Linearity in Operational Amplifiers
-- The differential pairs
$g_{meq}$ Enhancement with Driven Counterpart Circuit

\[ A_{V_0} = \frac{g_{m1} + g_{m3}}{g_{o1} + g_{o3}} \]

\[ GB = \frac{g_{m1} + g_{m3}}{C_L} \]

Both gain and GB enhancement
Recall:

Other Methods of Gain Enhancement

\[ A_{V0} = \frac{-g_{MQC}}{g_{OQC} + g_{OCC}} \]

Two Strategies:

1. Decrease denominator of \( A_{V0} \)
2. Increase numerator of \( A_{V0} \)

Consider again decreasing the denominator

\[ A_{V0} = \frac{-g_{MQC}}{g_{OQC} + g_{OCC} - g_{OX}} \]

Is it possible to come up with circuits that will provide a subtraction of conductance in the denominator?
Gain Enhancement with Regenerative Feedback

But - can we easily build circuits with this property? 

But – the inverting amplifier may be more difficult to build than the op amp itself!

YES – simply by cross-coupling the outputs in a fully differential structure
Gain Enhancement with Regenerative Feedback

If \( g_{mP1} = g_{oF1} + g_{oP1} \), the dc gain will become infinite !!
Gain Enhancement with Regenerative Feedback

\[ A_{V0}(s) = \frac{-g_{mF1}}{sC_L + g_{oF1} + g_{oP1} - g_{mP1}} \]

\[ p = \frac{-g_{oF1} - g_{oP1} + g_{mP1}}{C_L} \]

If \( g_{mP1} > g_{oF1} + g_{oP1} \), the pole will be in the RHP!!

This will make the op amp unstable.

Positive Feedback is BAD!!

This is the major reason most have avoided using the structure!
Gain Enhancement with Regenerative Feedback

\[
-A_{V0}(s) = \frac{-g_{mF1}}{sC_L + g_{oF1} + g_{oP1} - g_{mP1}}
\]

\[
p = \frac{-g_{oF1} - g_{oP1} + g_{mP1}}{C_L}
\]

If \(g_{mP1} > g_{oF1} + g_{oP1}\), the pole will be in the RHP.

It can be shown that the feedback amplifier is usually stable even if the open-loop Op amp is unstable. **How?**

\[
A_{V0}(s) = \frac{A_{V0}\tilde{p}_1}{s + \tilde{p}_1}
\]

\[
A_{FB}(s) = \frac{-A_{V0}(s)}{1 - A_{V0}(s)\beta + (s + \tilde{p}_1 - \beta A_{V0}\tilde{p}_1)}
\]

\[
p_{FB} = -\tilde{p}_1(1 - \beta A_{V0}) = p_1(1 - \beta A_{V0})
\]

Since \(A_{V0} < 0\) for \(p_1 < 0\) and \(A_{V0} > 0\) for \(p_1 > 0\), it follows that

\[
p_{FB} = \begin{cases} p_1(1 + \beta|A_{V0}|) & \text{if } p_1 < 0 \\ p_1(1 - \beta|A_{V0}|) & \text{if } p_1 > 0 \end{cases}
\]
Gain Enhancement with Regenerative Feedback

The feedback performance can actually be enhanced if the open-loop amplifier is unstable. Why?

- Time required to get in settling window can be reduced with RHP pole
- But, if pole is too far in RHP, response will exit top of window
Alternate Positive Feedback Amplifier

Review from last lecture...
Another Positive Feedback Amplifier
Practical Comments about Positive Feedback Gain Enhancement

- Significant gain enhancement is possible but most avoid regenerative feedback because of unfounded concerns about closed-loop stability
- Accuracy and settling time can be improved with some regenerative feedback
- Will become more critical in emerging processes where $g_m/g_o$ ratios degrade and where supply voltages shrink thus limiting the longstanding cascode process
- Regenerative structures can have high sensitivities
- Signal swing quite limited in some of the most basic regenerative feedback structures
- Most useful in two-stage architecture where regenerative feedback is used in first stage (effects of signal swing are reduced by gain of second stage)
Summary of Methods of Gain Enhancement

Increasing the output impedance of the amplifier
    cascode, folded cascode, regulated cascode, positive feedback

Increasing the transconductance
    (current mirror op amp) but it didn’t really help because
    the output conductance increased proportionally

    Driving the counterpart circuit does offer some improvements in gain

Cascading gives a multiplicative gain effect
    (thousands of architectures but compensation is essential)
    usually limited to a two-level cascade because of too much
    phase accumulation
Operational Amplifier Architectures

Most of the popular operational amplifier architectures have been introduced.

Large number of different architectural choices exist with substantially different performance potential.

Choice of architecture is important but judicious use of DOF is essential to obtain good performance.

Few architectures offer a GB power efficiency that is better than that of the reference op amp.

Some variants of the basic amplifier structures such as buffered output stages are commonly used in some applications.
Observations about Op Amp Design

• Considerably different insight can often be obtained by viewing a circuit in multiple ways

• Various systematic procedures for designing op amps have been introduced

• It is important to understand the design space and to identify a good set of design variables
  – Design spaces can be explored in many different ways but the degrees of freedom are incredibly valuable resources and should be used judiciously

• Cascaded amplifiers offer potential for gain enhancement but compensation schemes to practically work with more than two levels of cascading have not yet emerged

• Positive feedback appears to provide a promising approach for building high gain amplifiers in low voltage processes but research is ongoing into how this concept can be fully utilized
Up to this point all analysis of the op amp has focused on small-signal gain characteristics

Linearity of the amplifier does play a role in linearity and spectral performance of feedback amplifiers

Linearity is of major concern when the op amp is used open-loop such as in OTA applications

A major source of linearity is often associated with the differential input pair

Will consider linearity of the input differential pairs
Signal Swing and Linearity

Signal swing identifies range over which signals can be applied and still maintain operation of devices in desired region of operation.

Some subset of the signal swing range will be quite linear.

Often that subset is close to the entire signal swing range.
Signal Swing and Linearity

Ideal Scenario:

Completely Linear over Input and Output Range
Realistic Scenario:

- Modest Nonlinearity throughout Input Range
- But operation will be quite linear over subset of this range
Signal Swing and Linearity

\[
\begin{align*}
V_{\text{OUT}} & \quad \text{Output Range} \\
V_{\text{IN}} & \quad \text{Input Range} \\
V_{\text{OUT}} & \quad \text{Linear Output Range} \\
V_{\text{IN}} & \quad \text{Linear Input Range}
\end{align*}
\]
Linearity of Amplifiers

Linearity of differential pair of major concern

Linearity of common-source amplifier is of major concern (since signals so small at output of differential pair)
Differential Input Pairs

MOS Differential Pair

Bipolar Differential Pair
MOS Differential Pair

\[ V_d = V_2 - V_1 \]

\[ I_{D1} = \frac{\mu C_{ox} W}{2L} \left( V_1 - V_s - V_T \right)^2 \]

\[ I_{D2} = \frac{\mu C_{ox} W}{2L} \left( V_2 - V_s - V_T \right)^2 \]

\[ I_{D1} + I_{D2} = I_T \]

\[ \sqrt{I_{D1}} \sqrt{\frac{2L}{\mu C_{ox} W}} = V_1 - V_s - V_T \]

\[ \sqrt{I_{D2}} \sqrt{\frac{2L}{\mu C_{ox} W}} = V_2 - V_s - V_T \]
MOS Differential Pair

\[ V_d = \sqrt{\frac{2L}{\mu C_{ox} W}} \left( \sqrt{I_T} - I_{D1} - \sqrt{I_{D1}} \right) \]

\[ V_d = \sqrt{\frac{2L}{\mu C_{ox} W}} \left( \sqrt{I_{D2}} - \sqrt{I_T - I_{D2}} \right) \]

What values of \( V_d \) will cause all of the current to be steered to the left or the right?

\[ V_{dx} = \pm \sqrt{\frac{2L}{\mu C_{ox} W}} \left( \sqrt{I_T} \right) \]
\[ V_d = \sqrt{\frac{2L}{\mu C_{ox} W}} \left( \sqrt{I_{D2}} - \sqrt{I_T - I_{D2}} \right) \]
Q-point Calculations

\[ \frac{I_T}{2} = \frac{\mu C_{ox} W}{2L} (V_{EB})^2 \]

\[ V_{EB} = \sqrt{I_T} \sqrt{\frac{L}{\mu C_{ox} W}} \]

Observe!!

\[ V_{dx} = \pm \sqrt{2} V_{EB} \]
$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}} \left( \sqrt{I_{D2}^2} - \sqrt{I_T - I_{D2}} \right)$

How linear is the amplifier?

$V_{EB}$ affects linearity
How linear is the amplifier?

Consider the fit line:

\[ I = mV_d + h \]

When \( V_d = 0 \), \( I = \frac{I_T}{2} \), thus

\[ h = \frac{I_T}{2} \]

\[ V_{d_{int}} = -\frac{h}{m} = -\frac{I_T}{2m} \]

\[ m = \frac{\partial I_{D1}}{\partial V_d}_{Q_{pt}} \]
How linear is the amplifier?

\[ I = mV_d + h \]

\[ V_{d_{int}} = -\frac{h}{m} = -\frac{I_T}{2m} \]

\[ m = \frac{\partial I_{D1}}{\partial V_d} \bigg|_{Q-pt} \]

\[ V_d = \sqrt{\frac{2L}{\mu C_{ox} W}} \left( \sqrt{I_T - I_{D1}} - \sqrt{I_{D1}} \right) \]

\[ \frac{\partial V_d}{\partial I_{D1}} = \sqrt{\frac{2L}{\mu C_{ox} W}} \left( \frac{1}{2} (I_T - I_{D1})^{-1/2} (-1) - \frac{1}{2} (I_{D1})^{-1/2} \right) \bigg|_{Q-point} \]

\[ \frac{\partial V_d}{\partial I_{D1}} = -2 \sqrt{\frac{L}{\mu C_{ox} W}} \frac{1}{\sqrt{I_T}} \]

\[ \sqrt{\frac{L}{\mu C_{ox} W}} = \frac{V_{EB1}}{\sqrt{I_T}} \]

\[ \frac{\partial V_d}{\partial I_{D1}} = -2 \frac{V_{EB1}}{I_T} \]

\[ m = \frac{\partial I_{D1}}{\partial V_d} \bigg|_{Q-pt} = -\frac{I_T}{2V_{EB1}} \]
How linear is the amplifier?

\[ V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m} = V_{EB1} \]

\[ I = -\left( \frac{I_T}{2V_{EB1}} \right) V_d + \frac{I_T}{2} \]
How linear is the amplifier?

It can be shown that a 1% deviation from the straight line occurs at

$$V_d \approx \frac{V_{EB}}{3}$$

and a 0.1% variation occurs at

$$V_d \approx \frac{V_{EB}}{10}$$
How linear is the amplifier?

1% Linear = 0.3\(V_{EB1}\)
How linear is the amplifier?

Deviation from Linear

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<th>Vd/VEB</th>
<th>( \theta )</th>
<th>Vd/VEB</th>
<th>( \theta )</th>
<th>Vd/VEB</th>
<th>( \theta )</th>
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**Bipolar Differential Pair**

\[ I_{C1} = J_S A_{E1} e^{\frac{V_1 - V_E}{V_t}} \]

\[ I_{C2} = J_S A_{E2} e^{\frac{V_2 - V_E}{V_t}} \]

\[ I_{C1} + I_{C2} = I_T \]

\[ V_1 = V_E + V_t \ln \left( \frac{I_{C1}}{J_S A_{E1}} \right) \]

\[ V_2 = V_E + V_t \ln \left( \frac{I_{C2}}{J_S A_{E2}} \right) \]

\[ V_d = V_t \left( \ln \left( \frac{I_{C2}}{J_S A_{E2}} \right) - \ln \left( \frac{I_{C1}}{J_S A_{E1}} \right) \right)_{A_{E1}=A_{E2}} = V_t \ln \left( \frac{I_{C2}}{I_{C1}} \right) \]

\[ V_d = V_t \left( \ln \left( \frac{I_{C2}}{J_S A_{E2}} \right) - \ln \left( \frac{I_{C1}}{J_S A_{E1}} \right) \right)_{A_{E1}=A_{E2}} = V_t \ln \left( \frac{I_{C2}}{I_{C1}} \right) \]
Bipolar Differential Pair

\[ V_d = V_2 - V_1 \]

\[ V_d = V_t \left( \ln \left( \frac{I_{C_2}}{J_s A_{E_2}} \right) - \ln \left( \frac{I_{C_1}}{J_s A_{E_1}} \right) \right)^{A_{E_1}=A_{E_2}} = V_t \ln \left( \frac{I_{C_2}}{I_{C_1}} \right) \]

\[ V_d = V_t \ln \left( \frac{I_T - I_{C_1}}{I_{C_1}} \right) \]

\[ V_d = V_t \ln \left( \frac{I_{C_2}}{I_T - I_{C_2}} \right) \]

At \( I_{C_1} = I_{C_2} = I_T/2 \), \( V_d = 0 \)

As \( I_{C_1} \) approaches 0, \( V_d \) approaches infinity

As \( I_{C_1} \) approaches \( I_T \), \( V_d \) approaches minus infinity

Transition much steeper than for MOS case
Transfer Characteristics of Bipolar Differential Pair

Transition much steeper than for MOS case
Asymptotic Convergence to 0 and $I_T$
Signal Swing and Linearity of Bipolar Differential Pair

\[ I_{FIT} = mV_d + h \]

\[ V_{dint} = - \frac{h}{m} = ? \]

\[ m = \left. \frac{\partial I_{C1}}{\partial V_d} \right|_{Q\text{-point}} \]

\[ \frac{\partial V_d}{\partial I_{C1}} \bigg|_{Q\text{-point}} = -V_t \frac{I_T}{I_{C1}(I_T - I_{C1})} \bigg|_{I_{C1} = \frac{I_T}{2}} \]

\[ \frac{\partial V_d}{\partial I_{C1}} \bigg|_{Q\text{-point}} = - \frac{4V_t}{I_T} \]

\[ I_{FIT} = - \frac{I_T}{4V_t} V_d + \frac{I_T}{2} \]

\[ V_{dint} = - \frac{h}{m} = 2V_t \]
Signal Swing and Linearity of Bipolar Differential Pair

for 1% deviation, \( V_d = 0.56V_t \)
for 0.1% deviation, \( V_d = 0.27V_t \)
Signal Swing and Linearity of Bipolar Differential Pair

$1\%$ linear $= .56 V_t$
End of Lecture 21