EE 435

Lecture 16

Compensation of Feedback Amplifiers
By inspection

\[ A_0 = \left( \frac{-g_{m1}}{g_{o2} + g_{o4}} \right) \left( \frac{g_{m5}}{g_{o5} + g_{o6}} \right) \]

\[ p_2 = \frac{g_{m5}}{C_L} \]

\[ p_1 = \frac{g_{o1} + g_{o5}}{C_C \left( \frac{g_{m5}}{g_{o5} + g_{o6}} \right)} \]

\[ GB = \frac{g_{m1}}{C_C} \]

Will also get these results from a more complete (and time consuming) analysis.

This analysis was based only upon finding the poles and will miss zeros if they exist.
Small Signal Analysis of Basic Two-Stage Op Amp

Differential Small Signal Equivalent

Norton Equivalent One-Port

Two-Port
Small Signal Analysis of Basic Two-Stage Op Amp

Differential Small Signal Equivalent
Small Signal Analysis of Basic Two-Stage Op Amp

Differential Small Signal Equivalent

\[
\begin{align*}
I_X &= V_X (g_{o2} + g_{o4}) + g_{m2} \frac{V_d}{2} + g_{m4} V_4 \\
V_4 (g_{m3} + g_{o1} + g_{o3}) + g_{m1} \left( - \frac{V_d}{2} \right) &= 0 \\
I_X &= V_X (g_{o2} + g_{o4}) + g_{m2} V_d \left( 1 + \frac{g_{m1} \left( \frac{g_{m4}}{g_{m2}} \right)}{g_{m3} + g_{o2} + g_{o3}} \right) \\
I_X &\approx V_X \left( g_{o2} + g_{o4} \right) + g_{m2} V_d 
\end{align*}
\]
Small Signal Analysis of Basic Two-Stage Op Amp

Differential Small Signal Equivalent

Since $M_1$ and $M_2$ are matched as are $M_3$ and $M_4$

$g_{md} = g_{m1}$

$g_{od} = g_{02} + g_{04}$
Small Signal Analysis of Basic Two-Stage Op Amp

Differential Small Signal Equivalent

\[ V_5 \quad g_{M5} V_5 \quad g_{o5} \quad V_6 \quad g_{M6} V_6 \quad g_{o6} \quad I_X \quad V_X \]

\[ g_{oo} = g_{o5} + g_{o6} \]

\[ g_{mo} = g_{m5} \]
Small Signal Analysis of Basic Two-Stage Op Amp

Differential Small Signal Equivalent
Small Signal Analysis of Basic Two-Stage Op Amp

Differential Small Signal Equivalent
Small Signal Analysis of Basic Two-Stage Op Amp

Differential Small Signal Equivalent

\[ V_{OUT}(sC_C + sC_L + g_{oo}) + g_{mo} V_2 = sC_C V_2 \]
\[ V_2(sC_C + g_{od}) + g_{md} V_d = sC_C V_{OUT} \]

Solving we obtain:

\[ V_{OUT} = V_d \frac{g_{md}(g_{mo} - sC_C)}{s^2C_C C_L + s[g_{mo} C_C + (C_C(g_{oo} + g_{od}) + C_L g_{od})] + g_{oo} g_{od}} \]

This simplifies to:

\[ V_{OUT} \approx V_d \frac{g_{md}(g_{mo} - sC_C)}{s^2C_C C_L + s g_{mo} C_C + g_{oo} g_{od}} \]
Small Signal Analysis of Basic Two-Stage Op Amp

Differential Small Signal Equivalent

Summary:

\[ A(s) = \frac{g_{md} (g_{m5} - sC_C)}{s^2 C_C C_L + s g_{m5} C_C + g_{oo} g_{od}} \]

where

\[ g_{md} = g_{m1} = g_{m2} \]

\[ g_{od} = g_{o2} + g_{o4} \]

\[ g_{oo} = g_{o5} + g_{o6} \]
Small Signal Analysis of Two-Stage Miller-Compensated Op Amp

\[ A(s) = \frac{g_{md}(g_{m5} - sC_C)}{s^2C_CC_L + sg_{m5}C_C + g_{oo}g_{od}} \]

Note this is of the form:

\[ A(s) = A_0 \left( \frac{s}{p_1 + 1} + 1 \right) \left( \frac{s}{p_2 + 1} + 1 \right) \]

This has two negative real-axis poles and one positive real-axis zero
How does the Gain of the Two-Stage Miller-Compensated Op Amp Compare with Internal Compensated Op Amp?

\[
A(s) = \frac{g_{md}(g_{m5} - sC_C)}{s^2C_CC_L + sg_{m5}C_C + g_{oo}g_{od}}
\]

\[
A(s) = A_0 \frac{s}{p_1} \frac{s}{p_2} \frac{1}{s + 1} \frac{s + 1}{s + 1}
\]

Compensation criteria:

\[
4\beta A_0 > \frac{p_2}{p_1} > 2\beta A_0
\]
Consider a second-order factor of a denominator polynomial, \( P(s) \),

\[
P(s) = s^2 + a_1 s + a_0
\]

Then \( P(s) \) can be expressed in several alternative but equivalent ways

\[
s^2 + s \frac{\omega_0}{Q} + \omega_0^2
\]

\[
s^2 + s2\xi^2 \omega_0^2
\]

\[
(s - p_1)(s - p_2)
\]

and if complex conjugate poles,

\[
(s + \alpha + j\beta)(s + \alpha - j\beta)
\]

These are all 2-parameter characterizations of the second-order factor and it is easy to map from any one characterization to any other.
Review of Basic Concepts

\[ s^2 + s \frac{\omega_0}{Q} + \omega_0^2 \]

\[ \sin \theta = \frac{1}{2Q} \]

\( \omega_0 \) = magnitude of pole

Q determines the angle of the pole

Observe: Q=0.5 corresponds to two identical real-axis poles
Q=.707 corresponds to poles making 45° angle with Im axis
Simple pole calculations for 2-stage op amp

Since the poles of the 2-stage op amp must be widely separated, a simple calculation of the poles from the characteristic polynomial is possible.

Assume $p_1$ and $p_2$ are the poles and $p_1 << p_2$

\[
D(s) = s^2 + a_1 s + a_0
\]

but

\[
D(s) = (s + p_1)(s + p_2) = s^2 + s(p_1 + p_2) + p_1 p_2 \approx s^2 + p_2 s + p_1 p_2
\]

thus

\[
p_2 = -a_1 \quad \text{and} \quad p_1 = -a_0 / a_1
\]
Can now use these results to calculate poles of Basic Two-stage Miller Compensated Op Amp

From small signal analysis:

\[ A(s) = \frac{g_{md}(g_{m5} - sC_C)}{s^2C_CC_L + sg_{m5}C_C + g_{oo}g_{od}} \]

\[ p_2 = \frac{g_{m5}}{C_L} \]

\[ p_1 = \frac{g_{oo}g_{od}}{g_{m5}C_C} \]

\[ A_0 = \frac{g_{m5}g_{md}}{g_{oo}g_{od}} \]

\[ GB = \frac{g_{m5}g_{md}}{g_{oo}g_{od}} \cdot p_1 = \frac{g_{m5}g_{md}}{g_{oo}g_{od}} \cdot \frac{g_{oo}g_{od}}{g_{m5}C_C} = \frac{g_{md}}{C_C} \]

\[ g_{md} = g_{m1} = g_{m2} \]

\[ g_{od} = g_{o2} + g_{o4} \]

\[ g_{oo} = g_{o5} + g_{o6} \]
From Previous Inspection

\[ A_o = \left( \frac{-g_{m1}}{g_{o2} + g_{o4}} \right) \left( \frac{g_{m5}}{g_{o5} + g_{o6}} \right) \]

\[ p_1 = \frac{g_{o2} + g_{o4}}{C_C \left( \frac{g_{m5}}{g_{o5} + g_{o6}} \right)} \quad p_2 = \frac{g_{m5}}{C_L} \]

\[ GB = \frac{g_{m1}}{C_C} \]

Note the simple results obtained from inspection agree with the more time consuming results obtained from a small signal analysis.
Feedback applications of the two-stage Op Amp

How does the amplifier perform with feedback?

How should the amplifier be compensated?
Feedback applications of the two-stage Op Amp

Open-loop Gain

\[ A(s) = \frac{N(s)}{D(s)} \]

Standard Feedback Gain

\[ A_{FB}(s) = \frac{A(s)}{1 + A(s)\beta(s)} = \frac{N(s)}{D(s) + N(s)\beta(s)} \]

\[ N_{FB}(s) = N(s) \]

\[ D_{FB}(s) = D(s) + \beta(s)N(s) \]

- Open-loop and closed-loop zeros identical
- Closed-loop poles different than open-loop poles
- Often \( \beta(s) \) is not dependent upon frequency
Feedback applications of the two-stage Op Amp

Open-loop Gain

\[ A(s) = \frac{N(s)}{D(s)} \]

Standard Feedback Gain

\[ A_{FB}(s) = \frac{A(s)}{1 + A(s)\beta(s)} = \frac{1}{1 + \frac{1}{A(s)\beta(s)}} \]

Alternate Feedback Gain

\[ A_{FB}(s) = \frac{1}{\beta_1(s)} = \frac{\beta(s)N(s)}{\beta_1(s)(D(s) + N(s)\beta(s))} \]

In either case, denominators are the same and characteristic equation defined by

\[ D_{FB}(s) = D(s) + \beta(s)N(s) \]

Often \( \beta(s) \) and \( \beta_1(s) \) are not dependent upon frequency and in this case

\[ N_{FB}(s) = N(s) \]
Basic Two-Stage Op Amp with Feedback

Open-loop gain

\[ A(s) = \frac{g_{md}(g_{mo} - sC_c)}{s^2C_C C_L + sC_C (g_{mo} - \beta g_{md}) + g_{oo} g_{od} + \beta g_{md} g_{mo}} \]

Standard feedback gain with constant \( \beta \)

\[ A_{FB}(s) = \frac{g_{md}(g_{mo} - sC_c)}{s^2C_C C_L + sC_C (g_{mo} - \beta g_{md}) + g_{oo} g_{od} + \beta g_{md} g_{mo}} \]

where \( g_{md} = g_{m1} \) \( g_{mo} = g_{m5} \)

\( g_{od} = g_{o2} + g_{o4} \) \( g_{oo} = g_{o5} + g_{o6} \)
Basic Two-Stage Op Amp

\[ A_{FB}(s) \equiv \frac{g_{md}(g_{m0} - sC_C)}{s^2C_CC_L + sC_C(g_{m0} - \beta g_{md}) + \beta g_{md}g_{m0}} \]

Pole Q = ?
Basic Two-Stage Op Amp

\[ A_{FB}(s) \approx \frac{g_{md}(g_{m0} - sC_C)}{s^2C_CC_L + sC_C(g_{m0} - \beta g_{md}) + \beta g_{md}g_{m0}} \]

It can be shown that

\[ Q = \sqrt{\frac{C_L}{C_C}} \sqrt{\beta} \sqrt{\frac{g_{mo}g_{md}}{g_{mo} - \beta g_{md}}} \]

\[ C_C = \frac{C_L\beta}{Q^2} \frac{g_{mo}g_{md}}{(g_{mo} - \beta g_{md})^2} \]

where \( g_{md} = g_{m1} \)

\( g_{oo} = g_{o5} + g_{o6} \)

and \( g_{od} = g_{o2} + g_{o4} \)

But what pole Q is desired? \(.707 < Q < 0.5\)

Right Half-Plane Zero in OL Gain (from Miller Compensation) Limits Performance (because it increases the pole Q and thus requires a larger C_C!)
End of Lecture 16