Compensation
Review from last lecture

Review of Basic Concepts

Consider a second-order factor of a denominator polynomial, \( P(s) \),

\[
P(s) = s^2 + a_1 s + a_0
\]

Then \( P(s) \) can be expressed in several alternative but equivalent ways

\[
s^2 + s \frac{\omega_0}{Q} + \omega_0^2
\]
\[
s^2 + s 2\xi \omega_0 + \omega_0^2
\]
\[
(s - p_1)(s - p_2)
\]

and if complex conjugate poles,

\[
(s + \alpha + j\beta)(s + \alpha - j\beta)
\]

These are all 2-parameter characterizations of the second-order factor and it is easy to map from any one characterization to any other.
Review from last lecture

Review of Basic Concepts

\[ s^2 + s \frac{\omega_0}{Q} + \omega_0^2 \]

\[ \sin \theta = \frac{1}{2Q} \]

\( \omega_0 \) = magnitude of pole

Q determines the angle of the pole
Feedback applications of the two-stage Op Amp

Open-loop Gain

\[ A(s) = \frac{N(s)}{D(s)} \]

Alternate Feedback Gain

\[ A_{FB}(s) = \frac{1}{\frac{\beta_1(s)}{1 + \frac{1}{A(s) \beta(s)}}} = \frac{\beta(s)N(s)}{\beta_1(s)D(s) + N(s) \beta(s)} \]

In either case, denominators are the same and characteristic equation defined by

\[ D_{FB}(s) = D(s) + \beta(s)N(s) \]

Often \( \beta(s) \) and \( \beta_1(s) \) are not dependent upon frequency and in this case

\[ N_{FB}(s) = N(s) \]
Basic Two-Stage Op Amp

\[ A_{FB}(s) \approx \frac{g_{md}(g_{m0} - sC_C)}{s^2C_CC_L + sC_C(g_{mo} - \beta g_{md}) + \beta g_{md}g_{mo}} \]

It can be shown that

\[ Q = \sqrt{\frac{C_L}{C_C}} \sqrt{\beta} \frac{\sqrt{g_{mo}g_{md}}}{g_{mo} - \beta g_{md}} \]

\[ C_C = \frac{C_L \beta}{Q^2} \frac{g_{mo}g_{md}}{(g_{mo} - \beta g_{md})^2} \]

where \( g_{md} = g_{m1} \), \( g_{mo} = g_{m5} \), \( g_{oo} = g_{o5} + g_{o6} \) and \( g_{od} = g_{o2} + g_{o4} \)

But what pole Q is desired?

Right Half-Plane Zero Limits Performance
Compensation

What is “compensation” or “frequency compensation” and what is the goal of compensation?

Nobody defines it or defines it correctly but everybody tries to do it!
Compensation (alt Frequency Compensation) is the manipulation of the poles and/or zeros of the open-loop amplifier so that the closed-loop amplifier will perform acceptably.

Note this definition does not mention stability, positive feedback, negative feedback, phase margin, or oscillation.

Note that acceptable performance is strictly determined by the user in the context of the specific application.
Compensation

Compensation is the manipulation of the poles and/or zeros of the open-loop amplifier so that the closed-loop amplifier will perform acceptably.

Acceptable performance is often application dependent and somewhat interpretation dependent.

Acceptable performance should include affects of process and temperature variations.

Although some think of compensation as a method of maintaining stability with feedback, acceptable performance generally dictates much more stringent performance than simply stability.

Compensation criteria are often an indirect indicator of some type of desired (but unstated) performance.

Varying approaches and criteria are used for compensation often resulting in similar but not identical performance.

Over compensation often comes at a considerable expense (increased power, decreased frequency response, increased area, …)
Compensation

Compensation requirements usually determined by closed-loop pole locations:

\[ D_{FB}(s) = D(s) + \beta(s)N(s) \]

- Often Phase Margin or Gain Margin criteria are used instead of pole Q criteria when compensating amplifiers (for historical reasons but must still be conversant with this approach)

- Nyquist plots are an alternative stability criteria that is used some in the design of amplifiers

- Phase Margin and Gain Margin criteria are directly related to the Nyquist Plots

- Compensation requirements are strongly \( \beta \) dependent

Characteristic Polynomial obtained from denominator term of basic feedback equation

\[ 1 + A(s)\beta(s) \]

\( A(s)\beta(s) \) defined to be the “loop gain” of a feedback amplifier
Pole Locations and Stability

Theorem: A system is stable iff all closed-loop poles lie in the open left half-plane.
Pole Locations and Stability

Theorem: A system is stable iff all closed-loop poles lie in the open left half-plane.

Note: Practically want to avoid having closed-loop amplifier poles close to the imaginary axis to provide reasonable stability margin, to minimize ringing in the time-domain, and to minimize peaking in the frequency domain.

$45^\circ$ pole-pair angle corresponds to $Q = \frac{1}{\sqrt{2}} = .707$

$90^\circ$ pole angle (on pole pair) corresponds to $Q = \frac{1}{2}$
Nyquist Plots

The Nyquist Plot is a plot of the Loop Gain ($A\beta$) versus $j\omega$ in the complex plane for $-\infty < \omega < \infty$

**Theorem:** A system is stable iff the Nyquist Plot does not encircle the point $-1+j0$.

Note: If there are multiple crossings of the real axis by the Nyquist Plot, the term encirclement requires a formal definition that will not be presented here.
Nyquist Plots

Example

Stable since -1+j0 is not encircled
Useful for determining stability when few computational tools are available
Legacy of engineers and mathematicians of pre-computer era
Nyquist Plots

Example

$$A(s) = \frac{100}{s + 1}$$

$$\beta = 1/2$$

$$A\beta(j\omega) = \frac{50}{j\omega + 1}$$

In this example, Nyquist plot is circle of radius 25
Nyquist Plots

\[ D_{FB}(s) = 1 + A(s)\beta(s) \]

-1+j0 is the image of ALL poles

The Nyquist Plot is the image of the entire imaginary axis and separates the image complex plane into two parts

Everything outside of the Nyquist Plot is the image of the LHP

Nyquist plot can be generated with pencil and paper
Review of Basic Concepts

Nyquist Plots

Conceptually would like to be sure Nyquist Plot does not get too close to -1+j0
Nyquist Plots

Review of Basic Concepts

Conceptually would like to be sure Nyquist Plot does not get too close to -1+j0

But identification of a suitable neighborhood is not natural
Review of Basic Concepts

Nyquist Plots

Conceptually would like to be sure Nyquist Plot does not get too close to \(-1+j0\)

But identification of a suitable neighborhood is not natural
Phase margin is $180^\circ$ – angle of $A\beta$ when the magnitude of $A\beta = 1$
Gain margin is 1 – magnitude of $A\beta$ when the angle of $A\beta = 180^\circ$
Nyquist Plots

**Theorem:** A system is stable iff the phase margin is positive

**Theorem:** A system is stable iff the gain margin is positive

The phase margin is often the parameter that is specified when compensating operational amplifiers

Phase margins of $45^\circ$ to $60^\circ$ or sometimes even $75^\circ$ are often used

The definition of phase margin does not depend upon the order of the system and is affected by the location of the zeros of the system

The phase margin is a function of $\beta$
Review of Basic Concepts

Nyquist Plots

Engineers have some comfort in how far an amplifier is from becoming stable when specifying phase margin criteria (but this is often not mathematically justifiable).

Pole Q criteria are generally much better to use than phase margin criteria but industry is heavily “phase-margin” entrenched!

Separate magnitude and phase plots are often used rather than Nyquist Plots when assessing phase margins or gain margins.

The magnitude and phase plots convey exactly the same information as Nyquist Plots but have a linear (or logarithmic) axis rather than the highly skewed imaginary axis of the Nyquist Plot.
Example

A feedback amplifier has a characteristic polynomial of

\[ D(s) = s^2 + 9000s + 1.8E3 \]

Without using the quadratic equation, determine the poles by inspection and determine the ratio of the two poles.
A feedback amplifier has a characteristic polynomial of

\[ D(s) = s^2 + 9000s + 1.8E3 \]

\[ P_h = -9000 \]

\[ P_L = -2 \]

Ratio = 4500
Analysis of two-stage op amps is very systematic and can be done by inspection if characteristics of quarter circuit are known.

Compensation is essential for stability when applying feedback.

Miller compensation is very useful for decreasing size of internal compensation capacitor but it does not act as a shunting capacitor at high frequencies.

Nyquist plots are a viable alternative for determining stability from the loop gain.

Nyquist plot is a mapping by the function $A\beta$ from the s-plane to the s-plane and the image of the imaginary axis is the Nyquist plot.

Phase margin (and sometimes gain margin) are widely used to specify compensation expectations but probably not as useful as pole-Q compensation criterion however legacy will keep these concepts around for a long time.
End of Lecture 16