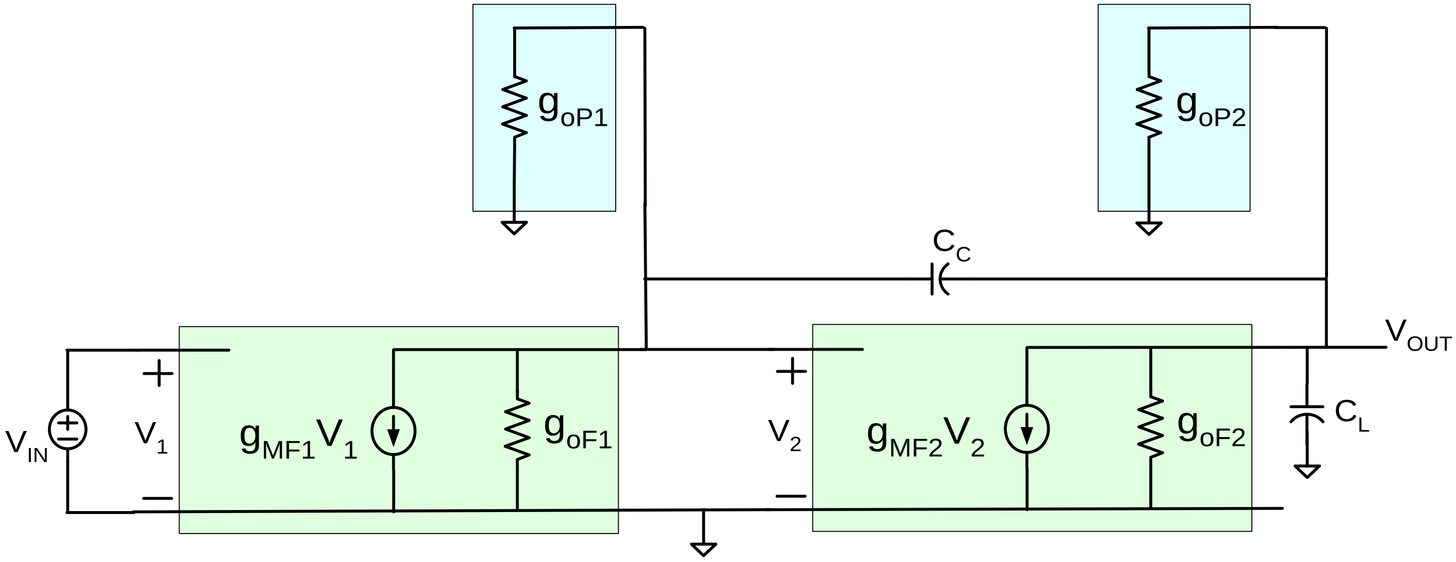
Miller Compensation

Compensation capacitance reduced by approximately the gain of the second stage!

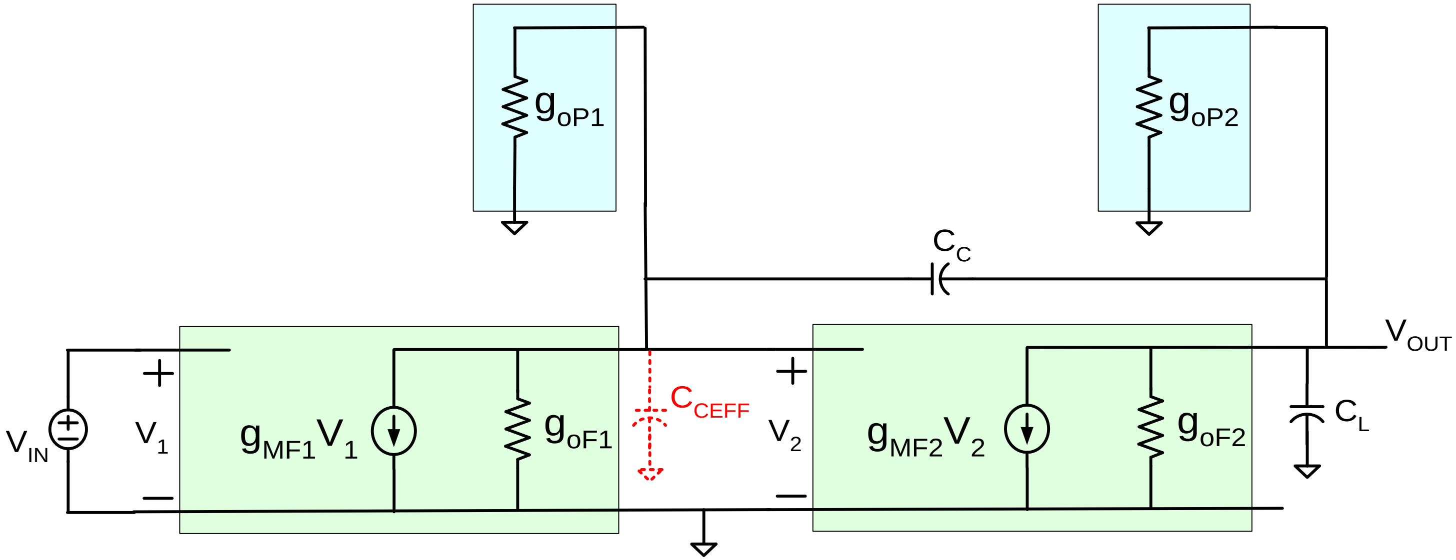
Since the gain of the second stage is not constant, however, a new analysis is needed

Analysis of Internally Miller-Compensated Two-Stage Op Amps





Analysis of Internally Miller-Compensated Two-Stage Op Amps

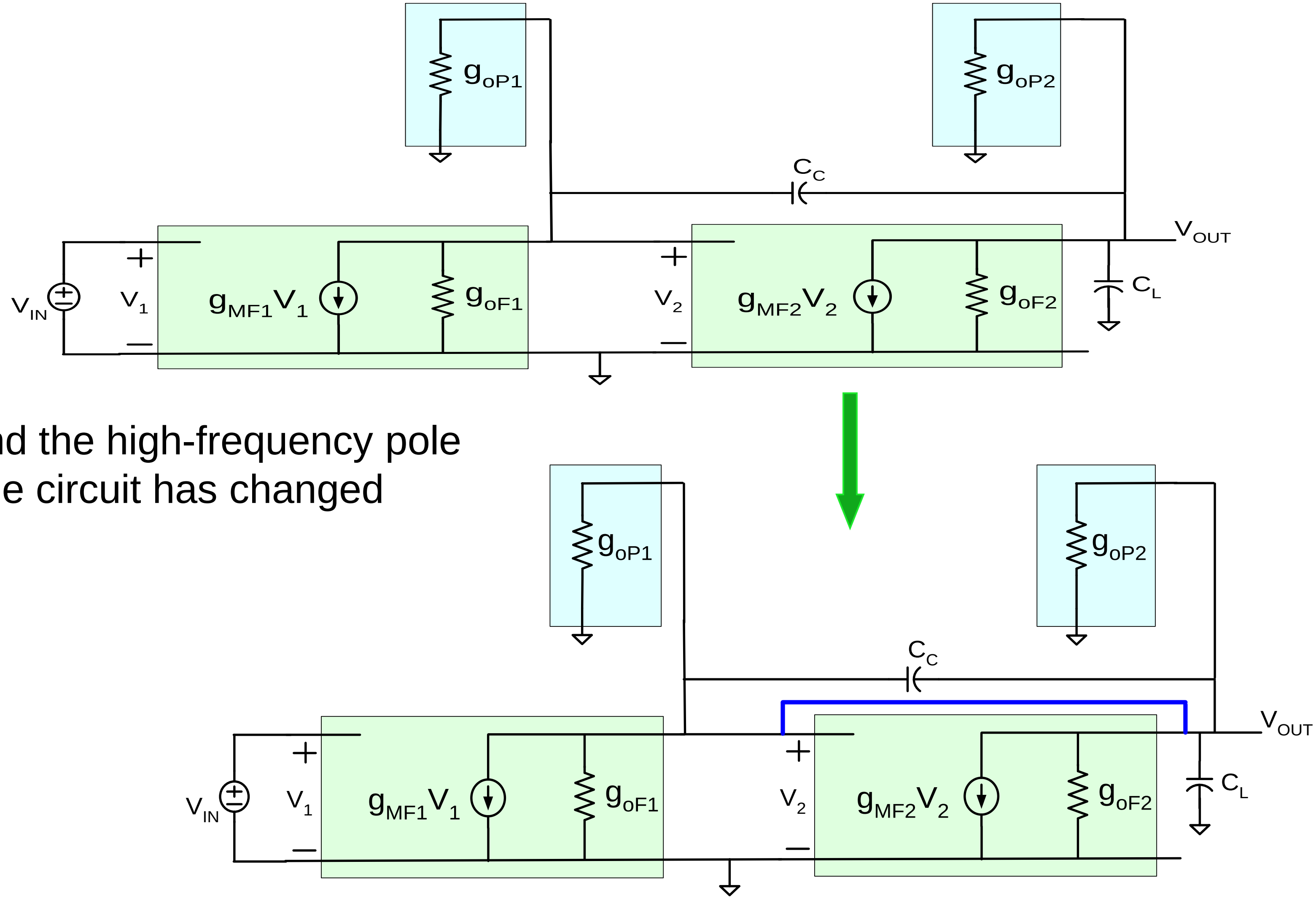


$$A_{V0} = \left(\frac{g_{mF1}}{g_{oF1} + g_{oP1}}\right)\left(\frac{g_{mF2}}{g_{oF2} + g_{oP2}}\right)$$

$$|p_1| = \frac{(g_{oF1} + g_{oP1})}{C_C} \quad \longrightarrow \quad |p_1| = (g_{oF1} + g_{oP1})\left(\frac{g_{oF2} + g_{oP2}}{C_C\ g_{mF2}}\right)$$

$$BW = |p_1|$$

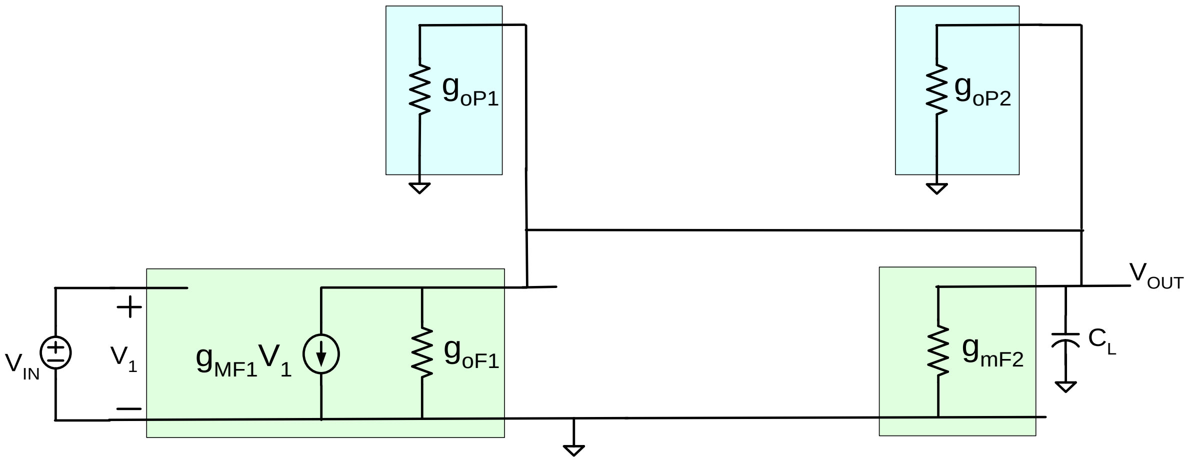
Analysis of Internally Miller-Compensated Two-Stage Op Amps



To find the high-frequency pole p_2, the circuit has changed

Note the F2 block is now "diode connected" at high frequencies

Analysis of Internally Miller-Compensated Two-Stage Op Amps



$$|p_2| = \frac{(g_{oF2}+g_{oP2})}{C_L} \rightarrow |p_2| = \frac{g_{mF2}}{C_L}$$

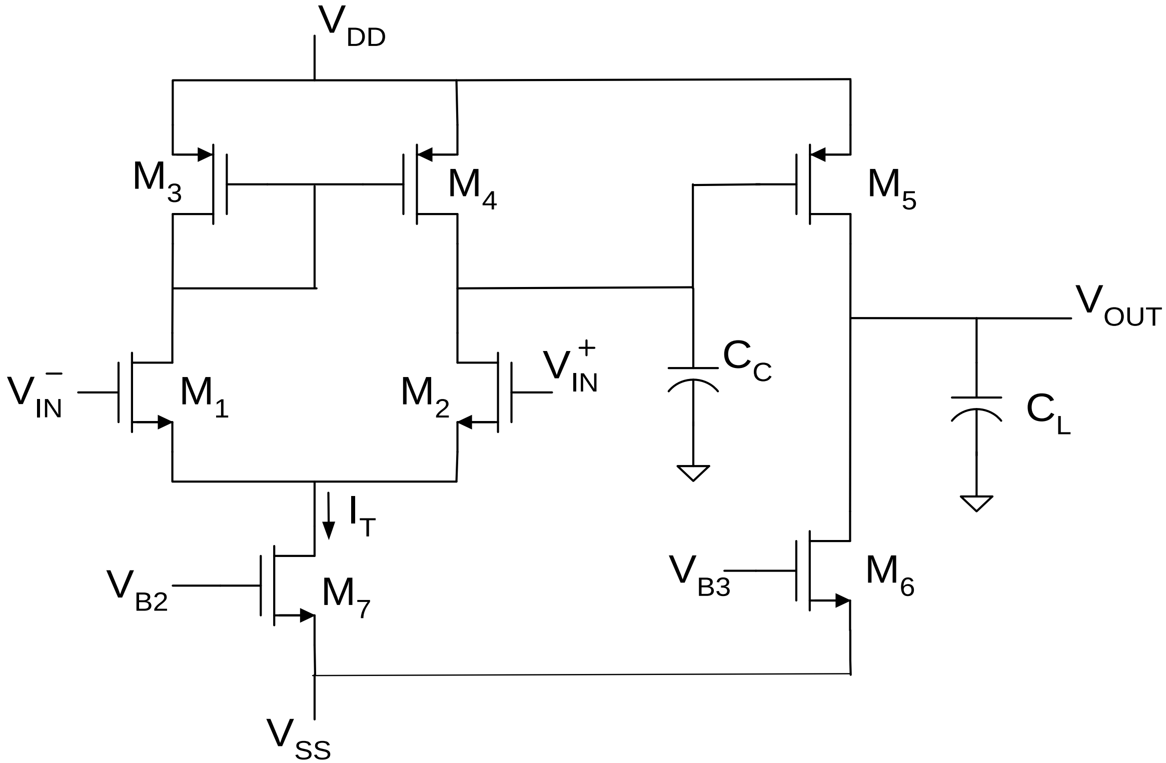
$$A_{V0} = \left(\frac{g_{mF1}}{g_{oF1}+g_{oP1}}\right)\left(\frac{g_{mF2}}{g_{oF2}+g_{oP2}}\right) \qquad BW = (g_{oF1}+g_{oP1})\left(\frac{g_{oF2}+g_{oP2}}{C_C\ g_{mF2}}\right)$$

$$GB = \frac{g_{mF1}g_{mF2}}{(g_{oF2}+g_{oP2})C_C} \rightarrow GB = \frac{g_{mF1}}{C_C}$$

Has the GB decreased?

No, because the C_C decreased by the same factor!

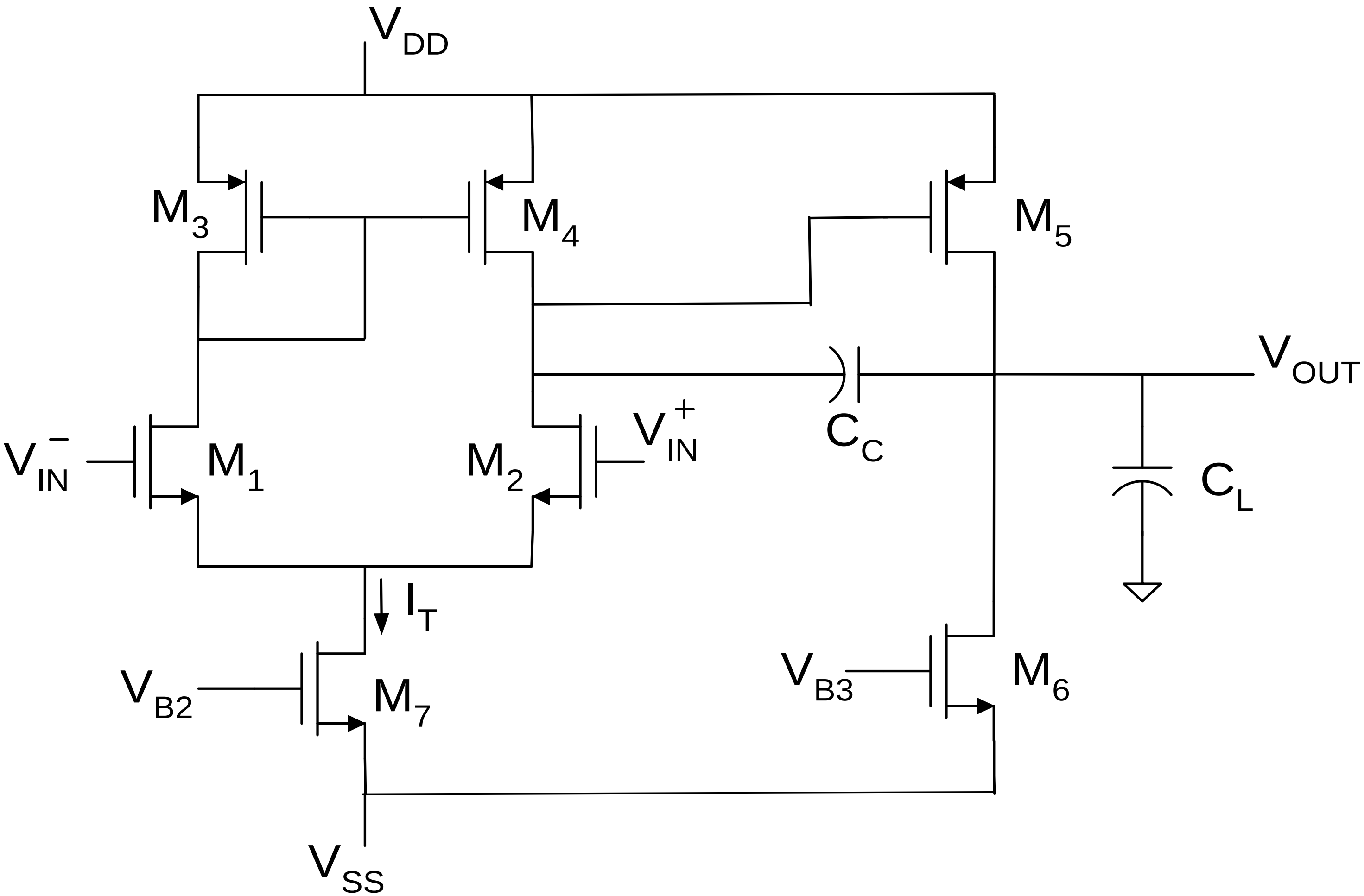
Basic Two-Stage Op Amp



- One of the most widely used op amp architectures
- Essentially just a cascade of two common-source stages
- Compensation Capacitor C_C used to get wide pole separation
- Two poles in amplifier
- No universally accepted strategy for designing this seemingly simple amplifier

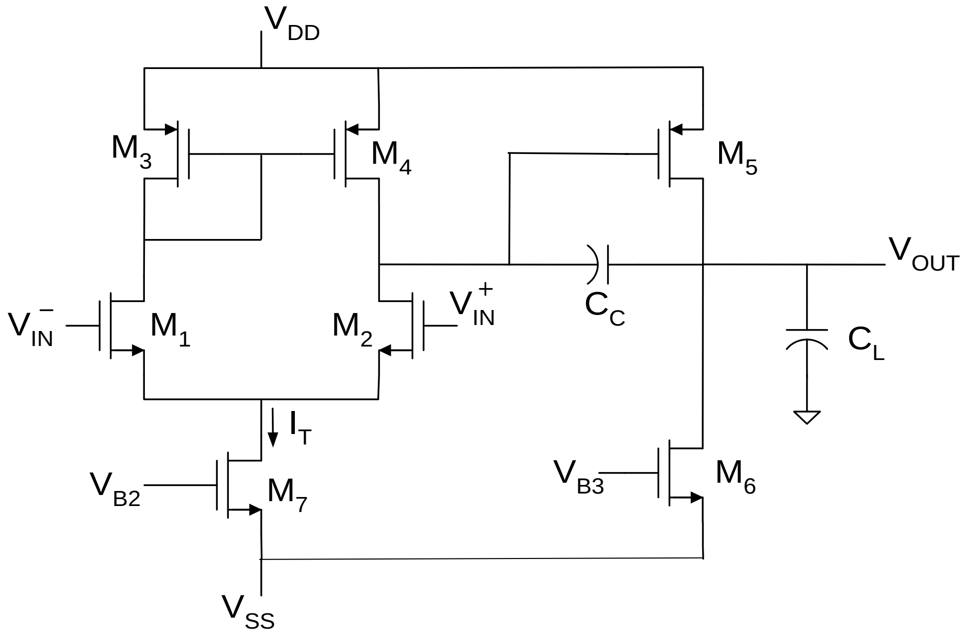
Pole spread $\propto \beta A_{01} A_{02}$ makes C_C unacceptably large

Basic Two-Stage Op Amp (with Miller Compensation)



- o Reduces C_C by approximately A_{02}
- o Pole spread $\propto \beta A_{01} A_{02}$ makes size of C_C manageable

Basic Two-Stage Miller Compensated Op Amp



By inspection

$$A_o = \left(\frac{-g_{m1}}{g_{o2}+g_{o4}}\right)\left(\frac{g_{m5}}{g_{o5}+g_{o6}}\right) \qquad p_2 = \frac{g_{m5}}{C_L}$$

$$p_1 = \frac{g_{o1}+g_{o5}}{C_C\left(\frac{g_{m5}}{g_{05}+g_{o6}}\right)} \qquad GB = \frac{g_{m1}}{C_C}$$

Will also get these results from a more complete (and time consuming) analysis

This analysis was based only upon finding the poles and will miss zeros if they exist

End of Lecture 13