Compensation of Cascaded Amplifier Structures
### Two-stage Architectural Choices

<table>
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<th>Stage 1</th>
<th>Stage 2</th>
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<td>Common Source</td>
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<td>Cascode</td>
<td>Cascode</td>
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<tr>
<td>Regulated Cascode</td>
<td>Regulated Cascode</td>
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<tr>
<td>Folded Cascode</td>
<td>Folded Cascode</td>
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<tr>
<td>Folded Regulated Cascode</td>
<td>Folded Regulated Cascode</td>
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<tr>
<td>Current Mirror</td>
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<tr>
<td>Differential Input</td>
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<tr>
<td>Single Ended Input</td>
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<tr>
<td>Tail Voltage</td>
<td>Tail Voltage</td>
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<tr>
<td>Tail Current</td>
<td>Tail Current</td>
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<tr>
<td>Output Compensated</td>
<td>Internally Compensated</td>
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<tr>
<td>Plus n-channel or p-channel on each stage</td>
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</table>

Which of these 2304 choices can be used to build a good op amp?

**All of them !!**
Basic Two-Stage Op Amp

- One of the most widely used op amp architectures
- Essentially just a cascade of two common-source stages
- Compensation Capacitor $C_C$ used to get wide pole separation
- Two poles in amplifier
- No universally accepted strategy for designing this seemingly simple amplifier
- Pole spread $\propto \beta A_{01} A_{02}$ makes $C_C$ unacceptably large
If $V_2 = -AV_1$ then for $A$ large:

$$C_{1EQ} = C(1 + A) \approx CA$$

$$C_{2EQ} = C\left(1 + \frac{1}{A}\right) \approx C$$

- If $A$ changes with frequency, $C_{1EQ}$ and $C_{2EQ}$ are no longer pure capacitors.
- More useful for giving a concept than for accurate actual analysis because of frequency dependence of $A$. 
Miller Capacitance - Review

Miller Capacitor

\[ Z_{IN} = \frac{V_X}{I_X} = \frac{1}{s[C(1+A)]} \]

If \( A(s) = \frac{A_0}{s+1} \)

\[ G_{IN} = s[C(1+A)] = sC \left( \frac{s}{p} + 1 + A_0 \right) \]

Does not behave as a capacitor for \( \omega > p \).
Basic Two-Stage Op Amp (with Miller Compensation)

- Reduces $C_C$ by approximately $A_{02}$
- Pole spread $\propto \beta A_{01} A_{02}$ makes size of $C_C$ manageable
By inspection

\[ A_o = \left( \frac{-g_{m1}}{g_{o2} + g_{o4}} \right) \left( \frac{g_{m5}}{g_{o5} + g_{o6}} \right) \]

\[ p_2 = \frac{g_{m5}}{C_L} \]

\[ p_1 = \frac{g_{o1} + g_{o5}}{C_C \left( \frac{g_{m5}}{g_{o5} + g_{o6}} \right)} \]

\[ GB = \frac{g_{m1}}{C_C} \]

Will also get these results from a more complete (and time consuming) analysis.

This analysis was based only upon finding the poles and will miss zeros if they exist.
Small Signal Analysis of Basic Two-Stage Op Amp

Differential Small Signal Equivalent

Norton Equivalent One-Port

Two-Port
Small Signal Analysis of Basic Two-Stage Op Amp

Differential Small Signal Equivalent
Small Signal Analysis of Basic Two-Stage Op Amp

Differential Small Signal Equivalent

\[ I_X = V_X \left( g_{o2} + g_{o4} \right) + g_{m2} \frac{V_d}{2} + g_{m4} V_4 \]

\[ V_4 \left( g_{m3} + g_{o1} + g_{o3} \right) + g_{m1} \left( -\frac{V_d}{2} \right) = 0 \]

\[ I_X = V_X \left( g_{o2} + g_{o4} \right) + g_{m2} V_d \left( 1 + \frac{g_{m1} \left( \frac{g_{m4}}{g_{m2} \left( g_{m3} + g_{o2} + g_{o3} \right)} \right)}{2} \right) \]

\[ I_X \approx V_X \left( g_{o2} + g_{o4} \right) + g_{m2} V_d \]
Small Signal Analysis of Basic Two-Stage Op Amp

Differential Small Signal Equivalent

\[ I_X \approx V_X (g_{o2} + g_{o4}) + g_{m2} V_d \]

Since \( M_1 \) and \( M_2 \) are matched as are \( M_3 \) and \( M_4 \)

\[ g_{md} = g_{m1} \]
\[ g_{od} = g_{o2} + g_{o4} \]
Small Signal Analysis of Basic Two-Stage Op Amp

Differential Small Signal Equivalent

\[ g_{oo} = g_{o5} + g_{o6} \]
\[ g_{mo} = g_{m5} \]
Small Signal Analysis of Basic Two-Stage Op Amp

Differential Small Signal Equivalent

\[ V_2 \approx g_m V_2 + g_{oo} \]

Diagram showing the equivalent circuit with transistors M5 and M6.
Small Signal Analysis of Basic Two-Stage Op Amp

Differential Small Signal Equivalent

\[ g_{md} V_d \quad g_{Od} \quad g_{mO} V_2 \quad g_{OO} \]

\[ V_{OUT} \]

\[ C_L \]

\[ C_C \]
Small Signal Analysis of Basic Two-Stage Op Amp

Differential Small Signal Equivalent

\[
\begin{align*}
V_{\text{OUT}} (sC_C + sC_L + g_{oo}) + g_{mo} V_2 &= sC_C V_2 \\
V_2 (sC_C + g_{od}) + g_{md} V_d &= sC_C V_{\text{OUT}}
\end{align*}
\]

Solving we obtain:

\[
V_{\text{OUT}} = \frac{V_d g_{md} (g_{mo} - sC_C)}{s^2 C_C C_L + s [g_{mo} C_C + (C_C (g_{oo} + g_{od}) + C_L g_{od})] + g_{oo} g_{od}}
\]

This simplifies to:

\[
V_{\text{OUT}} \approx \frac{V_d g_{md} (g_{mo} - sC_C)}{s^2 C_C C_L + s g_{mo} C_C + g_{oo} g_{od}}
\]
Small Signal Analysis of Basic Two-Stage Op Amp

Differential Small Signal Equivalent

Summary:

\[
A(s) = \frac{g_{md}(g_{m5} - sC_C)}{s^2C_CC_L + sg_{m5}C_C + g_{oo}g_{od}}
\]

where

\[
g_{md} = g_{m1} = g_{m2}
\]

\[
g_{od} = g_{o2} + g_{o4}
\]

\[
g_{oo} = g_{o5} + g_{o6}
\]
Small Signal Analysis of Two-Stage Miller-Compensated Op Amp

\[ A(s) = \frac{g_{md}(g_{m5} - sC_C)}{s^2C_CC_L + sg_{m5}C_C + g_{oo}g_{od}} \]

Note this is of the form:

\[ A(s) = A_0 \frac{s + 1}{s + 1} \frac{s + 1}{p_1} \frac{s + 1}{p_2} \]

This has two negative real-axis poles and one positive real-axis zero.
How does the Gain of the Two-Stage Miller-Compensated Op Amp Compare with Internal Compensated Op Amp?

\[ A(s) = \frac{g_{md}(g_{m5} - sC_C)}{s^2C_CC_L + sg_{m5}C_C + g_{oo}g_{od}} \]

\[ A(s) = A_0 \frac{s}{z_1} + 1 \]

Compensation criteria:

must be developed

\[ 4\beta A_0 > \frac{p_2}{p_1} > 2\beta A_0 \]
Review of Basic Concepts

Consider a second-order factor of a denominator polynomial, $P(s)$,

$$ P(s) = s^2 + a_1 s + a_0 $$

Then $P(s)$ can be expressed in several alternative but equivalent ways

$$ s^2 + s \frac{\omega_0}{Q} + \omega_0^2 $$

$$ s^2 + s 2\xi \omega_0^2 $$

$$(s - p_1)(s - p_2)$$

and if complex conjugate poles,

$$(s + \alpha + j\beta)(s + \alpha - j\beta)$$

These are all 2-paramater characterizations of the second-order factor and it is easy to map from any one characterization to any other.
Review of Basic Concepts

\[ s^2 + s \frac{\omega_0}{Q} + \omega_0^2 \]

\[ \sin \theta = \frac{1}{2Q} \]

\( \omega_0 = \) magnitude of pole

\( Q \) determines the angle of the pole

Observe:
- \( Q=0.5 \) corresponds to two identical real-axis poles
- \( Q=0.707 \) corresponds to poles making 45° angle with Im axis
Simple pole calculations for 2-stage op amp

Since the poles of the 2-stage op amp must be widely separated, a simple calculation of the poles from the characteristic polynomial is possible.

Assume \( p_1 \) and \( p_2 \) are the poles and \( p_1 \ll p_2 \)

\[
D(s) = s^2 + a_1 s + a_0
\]

but

\[
D(s) = (s + p_1)(s + p_2) = s^2 + s(p_1 + p_2) + p_1 p_2 \approx s^2 + p_2 s + p_1 p_2
\]

determines \( p_2 \)

determines \( p_1 \)

thus

\[
p_2 = -a_1 \quad \text{and} \quad p_1 = -\frac{a_0}{a_1}
\]
Can now use these results to calculate poles of Basic Two-stage Miller Compensated Op Amp

From small signal analysis:

\[
A(s) = \frac{g_{md}(g_{m5} - sC_C)}{s^2C_CC_L + sg_{m5}C_C + g_{oo}g_{od}}
\]

\[
p_2 = \frac{g_{m5}}{C_L}
\]

\[
p_1 = \frac{g_{oo}g_{od}}{g_{m5}C_C}
\]

\[
A_0 = \frac{g_{m5}g_{md}}{g_{oo}g_{od}}
\]

\[
GB = \frac{g_{m5}g_{md}}{g_{oo}g_{od}} \bullet p_1 = \frac{g_{m5}g_{md}}{g_{oo}g_{od}} \bullet \frac{g_{oo}g_{od}}{g_{m5}C_C} = \frac{g_{md}}{C_C}
\]

\[
g_{md} = g_{m1} = g_{m2}
\]

\[
g_{od} = g_{o2} + g_{o4}
\]

\[
g_{oo} = g_{o5} + g_{o6}
\]
Note the simple results obtained from inspection agree with the more time consuming results obtained from a small signal analysis.
Feedback applications of the two-stage Op Amp

How does the amplifier perform with feedback?

How should the amplifier be compensated?
Feedback applications of the two-stage Op Amp

Open-loop Gain

\[ A(s) = \frac{N(s)}{D(s)} \]

Standard Feedback Gain

\[ A_{FB}(s) = \frac{A(s)}{1 + A(s)\beta(s)} = \frac{N(s)}{D(s) + N(s)\beta(s)} \text{ defn } \frac{N_{FB}(s)}{D_{FB}(s)} \]

\[ N_{FB}(s) = N(s) \]
\[ D_{FB}(s) = D(s) + \beta(s)N(s) \]

- Open-loop and closed-loop zeros identical
- Closed-loop poles different than open-loop poles
- Often \( \beta(s) \) is not dependent upon frequency
Feedback applications of the two-stage Op Amp

Open-loop Gain

\[ A(s) = \frac{N(s)}{D(s)} \]

Standard Feedback Gain

\[ A_{FB}(s) = \frac{A(s)}{1 + A(s)\beta(s)} = \frac{1}{1 + \frac{1}{A(s)\beta(s)}} \]

Alternate Feedback Gain

\[ A_{FB}(s) = \frac{1}{\beta_1(s)} = \frac{\beta(s)N(s)}{\beta_1(s)} = \frac{\beta(s)}{\beta_1(s)} \frac{N(s)}{D(s) + N(s)\beta(s)} \]

In either case, denominators are the same and characteristic equation defined by

\[ D_{FB}(s) = D(s) + \beta(s)N(s) \]

Often \( \beta(s) \) and \( \beta_1(s) \) are not dependent upon frequency and in this case

\[ N_{FB}(s) = N(s) \]
Basic Two-Stage Op Amp with Feedback

Open-loop gain

\[ A(s) = \frac{g_{md}(g_{mo} - sC_C)}{s^2C_CC_L + sC_C(g_{mo} - \beta g_{md}) + g_{oo}g_{od} + \beta g_{md}g_{mo}} \]

Standard feedback gain with constant \( \beta \)

\[ A_{FB}(s) = \frac{g_{md}(g_{mo} - sC_C)}{s^2C_CC_L + sC_C(g_{mo} - \beta g_{md}) + g_{oo}g_{od} + \beta g_{md}g_{mo}} \]

where

\[ g_{md} = g_{m1} \]
\[ g_{mo} = g_{m5} \]
\[ g_{od} = g_{o2} + g_{o4} \]
\[ g_{oo} = g_{o5} + g_{o6} \]
Basic Two-Stage Op Amp

\[ A_{FB}(s) \approx \frac{g_{md}(g_{m0} - sC_c)}{s^2C_C C_L + sC_C(g_{m0} - \beta g_{md}) + \beta g_{md}g_{m0}} \]

Pole Q = ?
Basic Two-Stage Op Amp

\[ A_{FB}(s) \approx \frac{g_{md} \left( g_{m0} - sC_C \right)}{s^2 C_C C_L + sC_C \left( g_{m0} - \beta g_{md} \right) + \beta g_{md} g_{m0}} \]

It can be shown that

\[ Q = \sqrt{\frac{C_L}{C_C}} \sqrt{\beta} \frac{\sqrt{g_{m0} g_{md}}}{g_{m0} - \beta g_{md}} \]

\[ C_C = \frac{C_L \beta}{Q^2 \left( g_{m0} - \beta g_{md} \right)^2} \]

where \( g_{md} = g_{m1} \), \( g_{m0} = g_{m5} \), \( g_{oo} = g_{o5} + g_{o6} \), and \( g_{od} = g_{o2} + g_{o4} \)

But what pole \( Q \) is desired? \( .707 < Q < 0.5 \)

Right Half-Plane Zero in OL Gain (from Miller Compensation) Limits Performance
What is “compensation” or “frequency compensation”?

From Wikipedia: In electrical engineering, frequency compensation is a technique used in amplifiers, and especially in amplifiers employing negative feedback. It usually has two primary goals: To avoid the unintentional creation of positive feedback, which will cause the amplifier to oscillate, and to control overshoot and ringing in the amplifier’s step response.

From Martin and Johns – no specific definition but makes comparisons with “optimal compensation” which also is not defined.

From Allen and Holberg (p 243) The goal of compensation is to maintain stability when negative feedback is applied around the op amp.
Compensation

From Gray and Meyer (p634) Thus if this amplifier is to be used in a feedback loop with loop gain larger than $a_0 f_1$, efforts must be made to increase the phase margin. This process is known as compensation.

From Sedra and Smith (p 90) This process of modifying the open-loop gain is termed frequency compensation, and its purpose is to ensure that op-amp circuits will be stable (as opposed to oscillatory).

From Razavi (p355) Typical op amp circuit contain many poles. In a folded-cascode topology, for example, both the folding node and the output node contribute poles. For this reason, op amps must usually be “compensated”, that is, their open-loop transfer function must be modified such that the closed-loop circuit is stable and the time response is well-behaved.
Compensation

What is “compensation” or “frequency compensation” and what is the goal of compensation?

Nobody defines it or defines it correctly but everybody tries to do it!
Compensation

Compensation (alt Frequency Compensation) is the manipulation of the poles and/or zeros of the open-loop amplifier so that when feedback is applied, the closed-loop amplifier will perform acceptably.

Note this definition does not mention stability, positive feedback, negative feedback, phase margin, or oscillation.

Note that acceptable performance is strictly determined by the user in the context of the specific application.
End of Lecture 14