Linearity in Operational Amplifiers
  -- The differential pairs
  -- Programmable gain open-loop transconductors
Alternate Positive Feedback Amplifier

\[ V_{b1} \rightarrow Q7 \rightarrow V_{b2} \rightarrow Q5 \rightarrow Q3 \rightarrow Q4 \rightarrow Q6 \rightarrow V_{DD} \]

\[ V_{in} \rightarrow Q1 \rightarrow Q2 \rightarrow V_{out} \]

\[ C_L \]

Review from last lecture...
Alternate Positive Feedback Amplifier

\[ A_{VO} = \frac{(1/2)g_{m1}}{g_{o2} + g_{o4} + g_{o6} - g_{m4}} \]

\[ A(s) = \frac{(1/2)g_{m1}}{sC_L + [g_{o2} + g_{o4} + g_{o6} - g_{m4}]} \]

- Requires precise matching of \( g_{m4} \) to \((g_{o2}+g_{o4}+g_{o6})\) for good gain enhancement
- Difficult to match \( g_m \) terms to \( g_o \)-type terms
Practical Comments about Positive Feedback Gain Enhancement

- Significant gain enhancement is possible but most avoid regenerative feedback because of unfounded concerns about closed-loop stability.

- Accuracy and settling time can be improved with some regenerative feedback.

- Will become more critical in emerging processes where $g_m/g_o$ ratios degrade and where supply voltages shrink thus limiting the longstanding cascode process.

- Regenerative structures can have high sensitivities.

- Signal swing quite limited in some of the most basic regenerative feedback structures.

- Most useful in two-stage architecture where regenerative feedback is used in first stage (effects of signal swing are reduced by gain of second stage).
Linearity of Amplifiers

- Linearity of differential pair of major concern
- Linearity of common-source amplifier is of major concern (since signals so small at output of differential pair)

Review from last lecture.
Differential Input Pairs

MOS Differential Pair

Bipolar Differential Pair

Review from last lecture.
MOS Differential Pair

\[ I_{D1} = \frac{\mu C_{ox} W}{2L} (V_1 - V_S - V_T)^2 \]
\[ I_{D2} = \frac{\mu C_{ox} W}{2L} (V_2 - V_S - V_T)^2 \]
\[ I_{D1} + I_{D2} = I_T \]

\[ \sqrt{I_{D1}} \sqrt{\frac{2L}{\mu C_{ox} W}} = V_1 - V_S - V_T \]
\[ \sqrt{I_{D2}} \sqrt{\frac{2L}{\mu C_{ox} W}} = V_2 - V_S - V_T \]

\[ V_d = \sqrt{\frac{2L}{\mu C_{ox} W}} \left( \sqrt{I_T - I_{D1}} - \sqrt{I_{D1}} \right) \]
\[ V_d = \sqrt{\frac{2L}{\mu C_{ox} W}} \left( \sqrt{I_{D2}} - \sqrt{I_T - I_{D2}} \right) \]
MOS Differential Pair

\[
V_d = \sqrt{\frac{2L}{\mu C_{ox} W}} \left(\sqrt{I_T} - I_{D1} - \sqrt{I_{D1}}\right)
\]

\[
V_d = \sqrt{\frac{2L}{\mu C_{ox} W}} \left(\sqrt{I_{D2}} - \sqrt{I_T} - I_{D2}\right)
\]

What values of \( V_d \) will cause all of the current to be steered to the left or the right?

\[
V_{dx} = \pm \sqrt{\frac{2L}{\mu C_{ox} W}} \left(\sqrt{I_T}\right)
\]
\[ V_d = \frac{2L}{\mu C_{ox} W} \left( \sqrt{I_{D2}} - \sqrt{I_T - I_{D2}} \right) \]
Q-point Calculations

\[
\frac{I_T}{2} = \frac{\mu C_{ox} W}{2L} (V_{EB})^2
\]

\[
V_{EB} = \sqrt{I_T} \sqrt{\frac{L}{\mu C_{ox} W}}
\]

Observe !!

\[
V_{dx} = \pm \sqrt{2} V_{EB}
\]
\[ V_d = \sqrt{\frac{2L}{\mu C_{ox} W}} \left( \sqrt{I_{D2}} - \sqrt{I_T} - I_{D2} \right) \]

\[ V_{EB} \text{ affects linearity} \]

How linear is the amplifier?
How linear is the amplifier?

\[ V_d = \sqrt{\frac{2L}{\mu C_{ox} W}} \left( \sqrt{l_T - l_{D1}} - \sqrt{l_{D1}} \right) \]

Consider the fit line:

\[ I = m V_d + h \]

When \( V_d = 0 \), \( I = I_T / 2 \), thus

\[ h = \frac{I_T}{2} \]

\[ V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m} \]

\[ m = \left. \frac{\partial I_{D1}}{\partial V_d} \right|_{Q\text{-pt}} \]
How linear is the amplifier?

\[ I = mV_d + h \]

\[ I_{D1} \quad I_T \]

\[ V_{d_{int}} = \frac{-h}{m} = \frac{-I_T}{2m} \]

\[ \sqrt{2V_{EB1}} \]

\[ m = \frac{\partial I_{D1}}{\partial V_d} \bigg|_{Q-\text{pt}} \]

\[ V_d = \sqrt{\frac{2L}{\mu C_{ox} W}} \left( \sqrt{I_T - I_{D1}} - \sqrt{I_{D1}} \right) \]

\[ \frac{\partial V_d}{\partial I_{D1}} = \frac{2L}{\sqrt{\mu C_{ox} W}} \left( \frac{1}{2} (I_T - I_{D1})^{-1/2} (1) - \frac{1}{2} (I_{D1})^{-1/2} \right) \bigg|_{Q-\text{point}} \]

\[ \frac{\partial V_d}{\partial I_{D1}} = -2 \frac{L}{\sqrt{\mu C_{ox} W}} \frac{1}{\sqrt{I_T}} \]

\[ \frac{L}{\sqrt{\mu C_{ox} W}} = \frac{V_{EB1}}{\sqrt{I_T}} \]

\[ \frac{\partial V_d}{\partial I_{D1}} = -2 \frac{V_{EB1}}{I_T} \]

\[ m = \frac{\partial I_{D1}}{\partial V_d} \bigg|_{Q-\text{pt}} = -\frac{I_T}{2V_{EB1}} \]
How linear is the amplifier?

\[
V_{\text{dint}} = -\frac{h}{m} = -\frac{I_T}{2m} = V_{EB1}
\]

\[
I = -\left(\frac{I_T}{2V_{EB1}}\right)V_d + \frac{I_T}{2}
\]
How linear is the amplifier?

It can be shown that a 1% deviation from the straight line occurs at

\[ V_d \approx \frac{V_{EB}}{3} \]

and a 0.1% variation occurs at

\[ V_d \approx \frac{V_{EB}}{10} \]
How linear is the amplifier?

1% Linear = 0.3V_{EB1}
How linear is the amplifier?

Deviation from Linear

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<th>Vd/VEB</th>
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<td>0.6</td>
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</table>
Bipolar Differential Pair

\[ I_{C1} = J_S A_{E1} e^{\frac{V_1 - V_E}{V_t}} \]
\[ I_{C2} = J_S A_{E2} e^{\frac{V_2 - V_E}{V_t}} \]
\[ I_{C1} + I_{C2} = I_T \]

\[ V_1 = V_E + V_t \ln \left( \frac{I_{C1}}{J_S A_{E1}} \right) \]
\[ V_2 = V_E + V_t \ln \left( \frac{I_{C2}}{J_S A_{E2}} \right) \]

\[ V_d = V_t \left( \ln \left( \frac{I_{C2}}{J_S A_{E2}} \right) - \ln \left( \frac{I_{C1}}{J_S A_{E1}} \right) \right) = V_t \ln \left( \frac{I_{C2}}{I_{C1}} \right) \]
Bipolar Differential Pair

\[ V_d = V_2 - V_1 \]

\[ V_d = V_t \left( \ln \left( \frac{I_{C2}}{J_s A_{E2}} \right) - \ln \left( \frac{I_{C1}}{J_s A_{E1}} \right) \right)^{A_{E1} = A_{E2}} = V_t \ln \left( \frac{I_{C2}}{I_{C1}} \right) \]

\[ V_d = V_t \ln \left( \frac{I_T - I_{C1}}{I_{C1}} \right) \]

\[ V_d = V_t \ln \left( \frac{I_{C2}}{I_T - I_{C2}} \right) \]

At \( I_{C1} = I_{C2} = I_T/2 \), \( V_d = 0 \)

As \( I_{C1} \) approaches 0, \( V_d \) approaches infinity

As \( I_{C1} \) approaches \( I_T \), \( V_d \) approaches minus infinity

Transition much steeper than for MOS case
Transfer Characteristics of Bipolar Differential Pair

Transition much steeper than for MOS case
Asymptotic Convergence to 0 and $I_T$
Signal Swing and Linearity of Bipolar Differential Pair

\[ I_{\text{FIT}} = mV_d + h \]

\[ m = \left. \frac{\partial I_{c1}}{\partial V_d} \right|_{Q-\text{point}} \]

\[ \frac{\partial V_d}{\partial I_{c1}} \bigg|_{Q-\text{point}} = -V_t \frac{I_T}{I_{c1}(I_T - I_{c1})} \bigg|_{I_{c1} = I_T/2} \]

\[ \frac{\partial V_d}{\partial I_{c1}} \bigg|_{Q-\text{point}} = -4V_t \frac{I_T}{I_T} \]

\[ I_{\text{FIT}} = -\frac{I_T}{4V_t} V_d + \frac{I_T}{2} \]

\[ V_{\text{dint}} = -\frac{h}{m} = 2V_t \]
Signal Swing and Linearity of Bipolar Differential Pair

for 1% deviation, $V_d = 0.56V_t$

for 0.1% deviation, $V_d = 0.27V_t$
Signal Swing and Linearity of Bipolar Differential Pair

1% linear = .56V_t
Applications as a programmable OTA

The current-dependence of the $g_m$ of the differential pair is often used to program the transconductance of an OTA with the tail bias current $I_{ABC}$

MOS

$$g_m = \sqrt{I_{ABC}} \sqrt{2uC_{OX} \frac{W}{L}}$$

Two decade change in current for every decade change in $g_m$

$$g_m = uC_{OX} \frac{W}{L} V_{EB}$$

One decade decrease in signal swing for every decade decrease in $g_m$

Limited $g_m$ adjustment possibility

BJT

$$g_m = \frac{I_{ABC}}{V_t}$$

One decade change in current for every decade change in $g_m$

No change in signal swing when $g_m$ is changed

Large $g_m$ adjustment possible
Signal Swing and Linearity Summary

- Signal swing of MOSFET can be rather large if $V_{EB}$ is large but this limits gain
- Signal swing of MOSFET degrades significantly if $V_{EB}$ is changed for fixed W/L
- Bipolar swing is very small but independent of $g_m$
- Multiple-decade adjustment of bipolar $g_m$ is practical
- Even though bipolar input swing is small, since gain is often very large, this small swing does usually not limit performance in feedback applications
Linearity of Common-Source Amplifier

For convenience, will consider situation where current source biasing is ideal
Linearity of Common-Source Amplifier
Linearity of Common-Source Amplifier

\[ I_B = \frac{\mu C_{OX} W}{2L} (V_{iS} - V_{SS} - V_T)^2 (1 + \lambda [V_{OUT} - V_{SS}]) \]

\[ I_B = \beta (V_{iS} - V_{EB})^2 (1 + \lambda [V_{OS} + V_{OQ} - V_{SS}]) \]

\[ V_{OS} = V_{SS} - V_{OQ} - \left( \frac{I_B}{\beta V_{EB}^2 \left( 1 - \frac{V_{iS}}{V_{EB}} \right)^2} \right) - 1 \]
Linearity of Common-Source Amplifier

\[ \nu_{OS} = \nu_{SS} - \nu_{OQ} \left( \frac{l_B}{\lambda \beta V_{EB}^2 \left( 1 - \frac{\nu_{iS}}{V_{EB}} \right)^2} \right)^{-1} \]

\[ \nu_{OS} \approx \nu_{SS} - \nu_{OQ} \left( \frac{l_B}{\beta V_{EB}^2} \right)^{-1} \]

\[ \nu_{OS} \approx \nu_{SS} - \nu_{OQ} \left( \frac{l_B}{\lambda \beta V_{EB}^2} \left( 1 + \frac{\nu_{iS}}{V_{EB}} \right)^2 \right)^{-1} \]

\[ \nu_{OS} \approx \left[ \nu_{SS} - \nu_{OQ} + \frac{1}{\lambda} \left( \frac{l_B}{\beta V_{EB}^2} \right)^{-1} \right] - \frac{l_B}{\lambda \beta V_{EB}^2} \left( 2 \frac{\nu_{iS}}{V_{EB}} + \left( \frac{\nu_{iS}}{V_{EB}} \right)^2 \right) \]
Linearity of Common-Source Amplifier

\[
\nu_{OS} \approx \left[ V_{SS} - V_{OQ} + \frac{1}{\lambda} \left( \frac{i_B}{\beta V_{EB}^2} \right) - 1 \right] - \frac{i_B}{\lambda \beta V_{EB}^2} \left( 2 \frac{\nu_{IS}}{V_{EB}} + \left( \frac{\nu_{IS}}{V_{EB}} \right)^2 \right)
\]

\[
\nu_{OS} \approx - \left( 2 \frac{\nu_{IS}}{\lambda V_{EB}} + \frac{1}{\lambda} \left( \frac{\nu_{IS}}{V_{EB}} \right)^2 \right)
\]

\[
\nu_{OS} \approx - \frac{2}{\lambda V_{EB}} \left( \nu_{IS} + \frac{1}{2V_{EB}} \nu_{IS}^2 \right)
\]

Is this a linear or nonlinear relationship?
Linearity of Common-Source Amplifier

\[ v_{OS} \approx \frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right) \]

Is this a linear or nonlinear relationship?

when \( v_{iS} = -V_{EB} \) (the minimum value of \( v_{iS} \) to maintain saturation operation)

the error in \( V_{OS} \) will be \( V_{EB}/2 \) which is -50% !

Is this a linear or nonlinear relationship?
Linearity of Common-Source Amplifier

\[ v_{OS} \cong -\frac{2}{\lambda V_{EB}} \left( v_iS + \frac{1}{2V_{EB}} v_iS^2 \right) \]

Is this a linear or nonlinear relationship?

Note this is a high gain amplifier

Over what output voltage range are we interested?
Linearity of Common-Source Amplifier

\[ v_{OS} \approx -\frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right) \]

Is this a linear or nonlinear relationship?

Linearity is reasonably good over practical input range

Practical input range is much less than \( V_{EB} \)

\[ V_{EB} = 1V \]
\[ \lambda = 0.1 \]
Linearity of Common-Source Amplifier

\[ v_{OS} \approx -\frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right) \]

Is this a linear or nonlinear relationship?

Fit Line

Can’t see nonlinearity in this plot
Linearity of Common-Source Amplifier

\[ v_{OS} \approx -\frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right) \]

Is this a linear or nonlinear relationship?

\[ V_{EB} = 1\text{V} \]
\[ \lambda = 0.01 \]

\[ v_{FIT} \approx -\frac{2}{\lambda V_{EB}} v_{iS} \]

\[ \epsilon = v_{FIT} - v_{oS} \]

\[ \epsilon \approx \frac{1}{\lambda V_{EB}^2} v_{iS}^2 \]
Linearity of Common-Source Amplifier

\[ v_{OS} \approx -\frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right) \]

Is this a linear or nonlinear relationship?

\[ \varepsilon \approx \frac{1}{\lambda V_{EB}^2} v_{iS}^2 \]

\[ V_{EB} = 1V \]
\[ \lambda = 0.01 \]
Linearity of Common-Source Amplifier

\[ v_{OS} \approx -\frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right) \]

Is this a linear or nonlinear relationship?

\( V_{EB} = 1 \text{V} \)
\( \lambda = 0.01 \)

\[ \varepsilon_{PCT} \approx \frac{\varepsilon}{v_{FIT}} \times 100\% = \left( \frac{1}{\lambda V_{EB}^2} \right) \frac{v_{iS}^2}{2v_{iS}} \times 100\% = \left( \frac{100\%}{2V_{EB}} \right) v_{iS} \]

\[ \varepsilon_{PCT} \approx \left( -\frac{\lambda \cdot 100\%}{4} \right) v_{OS} \]
Linearity of Common-Source Amplifier

\[ v_{OS} \approx -\frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right) \]

Is this a linear or nonlinear relationship?

\[ V_{EB} = 1V \]
\[ \lambda = 0.01 \]

\[ \varepsilon_{PCT} \approx \left( \frac{100\%}{2V_{EB}} \right) v_{iS} \]

or, in terms of \( v_{OS} \),

\[ \varepsilon_{PCT} \approx \left( -\frac{\lambda \cdot 100\%}{4} \right) v_{OS} \]

1% deviation for this example occurs at \( |v_{OS}| \approx 0.01 \frac{4}{\lambda} \approx 4V \)
Linearity of Common-Source Amplifier

The transconductance amplifier driving a load $C_L$ is performing as an integrator.

Integrators often used in filters where $|V_{OS}|$ is comparable to $|V_{IS}|$

Is this common-source amplifier linear or nonlinear?
Linearity of Common-Source Amplifier

High-Gain Amplifier

Transconductance Amplifier

\[ I_{\text{OUT}} = I_B - I_D \]

\[ I_{\text{OUT}} = I_B - \beta (v_{iS} + V_{EB})^2 (1 + \lambda [V_{OS} + V_{OQ} - V_{SS}]) \]

\[ I_{\text{OUT}} = \left[ I_B - \beta (V_{EB})^2 (1 + \lambda [V_{OQ} - V_{SS}]) \right] - \beta (v_{iS}^2 + 2v_{iS}V_{EB}) (1 + \lambda [V_{OS} + V_{OQ} - V_{SS}]) \]

Is this a linear or nonlinear relationship?
Linearity of Common-Source Amplifier

As an OTA

\[ g_m = \frac{2I_B}{V_{EB}} \]

\[ I_{OUT} \approx -\frac{2I_B}{V_{EB}} \left( \nu_i + \frac{1}{2V_{EB}} \nu_i^2 \right) \]

Is this a linear or nonlinear relationship?

At \( \nu_i = -V_{EB} \), the error in \( I_{OUT} \) will be -50%!
Is this common-source amplifier linear?

- Reasonably linear if used in high-gain applications and $V_{EB}$ is large (e.g. if $A_V=g_m/g_o=2/((\lambda V_{EB})=100$ and $V_o=1\text{V}$, $V_{in}=10\text{mV}$)

- Highly nonlinear when used in low-gain applications
Linearity of Common-Emitter Amplifier

Is this common-emitter amplifier linear?

- Very linear if used in high-gain applications
  (e.g. if $A_V = \frac{g_m}{g_0} = \frac{V_{AF}}{V_t} = 4000$ and $V_o = 1V$, $V_{in} = 250uV$)

- Highly nonlinear when used in low-gain applications
End of Lecture 20