EE 435

Lecture 21

Linearity in Basic Gain Stages
Offset Voltages
How linear is the amplifier?

1% Linear = 0.3V_{EB1}
Signal Swing and Linearity of Bipolar Differential Pair

Review from last lecture.

1% linear = 0.56V_t

2V_t
Applications as a programmable OTA

Large $g_m$ adjustment possible
Is changed
No change in signal swing when $g_m$

One decade change in $g_m$

\[
\frac{g_m}{I_{ABC}} = \frac{1}{V_{EB}}
\]

BJT

Limited $g_m$ adjustment possibility
Every decade decrease in $g_m$
One decade decrease in signal swing for

\[
g_m = \frac{C}{W} V_{EB}
\]

Decade change in $g_m$

Two decade change in current for every

\[
\frac{g_m}{I_{ABC}} = \frac{L}{2V_{EB}}
\]

MOS

No change in signal swing when $g_m$

The current-dependence of the $g_m$ of the differential pair is often used to program the transconductance of an OTA with the tail bias current $I_{ABC}$.

Review from last lecture...
Linearity of Common-Source Amplifier

For convenience, will consider situation where current source biasing is ideal.
Linearity of Common-Source Amplifier

\[ v_{OS} \approx \left( v_{SS} - v_{OQ} + \frac{1}{\lambda} \left( \frac{l_B}{\beta V_{EB}^2} \right) - 1 \right) - \frac{l_B}{\lambda \beta V_{EB}^2} \left( 2 \frac{v_iS}{V_{EB}} + \left( \frac{v_iS}{V_{EB}} \right)^2 \right) \]

\[ v_{OS} \approx - \left( 2 \frac{v_iS}{\lambda V_{EB}} + \frac{1}{\lambda} \left( \frac{v_iS}{V_{EB}} \right)^2 \right) \]

\[ v_{OS} \approx - \frac{2}{\lambda V_{EB}} \left( v_iS + \frac{1}{2V_{EB}} v_iS^2 \right) \]

Is this a linear or nonlinear relationship?
Linearity of Common-Source Amplifier

\[ V_{OS} \approx \frac{2}{\lambda V_{EB}} \left( V_{iS} + \frac{1}{2V_{EB}} V_{iS}^2 \right) \]

Is this a linear or nonlinear relationship?

when \( V_{iS} = -V_{EB} \) (the minimum value of \( V_{iS} \) to maintain saturation operation)

the error in \( V_{OS} \) will be \( V_{EB}/2 \) which is -50%

Is this a linear or nonlinear relationship?
Linearity of Common-Source Amplifier

\[ v_{OS} \approx -\frac{2}{\lambda V_{EB}} \left( v_iS + \frac{1}{2V_{EB}} v_iS^2 \right) \]

Is this a linear or nonlinear relationship?

Note this is a high gain amplifier

Over what output voltage range are we interested?
Linearity of Common-Source Amplifier

\[ v_{OS} \approx -\frac{2}{\lambda V_{EB}} \left( v_i + \frac{1}{2 V_{EB}} v_i^2 \right) \]

Is this a linear or nonlinear relationship?

Linearity is reasonably good over practical input range

Practical input range is much less than \( V_{EB} \)
Linearity of Common-Source Amplifier

\[ v_{OS} \approx \frac{2}{\lambda V_{EB}} \left( v_i + \frac{1}{2V_{EB}} v_i^2 \right) \]

Is this a linear or nonlinear relationship?

Can't see nonlinearity in this plot

\[ V_{EB} = 1V \]
\[ \lambda = 0.01 \]
Linearity of Common-Source Amplifier

\[ v_{OS} \approx -\frac{2}{\lambda V_{EB}} \left( v_iS + \frac{1}{2V_{EB}} v_{iS}^2 \right) \]

Is this a linear or nonlinear relationship?

\[ V_{EB} = 1V \]
\[ \lambda = 0.01 \]

\[ v_{FIT} \approx -\frac{2}{\lambda V_{EB}} v_iS \]
\[ \varepsilon = v_{FIT} - v_{oS} \]
\[ \varepsilon \approx \frac{1}{\lambda V_{EB}^2} v_{iS}^2 \]
Linearity of Common-Source Amplifier

\[ v_{OS} \approx -\frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right) \]

Is this a linear or nonlinear relationship?

\[ \varepsilon \approx \frac{1}{\lambda V_{EB}^2} v_{iS}^2 \]

\[ V_{EB} = 1V \]
\[ \lambda = 0.01 \]
Linearity of Common-Source Amplifier

\[ v_{OS} \approx -\frac{2}{\lambda V_{EB}} \left( v_i S + \frac{1}{2V_{EB}} v_i S^2 \right) \]

Is this a linear or nonlinear relationship?

\[ V_{EB} = 1V \]
\[ \lambda = 0.01 \]

\[ \varepsilon_{PCT} \approx \frac{\varepsilon}{v_{FIT}} 100\% = \frac{\varepsilon}{\frac{1}{2V_{EB}} v_i S^2} 100\% = \left( \frac{100\%}{2V_{EB}} \right) v_i S \]

\[ \varepsilon_{PCT} \approx \left( -\frac{\lambda \cdot 100\%}{4} \right) v_{OS} \]
Linearity of Common-Source Amplifier

\[ v_{OS} \approx -\frac{2}{\lambda V_{EB}} \left( v_{IS} + \frac{1}{2V_{EB}} v_{IS}^2 \right) \]

Is this a linear or nonlinear relationship?

\[ V_{EB} = 1V \]
\[ \lambda = 0.01 \]

\[ \varepsilon_{PCT} \approx \left( \frac{100\%}{2V_{EB}} \right) v_{IS} \]

or, in terms of \( v_{OS} \),

\[ \varepsilon_{PCT} \approx \left( \frac{-\lambda \cdot 100\%}{4} \right) v_{OS} \]

1% deviation for this example occurs at

\[ |v_{OS}| \approx 0.01 \frac{4}{\lambda} \approx 4V \]
Is this common-source amplifier linear or nonlinear?
Linearity of Common-Source Amplifier

Is this a linear or nonlinear relationship?
Linearity of Common-Source Amplifier

As an OTA

\[ I_{\text{OUT}} = \frac{2I_B}{V_{EB}} \]

\[ g_m = \frac{2I_B}{V_{EB}} \]

\[ I_{\text{OUT}} \approx -\frac{2I_B}{V_{EB}} \left( V_{iS} + \frac{1}{2V_{EB}} V_{iS}^2 \right) \]

Is this a linear or nonlinear relationship?

At \( V_{iS} = -V_{EB} \), the error in \( I_{\text{OUT}} \) will be -50%!
Is this common-source amplifier linear?

• Reasonably linear if used in high-gain applications and \( V_{EB} \) is large (e.g. if \( A_v = g_m / g_o = 2/(\lambda V_{EB}) = 100 \) and \( V_o = 1V, V_{in} = 10mV \))

• Highly nonlinear when used in low-gain applications
Linearity of Common-Emitter Amplifier

High-Gain Amplifier

Transconduction Amplifier

Is this common-emitter amplifier linear?

- Very linear if used in high-gain applications
  (e.g. if $A_v = g_m/g_0 = V_{AF}/V_t = 4000$ and $V_o = 1V$, $V_{in} = 250uV$)

- Highly nonlinear when used in low-gain applications
Offset Voltage

Two types of offset voltage:

- Systematic Offset Voltage
- Random Offset Voltage

Definition: The output offset voltage is the difference between the desired output and the actual output when $V_{\text{id}}=0$ and $V_{\text{ic}}$ is the quiescent common-mode input voltage.

$$V_{\text{OUTOFF}} = V_{\text{OUT}} - V_{\text{OUTDES}}$$

Note: $V_{\text{OUTOFF}}$ is dependent upon $V_{\text{ICQ}}$ although this dependence is usually quite weak and often not specified.
Offset Voltage

Definition: The input-referred offset voltage is the differential dc input voltage that must be applied to obtain the desired output when $V_{ic}$ is the quiescent common-mode input voltage.

Note: $V_{OFF}$ is usually related to the output offset voltage by the expression

$$V_{OFF} = \frac{V_{OUTOFF}}{A_C}$$

Note: $V_{OFF}$ is dependent upon $V_{ICQ}$ although this dependence is usually quite weak and often not specified.
Offset Voltage

Two types of offset voltage:

• Systematic Offset Voltage
• Random Offset Voltage

After fabrication it is impossible (difficult) to distinguish between the systematic offset and the random offset in any individual op amp.

Measurements of offset voltages for a large number of devices will provide mechanism for identifying systematic offset and statistical Characteristics of the random offset voltage.
Systematic Offset Voltage

Offset voltage that is present if all device and model parameters assume their nominal value

Easy to simulate the systematic offset voltage

Almost always the designer’s responsibility to make systematic offset voltage very small

Generally easy to make the systematic offset voltage small
Random Offset Voltage

Due to random variations in process parameters and device dimensions

Random offset is actually a random variable at the design level but deterministic after fabrication in any specific device

Distribution nearly Gaussian

Has zero mean

Characterized by its standard deviation or variance

Often strongly layout dependent

Due to both local random variations and correlated gradient effects

Will consider both effects separately

Gradient effects usually dominate if not managed

Good methods exist for driving gradient effects to small levels and will be discussed later

In what follows it will be assumed that gradient effects have been managed
Gradient and Random Effect
Offset Voltage

Can be modeled as a dc voltage source in series with the input
Offset Voltage

Effects of Offset Voltage - an example

Desired I/O relationship
Offset Voltage

Effects of Offset Voltage - an example

Desired I/O relationship

Actual I/O relationship due to offset
Offset Voltage

Effects can be reduced or eliminated by adding equal amplitude opposite Dc signal (many ways to do this)

Widely used in offset-critical applications

Comes at considerable effort and expense

Prefer to have designer make $V_{OS}$ small in the first place
Effects of Offset Voltage

• Deviations in performance will change from one instantiation to another due to the random component of the offset

• Particularly problematic in high-gain circuits

• A major problem in many other applications

• Not of concern in many applications as well
Offset Voltage Distribution

Typical histogram of offset voltage (binned) after fabrication
Offset Voltage Distribution

Typical histogram of offset voltage (binned) after fabrication

Mean is nearly 0 (actually the systematic offset voltage)
Offset Voltage Distribution

Typical histogram of offset voltage (binned) in shipped parts

Extreme offset parts have been sifted at test
Offset Voltage Distribution

Typical histogram of offset voltage (binned) in shipped parts

Low-offset parts sold at a premium

Extreme offset parts have been sifted at test
Offset Voltage Distribution

Pdf of zero-mean Gaussian distribution

Characterized by its standard deviation $\sigma$ or variance $\sigma^2$

Offset voltage often specified as the $1\sigma$ or $3\sigma$ value
Offset Voltage Distribution

Pdf of zero-mean Gaussian distribution

Percent between:

<table>
<thead>
<tr>
<th>Interval</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>±σ</td>
<td>68.3%</td>
</tr>
<tr>
<td>±2σ</td>
<td>95.5%</td>
</tr>
<tr>
<td>±3σ</td>
<td>99.73%</td>
</tr>
</tbody>
</table>
Source of Random Offset Voltages

Consider as an example:

\[ V_{DD} \quad V_{SS} \]

\[ R_1 \quad R_2 \quad M_1 \quad M_2 \quad V_{OUT} \]

\[ I_T \]

\[ V_{OUT} = V_{DD} - \left( \frac{I_T}{2} \right) R \]

Assume this is the desired output voltage

Ideally \( R_1 = R_2 = R \), \( M_1 \) and \( M_2 \) are matched
Source of Random Offset Voltages

Consider as an example:

If everything ideal except $R_2 = R + \Delta R$

$$V_{OUT} = V_{DD} - \left( \frac{I_T}{2} \right) [R + \Delta R]$$

$$\Delta V_{OUT} = - \left( \frac{I_T}{2} \right) \Delta R$$
Source of Random Offset Voltages

Consider as an example:

\[ A_V = -\frac{g_m}{2} \]
Source of Random Offset Voltages

Determine the offset voltage – i.e. value of \( V_X \) needed to obtain desired output

\[
A_V = -\frac{g_m}{2} R
\]

\[
V_{OUT} = \left[ V_{DD} - \left( \frac{I_T}{2} \right) R \right] - \left( \frac{I_T}{2} \right) \Delta R - A_V V_X
\]

\[
V_X = -\frac{1}{A_V} \left( \frac{I_T}{2} \right) \Delta R
\]
Source of Random Offset Voltages

Determine the offset voltage – i.e. value of $V_X$ needed to obtain desired output

$$A_V = -\frac{g_m}{2} R$$

$$V_X = \frac{-1}{A_V} \left( \frac{I_T}{2} \right) \Delta R$$

$$V_X = \frac{2}{g_m R} \left( \frac{I_T}{2} \right) \Delta R = \frac{I_T}{g_m} \left( \frac{I_T}{I_T/V_{EB}} \right) \frac{\Delta R}{R} = V_{EB} \frac{\Delta R}{R}$$

$$V_X = V_{EB} \frac{\Delta R}{R}$$
End of Lecture 21