EE 435

Lecture 19

Linearity of Bipolar Differential Pair
Linearity of Common Source Amplifier
Offset Voltages
How linear is the amplifier?

$$I_D1$$

$$V_{EB1}$$

$$\sqrt{2V_{EB1}}$$

1% Linear = 0.3$$V_{EB1}$$
How linear is the amplifier?

\[ I_{D2} = a_0 + a_1 \sin(\omega t) + a_3 \sin^3(\omega t) \]

\[ a_1 = \sqrt{\frac{\theta I_T}{2}} V_m \quad a_3 = -\left[ \frac{\theta^2}{4\sqrt{2} \sqrt{I_T}} \right] V_m^3 \]

THD = 20\log \left( \frac{\sqrt{\sum_{k=2}^{\infty} a_k^2}}{a_1} \right)

Substituting in we obtain

\[ \text{THD} = 20 \log \left( \frac{\theta V_m^2}{4 I_T} \right) \]

where \[ \theta = \frac{\mu C_{OX} W}{2L} \]

\[ I_T = \frac{\mu C_{OX} W}{L} V_{EB1}^2 \]

This expression gives little insight. But in terms of the practical parameters:

\[ \text{THD} = 20 \log \left( \frac{V_m^2}{8 V_{EB1}^2} \right) \]

Thus to minimize THD, want \( V_{EB} \) large and \( V_M \) small
Bipolar Differential Pair

\[ I_{C1} = J_S A_{E1} e^{\frac{V_1 - V_E}{V_t}} \]
\[ I_{C2} = J_S A_{E2} e^{\frac{V_2 - V_E}{V_t}} \]
\[ I_{C1} + I_{C2} = I_T \]

\[ V_1 = V_E + V_t \ln \left( \frac{I_{C1}}{J_S A_{E1}} \right) \]
\[ V_2 = V_E + V_t \ln \left( \frac{I_{C2}}{J_S A_{E2}} \right) \]

\[ V_d = V_t \left( \ln \left( \frac{I_{C2}}{J_S A_{E2}} \right) - \ln \left( \frac{I_{C1}}{J_S A_{E1}} \right) \right) \]
\[ A_{E1} = A_{E2} \]
\[ = V_t \ln \left( \frac{I_{C2}}{I_{C1}} \right) \]
Bipolar Differential Pair

\[
V_d = V_t \left( \ln \left( \frac{I_{C2}}{J_S A_{E2}} \right) - \ln \left( \frac{I_{C1}}{J_S A_{E1}} \right) \right)^{A_{E1}=A_{E2}} = V_t \ln \left( \frac{I_{C2}}{I_{C1}} \right)
\]

\[
V_d = V_t \ln \left( \frac{I_T - I_{C1}}{I_{C1}} \right)
\]

\[
V_d = V_t \ln \left( \frac{I_{C2}}{I_T - I_{C2}} \right)
\]

At \( I_{C1}=I_{C2}=I_T/2 \), \( V_d=0 \)

As \( I_{C1} \) approaches 0, \( V_d \) approaches infinity

As \( I_{C1} \) approaches \( I_T \), \( V_d \) approaches minus infinity

Transition much steeper than for MOS case
Transfer Characteristics of Bipolar Differential Pair

Transition much steeper than for MOS case
Asymptotic Convergence to 0 and $I_T$

\[ V_d = V_t \ln \left( \frac{I_T - I_{C1}}{I_{C1}} \right) \]
Signal Swing and Linearity of Bipolar Differential Pair

\[ I_{\text{FIT}} = mV_d + h \]

\[ m = \frac{\partial I_{C1}}{\partial V_d} \bigg|_{Q\text{-point}} \]

\[ \frac{\partial V_d}{\partial I_{C1}} \bigg|_{Q\text{-point}} = -V_t \cdot \frac{I_T}{I_{C1}(I_T - I_{C1})} \bigg|_{I_{C1}=\frac{I_T}{2}} \]

\[ I_{\text{FIT}} = -\frac{I_T}{4V_t} V_d + \frac{I_T}{2} \]

\[ V_{\text{dint}} = -\frac{h}{m} = ? \]

\[ V_{\text{dint}} = -\frac{h}{m} = 2V_t \]
Signal Swing and Linearity of Bipolar Differential Pair

for 1% deviation, $V_d = .56V_t$

for 0.1% deviation, $V_d = .27V_t$

$x\%$ Deviation
Signal Swing and Linearity of Bipolar Differential Pair

$1\%$ linear = $0.56V_t$
How linear is the amplifier?

Distortion in the differential pair is another useful metric for characterizing linearity of $I_{C1}$ and $I_{C2}$ with sinusoidal differential excitation.

Consider again the differential pair and assume excited differentially with

$$V_2 = \frac{V_d}{2} \quad V_1 = -\frac{V_d}{2}$$

and assume $V_d = V_m \sin(\omega t)$

Recall:

$$V_d = V_t \ln\left(\frac{I_T - I_{C1}}{I_{C1}}\right)$$

Thus can express as

$$\frac{V_d}{V_t} = \frac{I_T - I_{C1}}{I_{C1}}$$

$$e^{\frac{V_d}{V_t}} = \frac{I_T - I_{C1}}{I_{C1}}$$

$$I_{C1} = I_T \left(1 + e^{\frac{V_d}{V_t}}\right)^{-1}$$

$$V_d = V_2 - V_1$$
How linear is the amplifier?

\[ I_{C1} = I_T \left( 1 + e^{\frac{V_d}{V_t}} \right)^{-1} \]

\[ V_d = V_m \sin(\omega t) \]

Consider a Taylor’s Series Expansion

\[ I_{C1} = I_{C1} \bigg|_{V_d=0} + \frac{\partial I_{C1}}{\partial V_d} \bigg|_{V_d=0} V_d + \frac{1}{2!} \frac{\partial^2 I_{C1}}{\partial V_d^2} \bigg|_{V_d=0} V_d^2 + \frac{1}{3!} \frac{\partial^3 I_{C1}}{\partial V_d^3} \bigg|_{V_d=0} V_d^3 + H.O.T \]
How linear is the amplifier?

\[ V_d = V_m \sin(\omega t) \]

\[ I_{C1} = I_{C1}\bigg|_{V_d=0} + \frac{\partial I_{C1}}{\partial V_d} \bigg|_{V_d=0} V_d + \frac{1}{2!} \frac{\partial^2 I_{C1}}{\partial V_d^2} \bigg|_{V_d=0} V_d^2 + \frac{1}{3!} \frac{\partial^3 I_{C1}}{\partial V_d^3} \bigg|_{V_d=0} V_d^3 + H.O.T. \]

\[ \frac{\partial I_{C1}}{\partial V_d} = -\frac{I_T}{V_t} \left( 1 + \frac{V_d}{V_i} \right)^2 e^{\frac{V_d}{V_i}} \]

\[ \frac{\partial^2 I_{C1}}{\partial V_d^2} = -\frac{I_T}{V_t^2} \left[ \left( 1 + \frac{V_d}{V_i} \right)^2 e^{\frac{V_d}{V_i}} - 2 e^{\frac{V_d}{V_i}} \left( 1 + \frac{V_d}{V_i} \right)^{-3} \right] \]

\[ \frac{\partial^3 I_{C1}}{\partial V_d^3} = -\frac{I_T}{V_t^3} \left[ \left( 1 + \frac{V_d}{V_i} \right)^2 e^{\frac{V_d}{V_i}} - 2 e^{\frac{V_d}{V_i}} \left( 1 + \frac{V_d}{V_i} \right)^{-3} \right] \]
How linear is the amplifier?

\[ V_d = V_m \sin(\omega t) \]

\[ I_{C1} = I_{C1}\big|_{V_d=0} + \frac{\partial I_{C1}}{\partial V_d}\big|_{V_d=0} V_d + \frac{1}{2!} \frac{\partial^2 I_{C1}}{\partial V_d^2}\big|_{V_d=0} V_d^2 + \frac{1}{3!} \frac{\partial^3 I_{C1}}{\partial V_d^3}\big|_{V_d=0} V_d^3 + H.O.T \]

\[
\left. \frac{\partial I_{C1}}{\partial V_d} \right|_{V_d=0} = -\frac{I_T}{V_t} \left( \frac{V_d}{V_t} \right)^2 \left( \frac{V_d}{V_t} \right) = -\frac{I_T}{4V_t} \left( \frac{V_d}{V_t} \right) \\
\left. \frac{\partial^3 I_{C1}}{\partial V_d^3} \right|_{V_d=0} = -\frac{I_T}{V_t^3} \left[ \left( \frac{V_d}{V_t} \right)^2 \left( \frac{V_d}{V_t} \right) - 2 \left( \frac{V_d}{V_t} \right)^3 \right] = -\frac{I_T}{V_t^3} \left[ \frac{V_d}{V_t} \right]^2 - 2 \left( \frac{V_d}{V_t} \right)^3 \\
\left. \frac{\partial^3 I_{C1}}{\partial V_d^3} \right|_{V_d=0} = -\frac{I_T}{8V_t^3} \left[ \frac{V_d}{V_t} \right]^2 - 2 \left( \frac{V_d}{V_t} \right)^3 + 6 \left( \frac{V_d}{V_t} \right)^4 - 4 \left( \frac{V_d}{V_t} \right)^3 = \frac{I_T}{8V_t^3} \]

\[
\left. \frac{\partial I_{C1}}{\partial V_d} \right|_{V_d=0} = -\frac{I_T}{4V_t} \\
\left. \frac{\partial^2 I_{C1}}{\partial V_d^2} \right|_{V_d=0} = 0 \\
\left. \frac{\partial^3 I_{C1}}{\partial V_d^3} \right|_{V_d=0} = \frac{I_T}{8V_t^3} 
\]
How linear is the amplifier?

\[ V_d = V_m \sin(\omega t) \]

\[ I_{C1} = I_{C1} \bigg|_{V_d=0} + \left. \frac{\partial I_{C1}}{\partial V_d} \right|_{V_d=0} V_d + \frac{1}{2!} \left. \frac{\partial^2 I_{C1}}{\partial V_d^2} \right|_{V_d=0} V_d^2 + \frac{1}{3!} \left. \frac{\partial^3 I_{C1}}{\partial V_d^3} \right|_{V_d=0} V_d^3 + H.O.T \]

\[ \left. \frac{\partial I_{C1}}{\partial V_d} \right|_{V_d=0} = -\frac{I_T}{4V_t} \]

\[ \left. \frac{\partial^2 I_{C1}}{\partial V_d^2} \right|_{V_d=0} = 0 \]

\[ \left. \frac{\partial^3 I_{C1}}{\partial V_d^3} \right|_{V_d=0} = \frac{I_T}{8V_t^3} \]

\[ I_{C1} \approx \frac{I_T}{2} - \frac{I_T}{4V_t} V_d + \frac{I_T}{48V_t^3} V_d^3 \]

\[ I_{C1} \approx \frac{I_T}{2} - \frac{I_T}{4V_t} V_m \sin(\omega t) + \frac{I_T}{48V_t^3} V_m^3 \sin^3(\omega t) \]

\[ \sin^3(\omega t) = \frac{3}{4} \sin(\omega t) - \frac{1}{4} \sin(3\omega t) \]
How linear is the amplifier?

\[ V_d = V_m \sin(\omega t) \]

\[ I_{C1} = I_{C1}\bigg|_{V_d=0} + \frac{\partial I_{C1}}{\partial V_d}\bigg|_{V_d=0} V_d + \frac{1}{2!} \frac{\partial^2 I_{C1}}{\partial V_d^2}\bigg|_{V_d=0} V_d^2 + \frac{1}{3!} \frac{\partial^3 I_{C1}}{\partial V_d^3}\bigg|_{V_d=0} V_d^3 + H.O.T \]

\[ I_{C1} \approx \frac{I_T}{2} - \frac{I_T}{4V_t} V_m \sin(\omega t) + \frac{I_T}{48V_t^3} V_m^3 \left[ \frac{3}{4} \sin(\omega t) - \frac{1}{4} \sin(3\omega t) \right] \]

\[ I_{C1} \approx \frac{I_T}{2} + \left[ \frac{3I_T}{4 \cdot 48V_t^3} V_m^3 - \frac{I_T}{4V_t} V_m \right] \sin(\omega t) - \frac{I_T}{4 \cdot 48V_t^3} V_m^3 \sin(3\omega t) \]

Thus:

\[ \text{THD} = 20 \log \left( \frac{V_m^2}{48V_t^2 - 3V_m^2} \right) \]

or, equivalently

\[ \text{THD} = -20 \log \left( 48 \left( \frac{V_t}{V_m} \right)^2 - 3 \right) \]

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<tr>
<th>( V_m/V_t )</th>
<th>THD (dB)</th>
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</thead>
<tbody>
<tr>
<td>2.5</td>
<td>-13.4049</td>
</tr>
<tr>
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<td>-33.0643</td>
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<tr>
<td>0.5</td>
<td>-45.5292</td>
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<tr>
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<td>0.025</td>
<td>-97.7069</td>
</tr>
<tr>
<td>0.01</td>
<td>-113.625</td>
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</table>
Comparison of Distortion in BJT and MOSFET Pairs

\[ V_d = V_m \sin(\omega t) \]

\[ \text{THD} = -20 \log \left( 48 \left( \frac{V_t}{V_m} \right)^2 - 3 \right) \]

\[ \text{THD} = -20 \log \left( 32 \left( \frac{V_{EB1}}{V_m} \right)^2 - 3 \right) \]

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</table>
Linearity and Signal Swing Comparison of Bipolar/MOS Differential Pair

1% linear = .56\(V_t\)

1% Linear = 0.3\(V_{EB1}\)

1% linear = \(0.3V_{EB1}\)

1% linear = 0.3\(V_{EB1}\)
Applications as a programmable OTA

The current-dependence of the $g_m$ of the differential pair is often used to program the transconductance of an OTA with the tail bias current $I_{ABC}$.

**MOS**

$$g_m = \sqrt{I_{ABC}} \sqrt{2uC_{OX}} \frac{W}{L}$$

Two decade change in current for every decade change in $g_m$

$$g_m = uC_{OX} \frac{W}{L} V_{EB}$$

One decade decrease in signal swing for every decade decrease in $g_m$

Limited $g_m$ adjustment possibility

**BJT**

$$g_m = \frac{I_{ABC}}{V_t}$$

One decade change in current for every decade change in $g_m$

No change in signal swing when $g_m$ is changed

Large $g_m$ adjustment possible
Signal Swing and Linearity Summary

- Signal swing of MOSFET can be rather large if $V_{EB}$ is large but this limits gain.
- Signal swing of MOSFET degrades significantly if $V_{EB}$ is changed for fixed W/L.
- Bipolar swing is very small but independent of $g_m$.
- Multiple-decade adjustment of bipolar $g_m$ is practical.
- Even though bipolar input swing is small, since gain is often very large, this small swing does usually not limit performance in feedback applications.
Linearity of Common-Source Amplifier

For convenience, will consider situation where current source biasing is ideal.
Linearity of Common-Source Amplifier

![Graph showing the linearity of a common-source amplifier with different gate-source voltages.](image)

- **VGS = 1V**
- **VGS = 1.5V**
- **VGS = 2V**
- **VGS = 2.5V**

The graph illustrates the relationship between drain-source voltage (VDS) and drain current (ID) for various gate-source voltages (VGS). The amplifier's performance is evaluated based on the linearity of its output current with respect to the input voltage.

The circuit diagram on the right shows the basic configuration of a common-source amplifier with input source voltage (ViS) and output voltage (VOUT). The drain-source voltage (VDS) and gate-source voltage (VGS) are key parameters affecting the amplifier's performance.

The graphs depict the current-voltage characteristics for different VGS values, indicating the amplifier's linearity and gain.

- **VGS = 1V**
- **VGS = 1.01V**
- **VGS = 0.99V**
Linearity of Common-Source Amplifier

\[ I_B = \frac{\mu C_{OX} W}{2L} (V_{iS} - V_{SS} - V_T)^2 (1 + \lambda [V_{OUT} - V_{SS}]) \]

\[ I_B = \beta (V_{iS} - V_{EB})^2 (1 + \lambda [V_{OS} + V_{OQ} - V_{SS}]) \]

\[ v_{OS} = V_{SS} - V_{OQ} - \frac{I_B}{\beta V_E^2(1 - \frac{V_{iS}}{V_{EB}})^2} \]

\[ \lambda \]
Linearity of Common-Source Amplifier

\[ \nu_{OS} = V_{SS} - V_{OQ} - \frac{\beta V_{EB}^2 \left( 1 - \frac{\nu_{iS}}{V_{EB}} \right)^2}{\lambda} \]

\[ \nu_{OS} \approx V_{SS} - V_{OQ} - \frac{\beta V_{EB}^2}{\lambda} \left( 1 + \frac{\nu_{iS}}{V_{EB}} \right)^2 \]

\[ \nu_{OS} \approx V_{SS} - V_{OQ} - \frac{i_B}{\lambda \beta V_{EB}^2} \left( 1 + 2 \frac{\nu_{iS}}{V_{EB}} + \left( \frac{\nu_{iS}}{V_{EB}} \right)^2 \right) - \frac{1}{\lambda} \]

\[ \nu_{OS} \approx \left[ V_{SS} - V_{OQ} + \frac{1}{\lambda} \left( \frac{i_B}{\beta V_{EB}^2} \right)^2 \right] - \frac{i_B}{\lambda \beta V_{EB}^2} \left( 2 \frac{\nu_{iS}}{V_{EB}} + \left( \frac{\nu_{iS}}{V_{EB}} \right)^2 \right) \]
Linearity of Common-Source Amplifier

\[ v_{OS} \approx \left[ V_{SS} - V_{OQ} + \frac{1}{\lambda} \left( \frac{l_B}{\beta V^2_{EB}} \right) - 1 \right] - \frac{l_B}{\lambda \beta V^2_{EB}} \left( 2 \frac{v_{iS}}{V_{EB}} + \left( \frac{v_{iS}}{V_{EB}} \right)^2 \right) \]

\[ v_{OS} \approx - \left( 2 \frac{v_{iS}}{\lambda V_{EB}} + \frac{1}{\lambda} \left( \frac{v_{iS}}{V_{EB}} \right)^2 \right) \]

\[ v_{OS} \approx - \frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right) \]

Is this a linear or nonlinear relationship?
Linearity of Common-Source Amplifier

\[ V_{OS} \approx \frac{2}{\lambda V_{EB}} \left( V_{iS} + \frac{1}{2V_{EB}} V_{iS}^2 \right) \]

Is this a linear or nonlinear relationship?

when \( V_{iS} = -V_{EB} \) (the minimum value of \( V_{iS} \) to maintain saturation operation)

the error in \( V_{OS} \) will be \( V_{EB}/2 \) which is 50%!

Is this a linear or nonlinear relationship?
Linearity of Common-Source Amplifier

\[ v_{OS} \approx -\frac{2}{\lambda V_{EB}} \left( v_i + \frac{1}{2V_{EB}} v_i^2 \right) \]

Is this a linear or nonlinear relationship?

Note this is a high gain amplifier

Over what output voltage range are we interested?
Linearity of Common-Source Amplifier

\[ \nu_{OS} \approx -\frac{2}{\lambda V_{EB}} \left( \nu_{iS} + \frac{1}{2V_{EB}} \nu_{iS}^2 \right) \]

Is this a linear or nonlinear relationship?

Linearity is reasonably good over practical input range

Practical input range is much less than \( V_{EB} \)
Linearity of Common-Source Amplifier

\[ V_{OS} \approx -\frac{2}{\lambda V_{EB}} \left( V_{IS} + \frac{1}{2V_{EB}} V_{iS}^2 \right) \]

Is this a linear or nonlinear relationship?

![Graph showing the relationship between \( V_{iS} \) and \( V_{OUT} \).](image)

Can’t see nonlinearity in this plot

**Diagram Details:**
- \( V_{DD} \) and \( V_{SS} \) supply voltages.
- \( I_B \) is the bias current.
- \( V_{OUT} \) is the output voltage.
- \( V_{iS} \) is the input voltage.
- \( \lambda = 0.01 \) is the parameter representing the nonlinearity of the amplifier.

**Plot Information:**
- Plot range: \(-0.025\) to \(0.025\) for \( V_{iS} \) and \(-5\) to \(5\) for \( V_{OUT} \).
- Fit line is drawn through the data points.
- \( V_{EB} = 1 \) \( V \) is shown.

**Equation Explanation:**
The equation \( V_{OS} \approx \) represents the offset voltage due to the nonlinearity of the amplifier. The terms inside the parentheses involve the input voltage \( V_{iS} \) and the second power of \( V_{iS} \), indicating the second-order nonlinearity of the amplifier.
Linearity of Common-Source Amplifier

\[ v_{OS} \approx -\frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right) \]

Is this a linear or nonlinear relationship?

\[ V_{EB} = 1 \text{V} \]
\[ \lambda = 0.01 \]

\[ v_{FIT} \approx -\frac{2}{\lambda V_{EB}} v_{iS} \]

\[ \varepsilon = v_{FIT} - v_{OS} \]

\[ \varepsilon \approx \frac{1}{\lambda V_{EB}^2} v_{iS}^2 \]
Linearity of Common-Source Amplifier

$$v_{OS} \approx -\frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right)$$

Is this a linear or nonlinear relationship?

$$\varepsilon \approx \frac{1}{\lambda V_{EB}^2} v_{iS}^2$$

$$V_{EB} = 1V$$

$$\lambda = 0.01$$
Linearity of Common-Source Amplifier

\[ v_{OS} \approx -\frac{2}{\lambda V_{EB}} \left( v_iS + \frac{1}{2V_{EB}} v_iS^2 \right) \]

Is this a linear or nonlinear relationship?

\[ V_{EB} = 1V \]
\[ \lambda = 0.01 \]

\[ \varepsilon_{PCT} \approx \frac{\varepsilon}{\varepsilon_{FIT}} \times 100\% = \left[ \frac{1}{\lambda V_{EB}^2} \frac{v_iS^2}{2v_iS} \right] \times 100\% \approx \left( \frac{100\%}{2V_{EB}} \right) v_iS \]

\[ \varepsilon_{PCT} \approx \left( -\frac{\lambda \cdot 100\%}{4} \right) v_{OS} \]
Linearity of Common-Source Amplifier

\[ v_{OS} \approx - \frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right) \]

Is this a linear or nonlinear relationship?

\[ V_{EB} = 1\, \text{V} \]
\[ \lambda = 0.01 \]

\[ \varepsilon_{PCT} \approx \left( \frac{100\%}{2V_{EB}} \right) v_{iS} \]

or, in terms of \( v_{OS} \),

\[ \varepsilon_{PCT} \approx \left( -\frac{\lambda \cdot 100\%}{4} \right) v_{OS} \]

1% deviation for this example occurs at

\[ |v_{OS}| \approx 0.01 \frac{4}{\lambda} \approx 4\, \text{V} \]
Linearity of Common-Source Amplifier

Is this common-source amplifier linear or nonlinear?

The transconductance amplifier driving a load $C_L$ is performing as an integrator

Integrators often used in filters where $|V_{OS}|$ is comparable to $|V_{iS}|$
Linearity of Common-Source Amplifier

High-Gain Amplifier

\[ I_{OUT} = I_B - I_D \]

\[ I_{OUT} = I_B - \beta (V_{iS} + V_{EB})^2 (1 + \lambda [V_{OS} + V_{OQ} - V_{SS}]) \]

\[ I_{OUT} = \left[I_B - \beta (V_{EB})^2 (1 + \lambda [V_{OQ} - V_{SS}])\right] - \beta (V_{iS}^2 + 2V_{iS} V_{EB}) (1 + \lambda [V_{OS} + V_{OQ} - V_{SS}]) \]

\[ I_{OUT} \approx -\beta (V_{iS}^2 + 2V_{iS} V_{EB}) \]

\[ I_{OUT} \approx -\frac{2I_B}{V_{EB}} \left(V_{iS} + \frac{1}{2V_{EB}} V_{iS}^2 \right) \]

Is this a linear or nonlinear relationship?
Linearity of Common-Source Amplifier

As an OTA

\[ I_{OUT} = \frac{2I_B}{V_{EB}} \left( V_{iS} + \frac{1}{2V_{EB}} V_{iS}^2 \right) \]

Is this a linear or nonlinear relationship?

At \( V_{iS} = -V_{EB} \), the error in \( I_{OUT} \) will be -50%!
Linearity of Common-Source Amplifier

Is this common-source amplifier linear?

- Reasonably linear if used in high-gain applications and $V_{EB}$ is large (e.g. if $A_V = g_m/g_o = 2/((\lambda V_{EB}) = 100$ and $V_o = 1V$, $V_{in} = 10mV$)
- Highly nonlinear when used in low-gain applications
Is this common-emitter amplifier linear?

- Very linear if used in high-gain applications (e.g. if $A_V = g_m/g_0 = V_{AF}/V_t = 4000$ and $V_o = 1V$, $V_{in} = 250uV$)
- Highly nonlinear when used in low-gain applications
Offset Voltage

Two types of offset voltage:

- Systematic Offset Voltage
- Random Offset Voltage

Definition: The output offset voltage is the difference between the desired output and the actual output when $V_{id}=0$ and $V_{ic}$ is the quiescent common-mode input voltage.

$V_{OUTOFF} = V_{OUT} - V_{OUTDES}$

Note: $V_{OUTOFF}$ is dependent upon $V_{ICQ}$, although this dependence is usually quite weak and often not specified.
Definition: The input-referred offset voltage is the differential dc input voltage that must be applied to obtain the desired output when $V_{ic}$ is the quiescent common-mode input voltage.

Note: $V_{OFF}$ is usually related to the output offset voltage by the expression

$$V_{OFF} = \frac{V_{OUTOFF}}{A_C}$$

Note: $V_{OFF}$ is dependent upon $V_{ICQ}$ although this dependence is usually quite weak and often not specified.
Offset Voltage

Two types of offset voltage:

- Systematic Offset Voltage
- Random Offset Voltage

After fabrication it is impossible (difficult) to distinguish between the systematic offset and the random offset in any individual op amp.

Measurements of offset voltages for a large number of devices will provide mechanism for identifying systematic offset and statistical characteristics of the random offset voltage.
Systematic Offset Voltage

Offset voltage that is present if all device and model parameters assume their nominal value

Easy to simulate the systematic offset voltage

Almost always the designer’s responsibility to make systematic offset voltage very small

Generally easy to make the systematic offset voltage small
Random Offset Voltage

Due to random variations in process parameters and device dimensions.

Random offset is actually a random variable at the design level but deterministic after fabrication in any specific device.

Distribution nearly Gaussian.

Has zero mean.

Characterized by its standard deviation or variance.

Often strongly layout dependent.

Due to both local random variations and correlated gradient effects.

Will consider both effects separately.

Gradient effects usually dominate if not managed.

Good methods exist for driving gradient effects to small levels and will be discussed later.

In what follows it will be assumed that gradient effects have been managed.