Gain Enhancement with Positive Feedback
Linearity in Operational Amplifiers
-- The differential pairs
$g_{meq}$ Enhancement with Driven Counterpart Circuit

\[ A_{V_0} = \frac{g_{m1} + g_{m3}}{g_{o1} + g_{o3}} \]

\[ GB = \frac{g_{m1} + g_{m3}}{C_L} \]

Both gain and GB enhancement
Other Methods of Gain Enhancement

Recall:

\[ A_{V0} = \frac{-g_{MQC}}{g_{OQC} + g_{OCC}} \]

Two Strategies:
1. Decrease denominator of \( A_{V0} \)
2. Increase numerator of \( A_{V0} \)

Consider again decreasing the denominator

\[ A_{V0} = \frac{-g_{MQC}}{g_{OQC} + g_{OCC} - g_{OX}} \]

Is it possible to come up with circuits that will provide a subtraction of conductance in the denominator?
Gain Enhancement with Regenerative Feedback

The gain can be made arbitrarily large by selecting $g_{mP1}$ appropriately.

The GB does not degrade!

But - can we easily build circuits with this property?
Gain Enhancement with Regenerative Feedback

But - can we easily build circuits with this property?

But – the inverting amplifier may be more difficult to build than the op amp itself!

YES – simply by cross-coupling the outputs in a fully differential structure
Gain Enhancement with Regenerative Feedback

\[ A_{V0}(s) = \frac{-g_{mF1}}{sC_L + g_{oF1} + g_{oP1} - g_{mP1}} \]

If \( g_{mP1} = g_{oF1} + g_{oP1} \), the dc gain will become infinite!!
Gain Enhancement with Regenerative Feedback

\[ A_{V0}(s) = \frac{-g_{mF1}}{sC_L + g_{OF1} + g_{OP1} - g_{mP1}} \]

\[ p = \frac{-g_{OF1} - g_{OP1} + g_{mP1}}{C_L} \]

If \( g_{mP1} > g_{OF1} + g_{OP1} \), the pole will be in the RHP!!

This will make the op amp unstable.

Positive Feedback is BAD!!

This is the major reason most have avoided using the structure!
Gain Enhancement with Regenerative Feedback

\[ AV_0(s) = \frac{-g_{mF1}}{sC_L + g_{oF1} + g_{oP1} - g_{mP1}} \]

\[ p = \frac{-g_{oF1} - g_{oP1} + g_{mP1}}{C_L} \]

If \( g_{mP1} > g_{oF1} + g_{oP1} \), the pole will be in the RHP !!

It can be shown that the feedback amplifier is usually stable even if the open-loop Op amp is unstable.

The feedback performance can actually be enhanced if the open-loop amplifier is unstable.

Considerable research has been ongoing for the past couple of years in using this approach and it shows considerable promise for gain enhancement in low voltage processes.
Gain Enhancement with Regenerative Feedback

If \( g_{mP1} > g_{oF1} + g_{oP1} \), the pole will be in the RHP!!

It can be shown that the feedback amplifier is usually stable even if the open-loop Op amp is unstable. **How?**

Recall: The numerator of \( A_{V0} \) does not change signs when the constant term in the denominator transitions from positive to negative.

\[
A_{V0}(s) = \begin{cases} 
\frac{A_{V0} \tilde{p}_1}{s + \tilde{p}_1} & \text{for } \tilde{p}_1 > 0 \\
-\frac{A_{V0} \tilde{p}_1}{s + \tilde{p}_1} & \text{for } \tilde{p}_1 < 0 
\end{cases}
\]
Gain Enhancement with Regenerative Feedback

It can be shown that the feedback amplifier is usually stable even if the open-loop Op amp is unstable.

**How?**

\[
A_V (s) = \begin{cases} 
\frac{A_{V0} \tilde{p}_1}{s + \tilde{p}_1} & \text{for } \tilde{p}_1 > 0 \\
-\frac{A_{V0} \tilde{p}_1}{s + \tilde{p}_1} & \text{for } \tilde{p}_1 < 0
\end{cases}
\]

\[
A_{FB} (s) = \begin{cases} 
\frac{A_{V0} \tilde{p}_1}{s + \tilde{p}_1 (1 + \beta A_{V0})} & \text{for } \tilde{p}_1 > 0 \\
-\frac{A_{V0} \tilde{p}_1}{s + \tilde{p}_1 (1 - \beta A_{V0})} & \text{for } \tilde{p}_1 < 0
\end{cases}
\]

\[
p_{FB} = \begin{cases} 
-\tilde{p}_1 (1 + \beta A_{V0}) = p_1 (1 + \beta A_{V0}) & \text{for } p_1 < 0 \\
-\tilde{p}_1 (1 - \beta A_{V0}) = p_1 (1 - \beta A_{V0}) & \text{for } p_1 > 0
\end{cases}
\]
Gain Enhancement with Regenerative Feedback

It can be shown that the feedback amplifier is usually stable even if the open-loop Op amp is unstable

How?

\[ p_{FB} = \begin{cases} -\tilde{p}_1 (1 + \beta A_{V0}) &= p_1 (1 + \beta A_{V0}) & \text{for } p_1 < 0 \\ -\tilde{p}_1 (1 - \beta A_{V0}) &= p_1 (1 - \beta A_{V0}) & \text{for } p_1 > 0 \end{cases} \]

Open-Loop and Closed-Loop Pole Plot for equal open-loop pole magnitudes
Gain Enhancement with Regenerative Feedback

\[ A_{V0} = \frac{-g_{mF1}}{sC_L + g_{oF1} + g_{oP1} - g_{mP1}} \]

\[ p = \frac{-g_{oF1} - g_{oP1} + g_{mP1}}{C_L} \]

If \( g_{mP1} > g_{oF1} + g_{oP1} \), the pole will be in the RHP !!

The feedback performance can actually be enhanced if the open-loop amplifier is unstable

Why?
Gain Enhancement with Regenerative Feedback

The feedback performance can actually be enhanced if the open-loop amplifier is unstable.

**Why?**
Gain Enhancement with Regenerative Feedback

The feedback performance can actually be enhanced if the open-loop amplifier is unstable.

Why?

- Time required to get in settling window can be reduced with RHP pole
- But, if pole is too far in RHP, response will exit top of window
Some Half-Circuits with Interesting Potential

\[
\begin{align*}
&V_A \rightarrow P_{1A}, \quad V_B \rightarrow P_{1B} \\
&V_{IN} \rightarrow F_1, \quad V_{OUT} \rightarrow C_L \\
&V_A \rightarrow P_{1A}, \quad V_B \rightarrow P_{1B} \\
&V_{IN} \rightarrow F_{1A}, \quad V_{INB} \rightarrow F_{1A} \\
&V_A \rightarrow P_{1}, \quad V_B \rightarrow P_{1} \\
&V_{IN} \rightarrow F_{1A}, \quad V_{INB} \rightarrow F_{1A} \\
&V_A \rightarrow P_{1A}, \quad V_B \rightarrow P_{1B} \\
&V_{IN} \rightarrow F_{1A}, \quad V_{OUT} \rightarrow C_L \\
&V_{IN} \rightarrow F_{1A}, \quad V_{OUT} \rightarrow C_L
\end{align*}
\]
Existing Positive Feedback Amplifier

\[ A_{VO} = \frac{(1/2)g_{ml}}{g_{o2} + g_{o4} + g_{o6} + g_{m6} - g_{m4}} \approx \frac{(1/2)g_{ml}}{g_{m6} - g_{m4}} \]

\[ A(s) = \frac{(1/2)g_{ml}}{sC_L + [g_{o2} + g_{o4} + g_{o6} + g_{m6} - g_{m4}]} \]
Existing Positive Feedback Amplifier

\[ A_{VO} = \frac{(1/2)g_{ml}}{g_{o2} + g_{o4} + g_{o6} + g_{m6} - g_{m4}} \approx \frac{(1/2)g_{ml}}{g_{m6} - g_{m4}} \]

\[ A(s) = \frac{(1/2)g_{ml}}{sC_L + [g_{o2} + g_{o4} + g_{o6} + g_{m6} - g_{m4}]} \]

• Requires precise matching of \( g_{m4} \) to \( (g_{o2} + g_{o4} + g_{o6} + g_{m6}) \) for good gain enhancement
• Difficult to match \( g_m \) terms to \( g_o \)-type terms
Alternate Positive Feedback Amplifier
Alternate Positive Feedback Amplifier

\[
A_{\text{vo}} = \frac{(1/2)g_{\text{ml}}}{g_{o2} + g_{o4} + g_{o6} - g_m}
\]

\[
A(s) = \frac{(1/2)g_{\text{ml}}}{sC_L + [g_{o2} + g_{o4} + g_{o6} - g_m]}
\]

• Requires precise matching of \(g_m\) to \((g_{o2}+g_{o4}+g_{o6})\) for good gain enhancement

• Difficult to match \(g_m\) terms to \(g_o\)-type terms
Another Positive Feedback Amplifier

- Regenerative feedback can be to either quarter circuit or counterpart circuit
- Regenerative feedback to cascode devices can significantly reduce the magnitude of the negative conductance term
Another Positive Feedback Amplifier

\[ V_1(g_{01} + g_{05} + g_{05A}) + g_{mi}V_{IN}/2 = V_0(g_{05} + g_{05A}) - g_{m5}(K_2V_2 + V_1) - g_{m5A}V_1 \]

\[ V_2(g_{03} + g_{07} + g_{07A}) = V_0(g_{07} + g_{07A}) + g_{m7}(-K_1V_1 - V_2) - g_{m7A}V_2 \]

\[ V_0(sC_L + g_{05} + g_{05A} + g_{07} + g_{07A}) = V_2(g_{07} + g_{07A}) + V_1(g_{05} + g_{05A}) + g_{m7}(K_1V_1 + V_2) \]

\[ + g_{m7A}V_2 + g_{m5}(K_2V_2 + V_1) + g_{m5A}V_1 \]

Small-signal half circuit

Ki=0 if cross-coupling absent, 1 if cross-coupling present
Another Positive Feedback Amplifier

\[ V_I\left( g_{o1} + g_{o5} + g_{o5A} \right) + g_{m1}V_{IN} / 2 = V_0 \left( g_{o5} + g_{o5A} \right) - g_{m5} \left( K_2 V_2 + V_1 \right) - g_{m5A} V_I \]

\[ V_2 \left( g_{o3} + g_{o7} + g_{o7A} \right) = V_0 \left( g_{o7} + g_{o7A} \right) + g_{m7} \left( -K_1 V_1 - V_2 \right) - g_{m7A} V_2 \]

\[ V_0 \left( sC_L + g_{o5} + g_{o5A} + g_{o7} + g_{o7A} \right) = V_2 \left( g_{o7} + g_{o7A} \right) + V_1 \left( g_{o5} + g_{o5A} \right) + g_{m7} \left( K_1 V_1 + V_2 \right) \\
+ g_{m7A} V_2 + g_{m5} \left( K_2 V_2 + V_1 \right) + g_{m5A} V_1 \]

Transfer function solution with MAPLE \( T(s) = N(s) / S(s) \)

\[
\text{num} := -(-K1 K2 g_{m5} g_{m7} + g_{m5} g_{m7} + g_{m7A} g_{m5} + g_{o7} g_{m5} + g_{o3} g_{m5} \\
+ g_{o7A} g_{m5} + K1 g_{o3} g_{m7} + g_{m5} g_{m7} + g_{o5} g_{m7} + g_{o5A} g_{m7} \\
+ g_{o5A} g_{o7A} + g_{o5} g_{o7A} + g_{o5A} g_{m7A} + g_{o5} g_{m7A} + g_{o5} g_{o7} \\
+ g_{o5} g_{o3} + g_{m5} g_{m7} + g_{m5} g_{o7} + g_{m5} g_{o3} + g_{o5} g_{o3} \\
+ g_{m5} g_{o7A} + g_{m5A} g_{o7}) g_{m1} \\
\]

\[
\text{den} := -g_{o1} g_{o7} g_{m5} K2 - g_{m7} K1 g_{o5} g_{o3} - g_{m7} K1 g_{o5} g_{o3} \\
- g_{o1} g_{o7A} g_{m5} K2 + (g_{m5} g_{m7A} + g_{m7A} g_{m5} + g_{o5} g_{o3} \\
+ g_{o5A} g_{m7A} + g_{m5} g_{m7} + g_{o1} g_{m7} + g_{o5} g_{m7} + g_{o5A} g_{m7} \\
+ g_{m5} g_{m7} - K1 K2 g_{m5} g_{m7} + g_{o1} g_{o7} + g_{m5} g_{o3} + g_{o5} g_{m7A} \\
+ g_{o3} g_{m5} + g_{o5} g_{o3} + g_{o5} g_{o7A} + g_{o5} g_{o7} + g_{o1} g_{o7A} \\
+ g_{o5A} g_{o7} + g_{m5} g_{o7A} + g_{o5A} g_{o7A} + g_{o7} g_{m5} + g_{m5} g_{o7} \\
+ g_{o7A} g_{m5} + g_{o1} g_{o3} + g_{o1} g_{m7A}) sCL + g_{m7} g_{o1} \\
+ g_{o5A} g_{o1} g_{o3} + g_{m7} g_{o5A} g_{o1} + g_{m5} g_{o7A} g_{o3} + g_{m5} g_{o7} g_{o3} \\
+ g_{o5} g_{o7} g_{o3} + g_{o5} g_{o7} g_{o3} + g_{m5} g_{o7} g_{o3} + g_{o1} g_{o7A} g_{o3} \\
+ g_{o1} g_{o7} g_{o3} + g_{o5A} g_{o1} g_{o7A} + g_{o5A} g_{o1} g_{o7} + g_{o5} g_{o1} g_{o3} \\
+ g_{o5A} g_{o1} g_{m7A} + g_{o5} g_{o7} g_{o3} + g_{o5} g_{o1} g_{m7A} + g_{o5} g_{o7A} g_{o3} \\
+ g_{o5} g_{o1} g_{o7A} + g_{m5} g_{o7A} g_{o3} + g_{o5A} g_{o7A} g_{o3} \\
\]

Ki=0 if cross-coupling absent, 1 if cross-coupling present
Another Positive Feedback Amplifier

\[ V_1 (g_{01} + g_{05} + g_{05A}) + g_{m1} V_{IN} / 2 = V_0 (g_{05} + g_{05A}) - g_{m5} (K_2 V_2 + V_1) - g_{m5A} V_1 \]

\[ V_2 (g_{03} + g_{07} + g_{07A}) = V_0 (g_{07} + g_{07A}) + g_{m7} (-K_1 V_1 - V_2) - g_{m7A} V_2 \]

\[ V_0 (sC_L + g_{05} + g_{05A} + g_{07} + g_{07A}) = V_2 (g_{07} + g_{07A}) + V_1 (g_{05} + g_{05A}) + g_{m7} (K_1 V_1 + V_2) \]

\[ + g_{m7A} V_2 + g_{m5} (K_2 V_2 + V_1) + g_{m5A} V_1 \]

\[ T(s) = N(s) / D(s) \]

Neglecting go terms compared to gm terms, simplifies to:

\[
\text{num} := (g_{m5h} g_{m7h} - K1 K2 g_{m5} g_{m7} + K1 g_{o3} g_{m7} + g_{m5h} g_{o3} + g_{o7h} g_{m5h} + g_{o5h} g_{m7h}) g_{m1}
\]

\[
\text{den} := (K1 K2 g_{m5} g_{m7} - g_{m5h} g_{m7h} - g_{o7h} g_{m5h} - g_{o1} g_{m7h} - g_{o5h} g_{m7h} - g_{m5h} g_{o3}) sC_L - g_{o7h} g_{o1} g_{o5h} - g_{o7h} g_{o1} g_{o3} - g_{o7h} g_{o5h} g_{o3} - g_{o1} g_{o5h} g_{m7h} - g_{o1} g_{o5h} g_{o3} - g_{o7h} g_{m5h} g_{o3} + g_{o5h} g_{m7} K1 g_{o3} + g_{o7h} g_{o1} g_{m5} K2
\]
Practical Comments about Positive Feedback Gain Enhancement

- Significant gain enhancement is possible but most designers avoid regenerative feedback because of unfounded concerns about closed-loop stability.

- Accuracy and settling time can be improved with some regenerative feedback.

- Will become more critical in emerging processes where $g_m/g_o$ ratios degrade and where supply voltages shrink thus limiting the longstanding cascode process.

- Regenerative structures can have high sensitivities.

- Signal swing quite limited in some of the most basic regenerative feedback structures.

- Most useful in two-stage architecture where regenerative feedback is used in first stage (effects of signal swing are reduced by gain of second stage).
Summary of Methods of Gain Enhancement

Increasing the output impedance of the amplifier
- cascode, folded cascode, regulated cascode, positive feedback

Increasing the transconductance
- (current mirror op amp) but it didn’t really help because the output conductance increased proportionally
- Driving the counterpart circuit does offer some improvements in gain

Cascading gives a multiplicative gain effect
- (thousands of architectures but compensation is essential)
- usually limited to a two-level cascade because of too much phase accumulation

One or more of these effects can be combined
Operational Amplifier Architectures

Most of the popular operational amplifier architectures have been introduced.

Large number of different architectural choices exist with substantially different performance potential.

Choice of architecture is important but judicious use of DOF is essential to obtain good performance.

Few architectures offer a GB power efficiency that is better than that of the reference op amp (but some two-stage amplifiers do).

Some variants of the basic amplifier structures such as buffered output stages are commonly used in some applications.
Observations about Op Amp Design

• Considerably different insight can often be obtained by viewing a circuit in multiple ways

• Various systematic procedures for designing op amps have been introduced

• It is important to understand the design space and to identify a good set of design variables
  – Design spaces can be explored in many different ways but the degrees of freedom are incredibly valuable resources and should be used judiciously

• Cascaded amplifiers offer potential for gain enhancement but compensation schemes to practically work with more than two levels of cascading have not yet emerged

• Positive feedback appears to provide a promising approach for building high gain amplifiers in low voltage processes but research is ongoing into how this concept can be fully utilized
Up to this point all analysis of the op amp has focused on small-signal gain characteristics.

Linearity of the amplifier does play a role in linearity and spectral performance of feedback amplifiers.

Linearity is of major concern when the op amp is used open-loop such as in OTA applications.

A major source of linearity is often associated with the differential input pair.

Will consider linearity of the input differential pairs.
Signal Swing and Linearity

Signal swing identifies range over which signals can be applied and still maintain operation of devices in desired region of operation.

Some subset of the signal swing range will be quite linear.

Often that subset is close to the entire signal swing range.
Signal Swing and Linearity

Ideal Scenario:

Completely Linear over Input and Output Range
Signal Swing and Linearity

Realistic Scenario:

• Modest Nonlinearity throughout Input Range
• But operation will be quite linear over subset of this range
Signal Swing and Linearity

\[ V_{\text{OUT}} \]

\[ V_{\text{IN}} \]

Output Range

Input Range

Linear Output Range

Linear Input Range
Linearity of Amplifiers

Linearity of differential pair of major concern

Linearity of common-source amplifier is of major concern (since signals so small at output of differential pair)
Differential Input Pairs

MOS Differential Pair

Bipolar Differential Pair
MOS Differential Pair

\[ I_{D1} = \frac{\mu C_{ox} W}{2L} (V_1 - V_S - V_T)^2 \]

\[ I_{D2} = \frac{\mu C_{ox} W}{2L} (V_2 - V_S - V_T)^2 \]

\[ I_{D1} + I_{D2} = I_T \]

\[ \sqrt{I_{D1}} \frac{2L}{\mu C_{ox} W} = V_1 - V_S - V_T \]

\[ \sqrt{I_{D2}} \frac{2L}{\mu C_{ox} W} = V_2 - V_S - V_T \]

\[ V_d = \sqrt{\frac{2L}{\mu C_{ox} W}} (\sqrt{I_T} - \sqrt{I_{D1}} - \sqrt{I_{D1}}) \]

\[ V_d = \sqrt{\frac{2L}{\mu C_{ox} W}} (\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}) \]

\[ V_d = V_2 - V_1 \]
MOS Differential Pair

\[
V_d = \sqrt{\frac{2L}{\mu C_{OX}W}} \left(\sqrt{I_T} - \sqrt{I_{D1}} - \sqrt{I_{D2}}\right)
\]

\[
V_d = \sqrt{\frac{2L}{\mu C_{OX}W}} \left(\sqrt{I_{D2}} - \sqrt{I_T} - \sqrt{I_{D2}}\right)
\]

What values of \(V_d\) will cause all of the current to be steered to the left or the right?

\[
V_{dx} = \pm \sqrt{\frac{2L}{\mu C_{OX}W}} \left(\sqrt{I_T}\right)
\]
\[ V_d = \sqrt{\frac{2L}{\mu C_{ox} W}} \left( \sqrt{I_{D2}} - \sqrt{I_T - I_{D2}} \right) \]
Q-point Calculations

\[ \frac{I_T}{2} = \frac{\mu C_{ox} W}{2L} (V_{EB})^2 \]

\[ V_{EB} = \sqrt{I_T} \sqrt{\frac{L}{\mu C_{ox} W}} \]

Observe !!

\[ V_{dx} = \pm \sqrt{2} V_{EB} \]
\[ V_d = \sqrt{\frac{2L}{\mu C_{ox} W}} \left( \sqrt{I_{D2}} - \sqrt{I_T - I_{D2}} \right) \]

- \( I_{D1} \) affects linearity
- \( I_{D2} \) affects linearity
- \( V_{EB} \) affects linearity

**How linear is the amplifier?**
How linear is the amplifier?

\[ V_d = \sqrt{\frac{2L}{\mu C_{ox} W}} \left( \sqrt{I_T} - I_{D1} - \sqrt{I_{D1}} \right) \]

Consider the fit line:

\[ I = mV_d + h \]

When \( V_d = 0 \), \( I = I_T / 2 \), thus

\[ h = \frac{I_T}{2} \]

\[ V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m} \]

\[ m = \left. \frac{\partial I_{D1}}{\partial V_d} \right|_{Q-p} \]
How linear is the amplifier?

\[ I = mV_d + h \]

\[ V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m} \]

\[ m = \frac{\partial I_{D1}}{\partial V_d} \bigg|_{Q\text{-pt}} \]

\[ V_d = \sqrt{\frac{2L}{\mu C_{ox} W}} \left( \sqrt{I_T - I_{D1}} - \sqrt{I_{D1}} \right) \]

\[ \frac{\partial V_d}{\partial I_{D1}} = -2 \sqrt{\frac{L}{\mu C_{ox} W}} \frac{1}{\sqrt{I_T}} \]

\[ \frac{\partial V_d}{\partial I_{D1}} = -2 \frac{V_{EB1}}{I_T} \]

\[ \sqrt{\frac{L}{\mu C_{ox} W}} = \frac{V_{EB1}}{\sqrt{I_T}} \]

\[ \frac{\partial V_d}{\partial I_{D1}} = -2 \frac{V_{EB1}}{I_T} \]

\[ m = \frac{\partial I_{D1}}{\partial V_d} \bigg|_{Q\text{-pt}} = -\frac{I_T}{2V_{EB1}} \]
How linear is the amplifier?

\[ V_{\text{dint}} = -\frac{h}{m} = -\frac{I_T}{2m} = V_{EB1} \]

\[ I = -\left(\frac{I_T}{2V_{EB1}}\right)V_d + \frac{I_T}{2} \]
How linear is the amplifier?

It can be shown that a 1% deviation from the straight line occurs at

\[ V_d \approx \frac{V_{EB}}{3} \]

and a 0.1% variation occurs at

\[ V_d \approx \frac{V_{EB}}{10} \]
How linear is the amplifier?

1% Linear = 0.3\(V_{EB1}\)
How linear is the amplifier?

![Deviation from Linear](image)

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<td>2.02</td>
<td>0.6</td>
<td>4.61</td>
</tr>
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</table>
How linear is the amplifier?

Distortion in the differential pair is another useful metric for characterizing linearity of $I_{D1}$ and $I_{D2}$ with sinusoidal differential excitation.

Consider again the differential pair and assume excited differentially with

$$V_2 = \frac{V_d}{2}, \quad V_1 = -\frac{V_d}{2}$$

and assume $V_d = V_m \sin(\omega t)$

Recall:

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}} \left( \sqrt{I_{D2}} - \sqrt{I_T} \right)$$

Define

$$\theta = \frac{\mu C_{OX} W}{2L}$$

Thus can express as

$$\sqrt{\theta} V_d = \sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}$$
How linear is the amplifier?

and assume $V_d = V_m \sin(\omega t)$

$$\theta = \frac{\mu C_{\text{ox}} W}{2L}$$

$$\sqrt{\theta V_d} = \sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}$$

Squaring, regrouping, and squaring we obtain

$$\theta V_d^2 = I_{D2} + (I_T - I_{D2}) - 2\sqrt{I_{D2}} \sqrt{I_T - I_{D2}}$$

$$\theta V_d^2 = I_T - 2\sqrt{I_{D2}} \sqrt{I_T - I_{D2}}$$

$$(\theta V_d^2 - I_T)^2 = 4I_{D2} (I_T - I_{D2})$$

This latter equation can be expressed as a second-order polynomial in $I_{D2}$ as

$$I_{D2}^2 - I_{D2} I_T + \left(\frac{\theta V_d^2 - I_T}{2}\right)^2 = 0$$
How linear is the amplifier?

and assume \( V_d = V_m \sin(\omega t) \)

\[
\theta = \frac{\mu C_{\text{ox}} W}{2L}
\]

\[
I_{D2}^2 - I_{D2} I_T + \left( \frac{\theta V_d^2 - I_T}{2} \right)^2 = 0
\]

Solving, we obtain

\[
I_{D2} = \frac{I_T}{2} + \sqrt{\left( \frac{I_T}{2} \right)^2 - \left( \frac{\theta V_d^2 - I_T}{2} \right)^2}
\]

\[
I_{D2} = \frac{I_T}{2} + \sqrt{\left( \frac{I_T}{2} \right)^2 - \left( \frac{\theta V_d^2}{2} \right)^2 - \left( \frac{I_T}{2} \right)^2 + \frac{\theta I_T}{2} V_d^2}
\]

\[
I_{D2} = \frac{I_T}{2} + \sqrt{\theta I_T \frac{V_d^2}{2} - \left( \frac{\theta V_d^2}{2} \right)^2}
\]
How linear is the amplifier?

and assume \( V_d = V_m \sin(\omega t) \)

\[ \theta = \frac{\mu C_{\text{OX}} W}{2L} \]

\[ I_{D2} = \frac{I_T}{2} + \sqrt{\frac{\theta I_T}{2}} V_d^2 - \left( \frac{\theta V_d^2}{2} \right)^2 \]

This can be expressed as

\[ I_{D2} = \frac{I_T}{2} + V_d \sqrt{\frac{\theta I_T}{2}} \left( 1 - \sqrt{1 - \frac{V_d^2}{2 I_T}} \right) \]

Using a Truncated Taylor’s series, we obtain

\[ I_{D2} \approx \frac{I_T}{2} + V_d \sqrt{\frac{\theta I_T}{2}} \left( 1 - \frac{V_d^2}{4 I_T} \right) \]

Note this has no second-order term thus the dominant distortion when \( V_d = V_m \sin(\omega t) \) will be a third-order harmonic.
How linear is the amplifier?

\[ I_{D2} \approx \frac{I_T}{2} + V_d \sqrt{\frac{\theta I_T}{2}} \left( 1 - V_d^2 \frac{\theta}{4I_T} \right) \]

\[ \theta = \frac{\mu C_{OX} W}{2L} \]

Substituting in \( V_d = V_m \sin(\omega t) \)

\[ I_{D2} \approx \frac{I_T}{2} + V_m \sin(\omega t) \sqrt{\frac{\theta I_T}{2}} \left( 1 - V_m^2 \sin^2(\omega t) \frac{\theta}{4I_T} \right) \]

\[ I_{D2} \approx \frac{I_T}{2} + \left[ V_m \sqrt{\frac{\theta I_T}{2}} \right] \sin(\omega t) - \left[ V_m^3 \frac{\theta^2}{4\sqrt{2} \sqrt{I_T}} \right] \sin^3(\omega t) \]

\[ \sin^3(\omega t) = \frac{3}{4} \sin(\omega t) - \frac{1}{4} \sin(3\omega t) \]

\[ I_{D2} \approx \frac{I_T}{2} + \left[ V_m \sqrt{\frac{\theta I_T}{2}} \right] \sin(\omega t) - \left[ V_m^3 \frac{\theta^2}{4\sqrt{2} \sqrt{I_T}} \right] \left[ \frac{3}{4} \sin(\omega t) - \frac{1}{4} \sin(3\omega t) \right] \]

\[ I_{D2} \approx \frac{I_T}{2} + \left[ V_m \sqrt{\frac{\theta I_T}{2}} - V_m^3 \frac{3\theta^2}{16\sqrt{2} \sqrt{I_T}} \right] \sin(\omega t) + \left[ V_m^3 \frac{\theta^2}{16 \sqrt{2} \sqrt{I_T}} \right] \left[ \sin(3\omega t) \right] \]
How linear is the amplifier?

\[ I_{D2} \approx \frac{I_T}{2} + V_d \sqrt{\frac{\theta I_T}{2}} \left(1 - V_d^2 \frac{\theta}{4I_T}\right) \quad \theta = \frac{\mu C_{OX} W}{2L} \]

\[ I_{b2} \approx \frac{I_T}{2} + \left[V_m \sqrt{\frac{\theta I_T}{2}} - V_m^3 \frac{3\theta^2}{16\sqrt{2}\sqrt{I_T}}\right] \sin(\omega t) + \left[V_m^3 \frac{\theta^2}{16\sqrt{2}\sqrt{I_T}}\right] \sin(3\omega t) \]

\[ I_{D2} \approx a_0 + a_1 \sin(\omega t) + a_3 (3\omega t) \]

\[ a_1 = \left[V_m \sqrt{\frac{\theta I_T}{2}} - V_m^3 \frac{3\theta^2}{16\sqrt{2}\sqrt{I_T}}\right] \quad a_3 = \left[V_m^3 \frac{\theta^2}{16\sqrt{2}\sqrt{I_T}}\right] V_m^3 \]
How linear is the amplifier?

\[ I_{D2} = a_0 + a_1 \sin(\omega t) + a_3 \sin(3\omega t) \]

\[ a_1 = \left[ V_m \sqrt{\frac{\theta I_T}{2}} - V_m^3 \frac{3\theta^2}{16\sqrt{2} \sqrt{I_T}} \right] \]

\[ a_3 = \frac{\theta^3}{16\sqrt{2} \sqrt{I_T}} V_m^3 \]

THD = \( 20 \log \left( \frac{\sqrt{\sum_{k=2}^{\infty} a_k^2}}{a_1} \right) \)

Substituting in we obtain

\[ \text{THD} = 20 \log \left( \frac{\theta^2}{16\sqrt{2} \sqrt{I_T}} \right) \left( \frac{V_m^3}{V_m \sqrt{\frac{\theta I_T}{2}} - V_m^3 \frac{3\theta^2}{16\sqrt{2} \sqrt{I_T}}} \right) \]

This expression gives little insight. But in terms of the practical parameters:

Thus to minimize THD, want \( V_{EB} \) large and \( V_M \) small.
How linear is the amplifier?

\[ I_{D2} = a_0 + a_1 \sin(\omega t) + a_3 \sin(3\omega t) \]

\[ \text{THD} = 20 \log \left( \frac{\frac{\theta^2}{16\sqrt{2}} V_m^3}{V_m \sqrt{\frac{\theta I_T}{2}} - V_m^3 \frac{3\theta^2}{16\sqrt{2} I_T}} \right) \]

Eliminating \( I_T \) and \( \theta \), we obtain

\[ \text{THD} = -20 \log \left( 32 \left( \frac{V_{EB1}}{V_m} \right)^2 - 3 \right) \]

Thus to minimize THD, want \( V_{EB} \) large and \( V_m \) small

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<th>( V_m/V_{EB1} )</th>
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End of Lecture 18