EE 435

Lecture 17

Compensation of Feedback Amplifiers
Can now use these results to calculate poles of Basic Two-stage Miller Compensated Op Amp

From small signal analysis:

\[
A(s) = \frac{g_{md}(g_{m5} - sC_C)}{s^2C_CC_L + sg_{m5}C_C + g_{oo}g_{od}}
\]

\[
p_2 = \frac{g_{m5}}{C_L}
\]

\[
p_1 = \frac{g_{oo}g_{od}}{g_{m5}C_C}
\]

\[
A_0 = \frac{g_{m5}g_{md}}{g_{oo}g_{od}}
\]

\[
GB = \frac{g_{m5}g_{md}}{g_{oo}g_{od}} \cdot p_1 = \frac{g_{m5}g_{md}}{g_{oo}g_{od}} \cdot \frac{g_{oo}g_{od}}{g_{m5}C_C} = \frac{g_{md}}{C_C}
\]

\[
g_{md} = g_{m1} = g_{m2}
\]

\[
g_{od} = g_{o2} + g_{o4}
\]

\[
g_{oo} = g_{o5} + g_{o6}
\]
Basic Two-Stage Op Amp

\[ A_{FB}(s) \approx \frac{g_{md}(g_{m0} - sC_C)}{s^2C_CC_L + sC_C(g_{m0} - \beta g_{md}) + \beta g_{md}g_{m0}} \]

Pole Q = ?
Basic Two-Stage Op Amp

\[ A_{FB}(s) \approx \frac{g_{md}(g_{m0} - sC_C)}{s^2C_CC_L + sC_C(g_{m0} - \beta g_{md}) + \beta g_{md}g_{m0}} \]

It can be shown that

\[ Q = \sqrt{\frac{C_L}{C_C}} \sqrt{\frac{\beta}{\sqrt{\frac{g_{m0}g_{md}}{g_{m0} - \beta g_{md}}}}} \]

\[ C_C = \frac{C_L\beta}{Q^2 \left(\frac{g_{m0}g_{md}}{g_{m0} - \beta g_{md}}\right)^2} \]

where \( g_{md} = g_{m1} \)
\( g_{m0} = g_{m5} \)
\( g_{oo} = g_{o5} + g_{o6} \)

and \( g_{od} = g_{o2} + g_{o4} \)

But what pole \( Q \) is desired? \(.707 < Q < 0.5\)

Right Half-Plane Zero in OL Gain (from Miller Compensation) Limits Performance
Compensation

What is “compensation” or “frequency compensation”?

From Wikipedia: In electrical engineering, frequency compensation is a technique used in amplifiers, and especially in amplifiers employing negative feedback. It usually has two primary goals: To avoid the unintentional creation of positive feedback, which will cause the amplifier to oscillate, and to control overshoot and ringing in the amplifier’s step response.

From Martin and Johns – no specific definition but makes comparisons with “optimal compensation” which also is not defined

From Allen and Holberg (p 243) The goal of compensation is to maintain stability when negative feedback is applied around the op amp.
Compensation

From Gray and Meyer (p634) Thus if this amplifier is to be used in a feedback loop with loop gain larger than $a_0f_1$, efforts must be made to increase the phase margin. This process is known as compensation.

From Sedra and Smith (p 90) This process of modifying the open-loop gain is termed frequency compensation, and its purpose is to ensure that op-amp circuits will be stable (as opposed to oscillatory).

From Razavi (p355) Typical op amp circuit contain many poles. In a folded-cascode topology, for example, both the folding node and the output node contribute poles. For this reason, op amps must usually be “compensated”, that is, their open-loop transfer function must be modified such that the closed-loop circuit is stable and the time response is well-behaved.
Compensation

What is “compensation” or “frequency compensation” and what is the goal of compensation?

Nobody defines it or defines it correctly but everybody tries to do it!
Compensation

Compensation (alt Frequency Compensation) is the manipulation of the poles and/or zeros of the open-loop amplifier so that when feedback is applied, the closed-loop amplifier will perform acceptably.

Note this definition does not mention stability, positive feedback, negative feedback, phase margin, or oscillation.

Note that acceptable performance is strictly determined by the user in the context of the specific application.
Approach to Studying Compensation

Will attempt to develop a correct understanding of the concept of compensation rather than plunge into a procedure for “doing compensation”

Compensation requires the use of some classical mathematical concepts
Compensation

Compensation is the manipulation of the poles and/or zeros of the open-loop amplifier so that when feedback is applied, the closed-loop amplifier will perform acceptably.

Acceptable performance is often application dependent and somewhat interpretation dependent.

Acceptable performance should include affects of process and temperature variations.

Although some think of compensation as a method of maintaining stability with feedback, acceptable performance generally dictates much more stringent performance than simply stability.

Compensation criteria are often an indirect indicator of some type of desired (but unstated) performance.

Varying approaches and criteria are used for compensation often resulting in similar but not identical performance.

Over compensation often comes at a considerable expense (increased power, decreased frequency response, increased area, …)
Compensation

Compensation requirements usually determined by closed-loop pole locations:

\[ D_{FB}(s) = D(s) + \beta(s)N(s) \]

- Often Phase Margin or Gain Margin criteria are used instead of pole Q criteria when compensating amplifiers (for historical reasons but must still be conversant with this approach)

- Nyquist plots are an alternative stability criteria that is used some in the design of amplifiers

- Phase Margin and Gain Margin criteria are directly related to the Nyquist Plots

- Compensation requirements are strongly \( \beta \) dependent

Characteristic Polynomial obtained from denominator term of basic feedback equation

\[ 1 + A(s)\beta(s) \]

\( A(s)\beta(s) \) defined to be the “loop gain” of a feedback amplifier
Pole Locations and Stability

Theorem: A system is stable iff all closed-loop poles lie in the open left half-plane.
Pole Locations and Stability

Theorem: A system is stable iff all closed-loop poles lie in the open left half-plane.

Note: Practically want to avoid having closed-loop amplifier poles close to the imaginary axis to provide reasonable stability margin, to minimize ringing in the time-domain, and to minimize peaking in the frequency domain.

45° pole-pair angle corresponds to

90° pole angle (on pole pair) corresponds to $Q = \frac{1}{2}$
Nyquist Plots

The Nyquist Plot is a plot of the Loop Gain ($A\beta$) versus $j\omega$ in the complex plane for $-\infty < \omega < \infty$.

**Theorem:** A system is stable iff the Nyquist Plot does not encircle the point $-1+j0$.

**Note:** If there are multiple crossings of the real axis by the Nyquist Plot, the term encirclement requires a formal definition that will not be presented here.
Nyquist Plots

Review of Basic Concepts

Example

Stable since -1+j0 is not encircled
Useful for determining stability when few computational tools are available
Legacy of engineers and mathematicians of pre-computer era
Nyquist Plots

Example

\[ A(s) = \frac{100}{s + 1} \]

\[ \beta = \frac{1}{2} \]

\[ A\beta(j\omega) = \frac{50}{j\omega + 1} \]

In this example, Nyquist plot is circle of radius 25
Nyquist Plots

Review of Basic Concepts

$D_{FB}(s) = 1 + A(s)\beta(s)$

-1+j0 is the image of ALL poles

The Nyquist Plot is the image of the entire imaginary axis and separates the image complex plane into two parts

Everything outside of the Nyquist Plot is the image of the LHP

Nyquist plot can be generated with pencil and paper

Important in the ‘30s - ‘60’s
Review of Basic Concepts

Nyquist Plots

Conceptually would like to be sure Nyquist Plot does not get too close to -1+j0
Nyquist Plots

Conceptually would like to be sure Nyquist Plot does not get too close to $-1+j0$

But identification of a suitable neighborhood is not natural.
Nyquist Plots

Review of Basic Concepts

Conceptually would like to be sure Nyquist Plot does not get too close to \(-1+j0\)

But identification of a suitable neighborhood is not natural
Phase margin is $180^\circ$ – angle of $A\beta$ when the magnitude of $A\beta = 1$
Gain margin is 1 – magnitude of $A\beta$ when the angle of $A\beta = 180^\circ$
Nyquist and Gain-Phase Plots convey identical information but gain-phase plots often easier to work with.

Note: The two plots do not correspond to the same system in this slide.
Nyquist and Gain-Phase Plots

Nyquist and Gain-Phase Plots convey identical information but gain-phase plots often easier to work with

Aβ plots change with different values of β
Often β is independent of frequency
   in this case Aβ plot is just a shifted version of A
   in this case phase of Aβ is equal to the phase of A

Instead of plotting Aβ, often plot |A| and phase of A and superimpose |β| and phase of β to get gain and phase margins
   do not need to replot |A| and phase of A to assess performance with different β
Gain and Phase Margin Examples

\[ T(s) = \frac{1000}{(s + 1)} \]

Magnitude in dB

Angle in degrees

-180°
Gain and Phase Margin Examples

Be aware of the multiple values of the arctan function!
Gain and Phase Margin Examples

Magnitude in dB

Phase Margin

Angle in degrees
Gain and Phase Margin Examples

Magnitude in dB

Phase Margin

Angle in degrees

$\beta^{-1}$
Gain and Phase Margin Examples

$$A(s) = \frac{1000}{(s+1)\left(\frac{s}{200}+1\right)}$$

Phase Margin:

- Magnitude in dB
- Angle in degrees

$$\beta^{-1}$$
Gain and Phase Margin Examples

\[ A(s) = \frac{1000}{(s+1)\left(\frac{s}{200} + 1\right)} \]

- **Magnitude in dB**
- **Angle in degrees**

**Phase Margin**

Unstable!
Gain and Phase Margin Examples

\[ T(s) = \frac{1000}{(s + 1)^3} \]

Magnitude in dB

\[ \beta^{-1} \]

Angle in degrees

-180°

Phase Margin
Gain and Phase Margin Examples

\[ T(s) = \frac{1000}{(s + 1)^3} \]

- **Gain Margin**: \( \beta^{-1} \)
- **Phase Margin**: \(-180^\circ\)
Gain and Phase Margin Examples

\[ T(s) = \frac{1000}{(s + 1)^3} \]

Magnitude in dB

Angle in degrees

Phase Margin

-180°
Gain and Phase Margin Examples

\[ T(s) = \frac{1000}{(s + 1)^3} \]

Gain Margin

-180°
Relationship between pole Q and phase margin

In general, the relationship between the phase margin and the pole Q is dependent upon the order of the transfer function and on the location of the zeros.

In the special case that the open loop amplifier is second-order low-pass, a closed form analytical relationship between pole Q and phase margin exists and this is independent of \( A_0 \) and \( \beta \).

\[
Q = \frac{\sqrt{\cos(\varphi_M)}}{\sin(\varphi_M)} \quad \varphi_M = \cos^{-1}\left(\sqrt{1 + \frac{1}{4Q^4} - \frac{1}{2Q^2}}\right)
\]

The region of interest is invariable only for \( 0.5 < Q < 0.7 \). Larger Q introduces unacceptable ringing and settling. Smaller Q slows the amplifier down too much.
Phase Margin vs Q

Second-order low-pass Amplifier
Phase Margin vs Q

Second-order low-pass Amplifier

![Graph showing the relationship between phase margin and pole Q. The graph plots Pole Q on the y-axis and Phase Margin on the x-axis. The data shows a decreasing trend as phase margin increases.]
Phase Margin vs Q

Second-order low-pass Amplifier

![Graph showing Phase Margin vs Q for a second-order low-pass amplifier.](image)
Magnitude Response of 2\textsuperscript{nd}-order Lowpass Function

$$Q_{\text{MAX}} \text{ for no peaking } = \frac{1}{\sqrt{2}} = .707$$

$$\xi = \frac{1}{2Q}$$

From Laker-Sansen Text
Phase Response of 2nd-order Lowpass Function

\[ \xi = \frac{1}{2Q} \]

From Laker-Sansen Text
Step Response of 2\textsuperscript{nd}-order Lowpass Function

$$Q_{\text{MAX}} \text{ for no overshoot } = \frac{1}{2}$$

$$\xi = \frac{1}{2Q}$$

From Laker-Sansen Text
Step Response of 2\textsuperscript{nd}-order Lowpass Function

\[ \xi = \frac{1}{2Q} \]

From Laker-Sansen Text
Compensation Summary

• Gain and phase margin performance often strongly dependent upon architecture
• Relationship between overshoot and ringing and phase margin were developed only for 2\textsuperscript{nd}-order lowpass gain characteristics and differ dramatically for higher-order structures
• Absolute gain and phase margin criteria are not robust to changes in architecture or order
• It is often difficult to correctly “break the loop” to determine the loop gain $A\beta$ with the correct loading on the loop (will discuss this more later)
End of Lecture 17