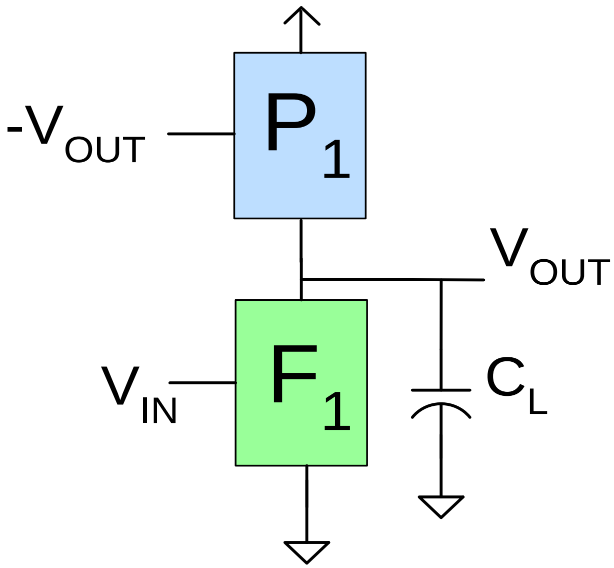
EE 435

Lecture 21

Linearity in Operational Amplifiers

-- The differential pairs

Gain Enhancement with Regenerative Feedback



$$A_{V0} = \frac{-g_{mF1}}{sC_L + g_{oF1} + g_{oP1} - g_{mP1}}$$

$$A_{V0} = \frac{g_{mF1}}{g_{oF1} + g_{oP1} - g_{mP1}}$$

$$BW = \frac{g_{oF1} + g_{oP1} - g_{mP1}}{C_L}$$

$$GB = \frac{g_{mF1}}{C_L}$$

The gain can be made arbitrarily large by selecting g_{mP1} appropriately

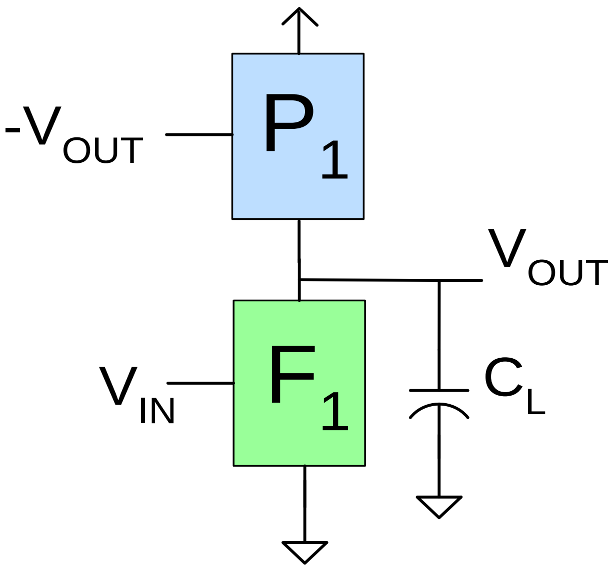
The GB does not degrade !

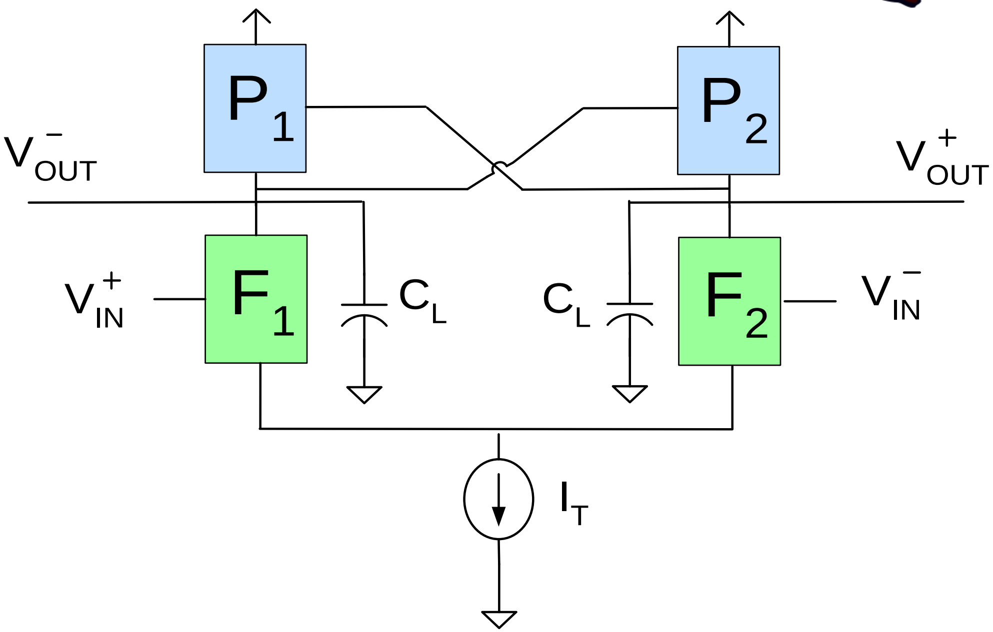
But - can we easily build circuits with this property?

Gain Enhancement with Regenerative Feedback

Review from last lecture

- But - can we easily build circuits with this property?



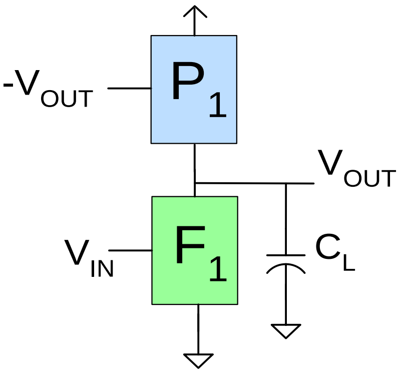


- But – the inverting amplifier may be more difficult to build than the op amp itself!

- YES – simply by cross-coupling the outputs in a fully differential structure

Review from last lecture

Gain Enhancement with Regenerative Feedback

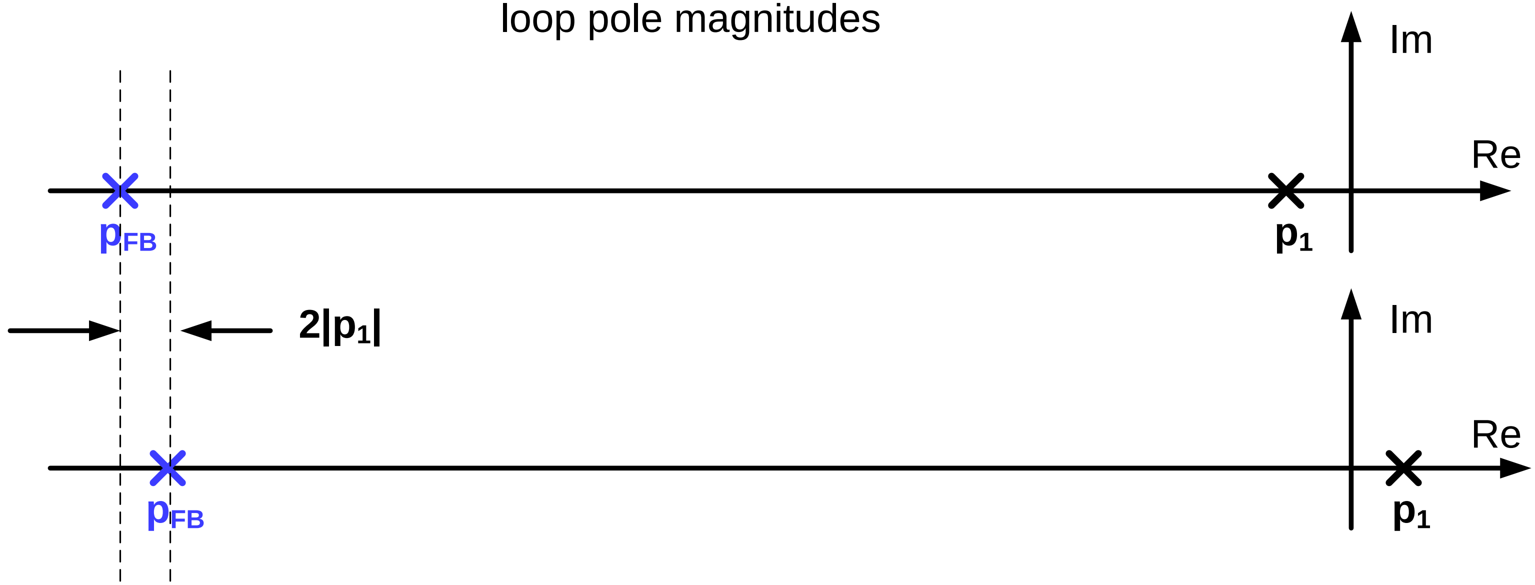


It can be shown that the feedback amplifier is usually stable even if the open-loop Op amp is unstable

How?

$$p_{FB} = \begin{cases} -\tilde{p}_1\left(1+\beta A_{V0}\right) = p_1\left(1+\beta A_{V0}\right) & \text{for } p_1 < 0 \\ -\tilde{p}_1\left(1-\beta A_{V0}\right) = p_1\left(1-\beta A_{V0}\right) & \text{for } p_1 > 0 \end{cases}$$

Open-Loop and Closed-Loop Pole Plot for equal open-loop pole magnitudes

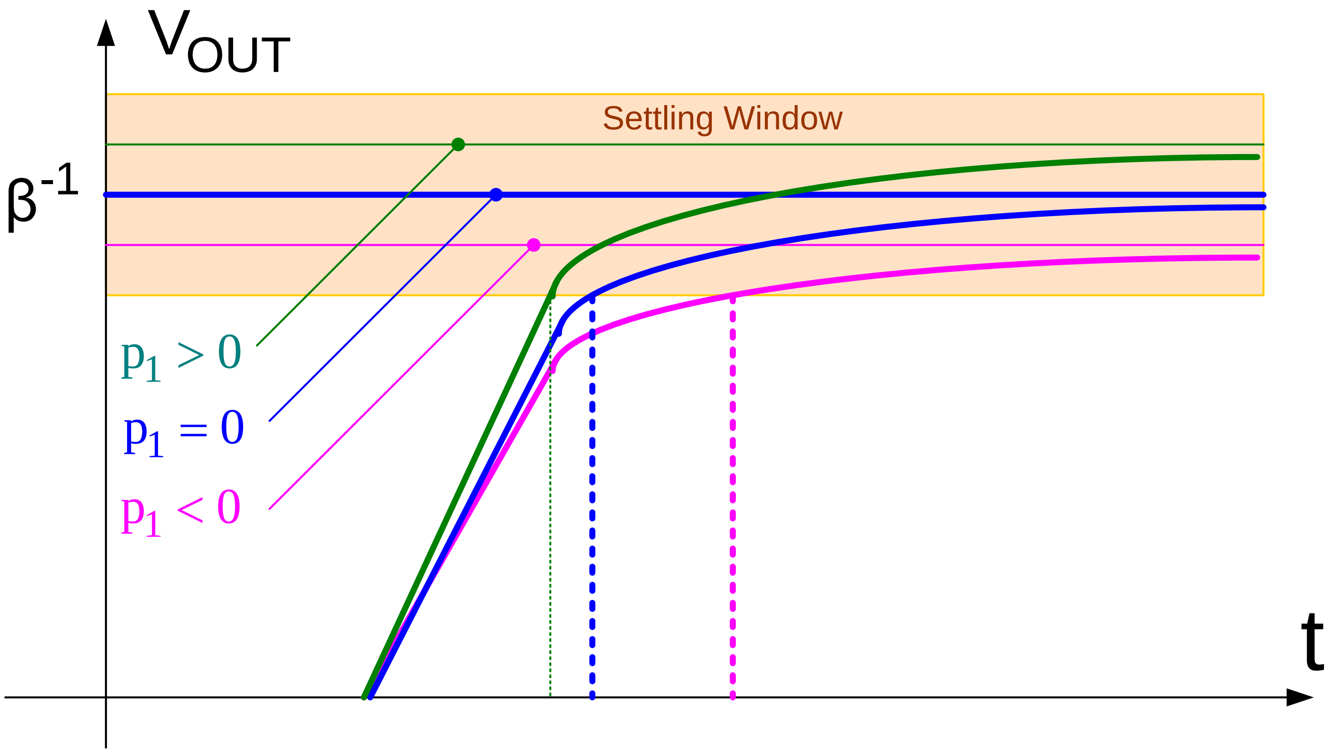


Review from last lecture

Gain Enhancement with Regenerative Feedback

The feedback performance can actually be enhanced if the open-loop amplifier is unstable

Why?



- Time required to get in settling window can be reduced with RHP pole
- But, if pole is too far in RHP, response will exit top of window

Up to this point all analysis of the op amp has focused on small-signal gain characteristics

Linearity of the amplifier does play a role in linearity and spectral performance of feedback amplifiers

Linearity is of major concern when the op amp is used open-loop such as in OTA applications

A major source of linearity is often associated with the differential input pair

Will consider linearity of the input differential pairs

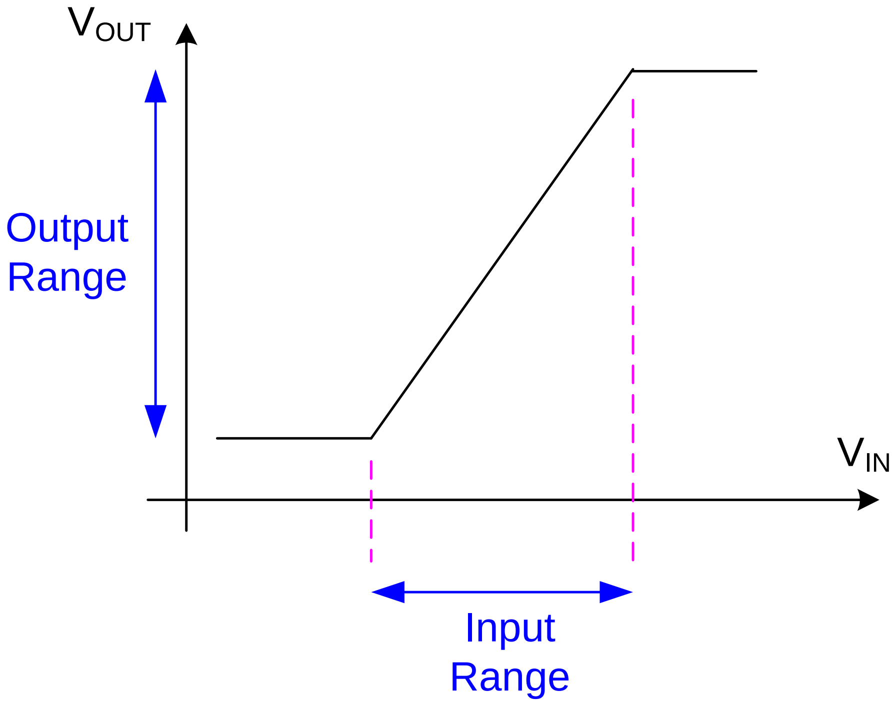
Signal Swing and Linearity

Signal swing identifies range over which signals can be applied and still maintain operation of devices in desired region of operation

Some subset of the signal swing range will be quite linear

Often that subset is close to the entire signal swing range

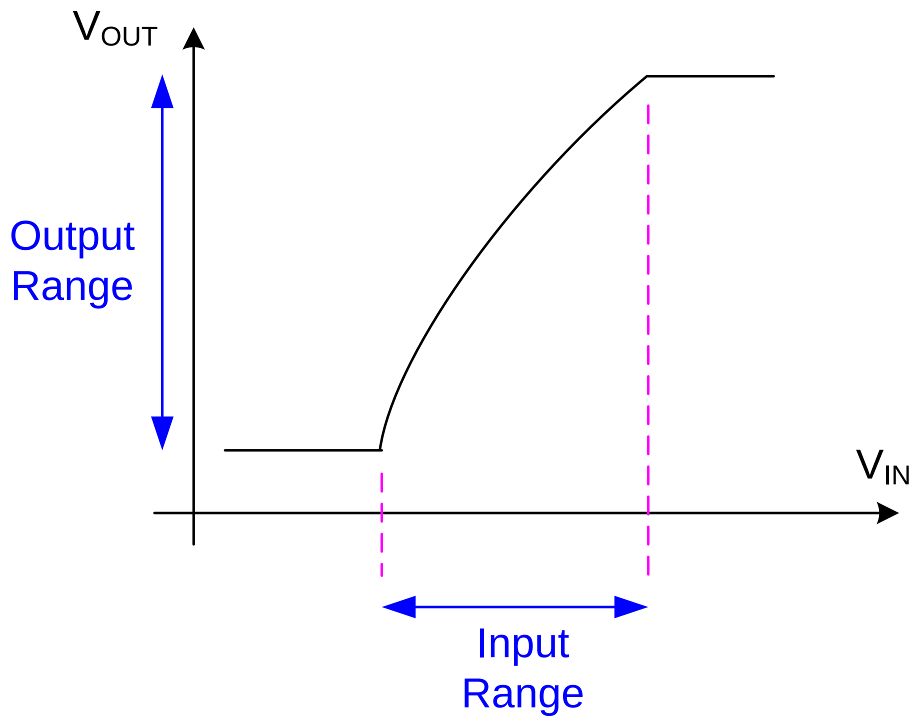
Signal Swing and Linearity



Ideal Scenario:

Completely Linear over Input and Output Range

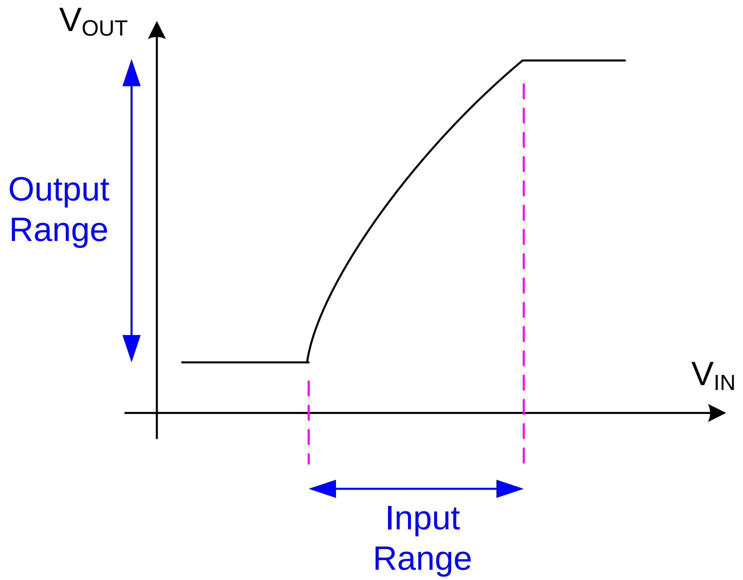
Signal Swing and Linearity

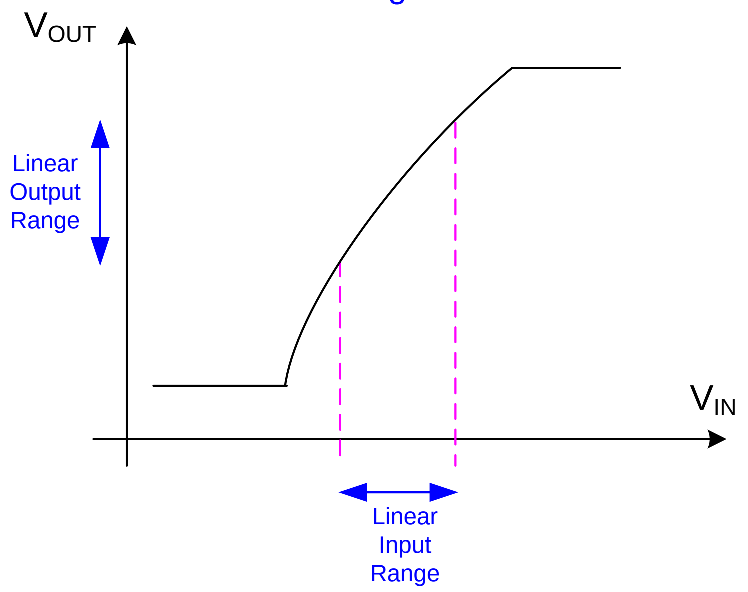


Realistic Scenario:

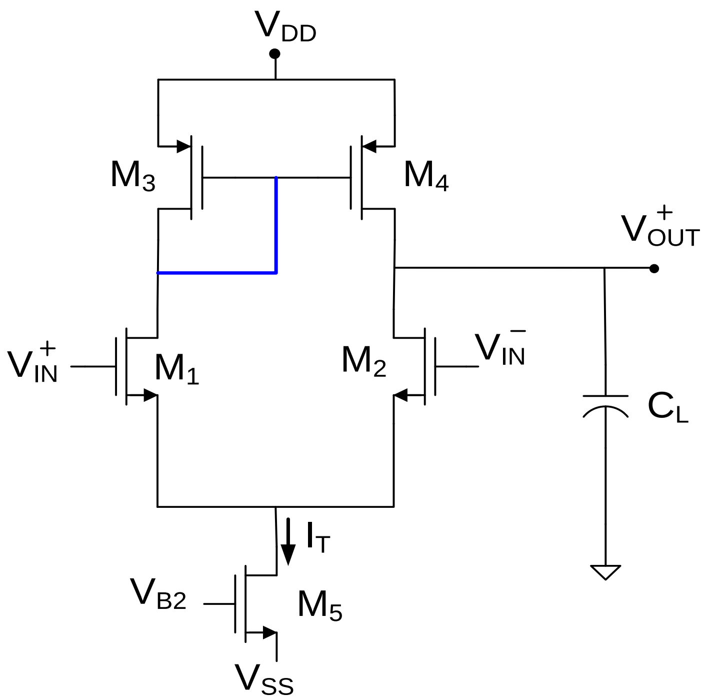
- Modest Nonlinearity throughout Input Range
- But operation will be quite linear over subset of this range

Signal Swing and Linearity

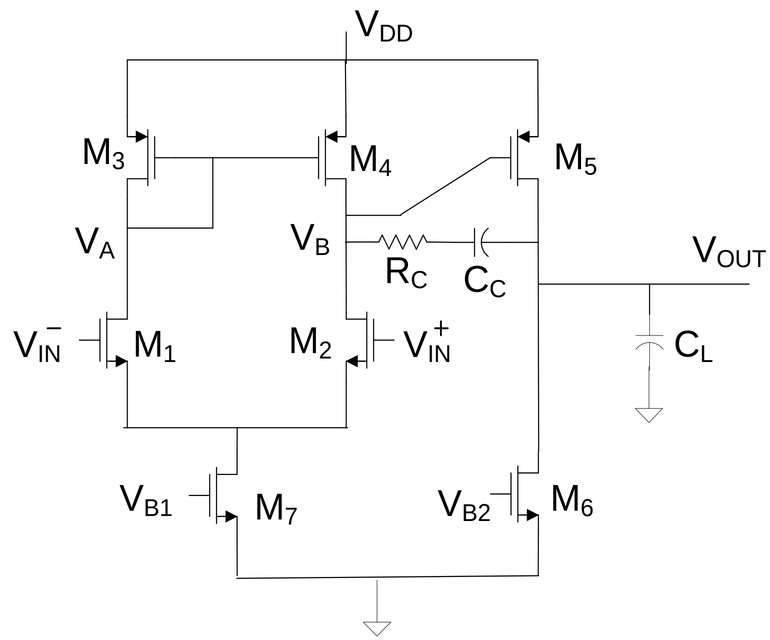




Linearity of Amplifiers



Single-Stage

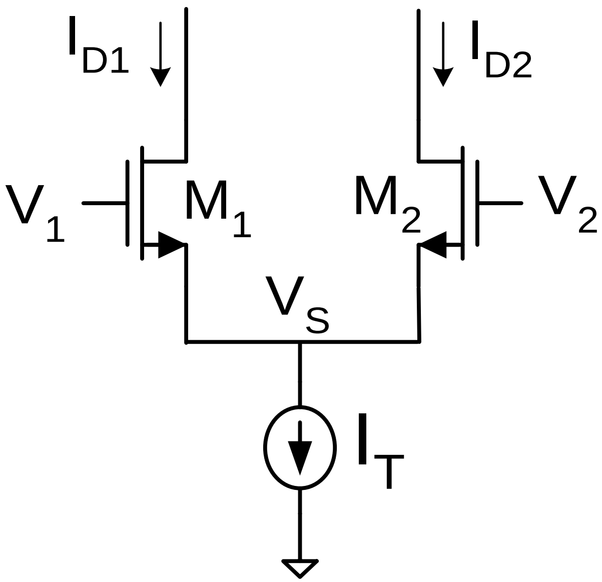


Two-Stage

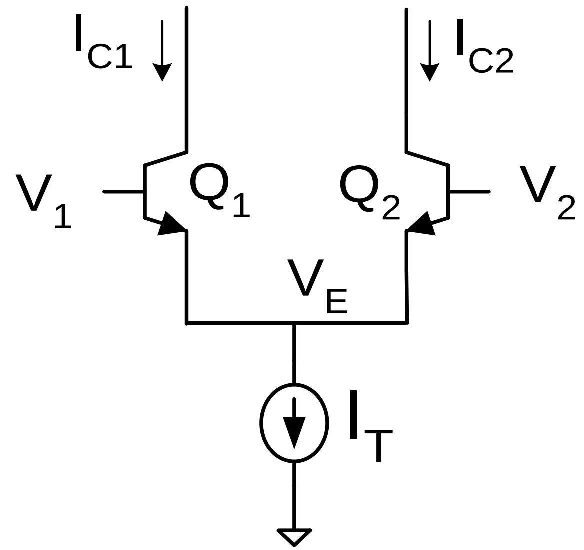
Linearity of differential pair of major concern

Linearity of common-source amplifier is of major concern (since signals so small at output of differential pair)

Differential Input Pairs

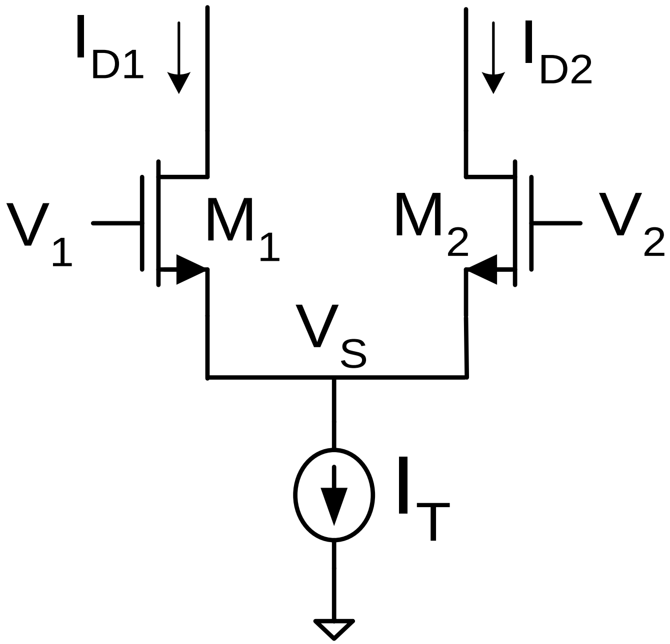


MOS Differential Pair



Bipolar Differential Pair

MOS Differential Pair



$$I_{D1} = \frac{\mu C_{OX} W}{2L}\left(V_1 - V_S - V_T\right)^2$$

$$I_{D2} = \frac{\mu C_{OX} W}{2L}\left(V_2 - V_S - V_T\right)^2$$

$$I_{D1} + I_{D2} = I_T$$

$$\sqrt{I_{D1}}\sqrt{\frac{2L}{\mu C_{OX} W}} = V_1 - V_S - V_T$$

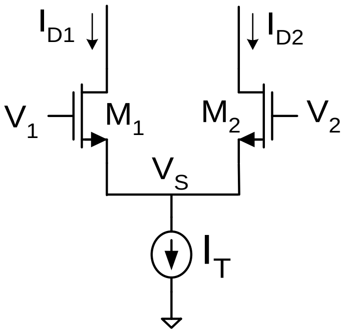
$$\sqrt{I_{D2}}\sqrt{\frac{2L}{\mu C_{OX} W}} = V_2 - V_S - V_T$$

$$V_d = V_2 - V_1$$

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right)$$

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$

MOS Differential Pair



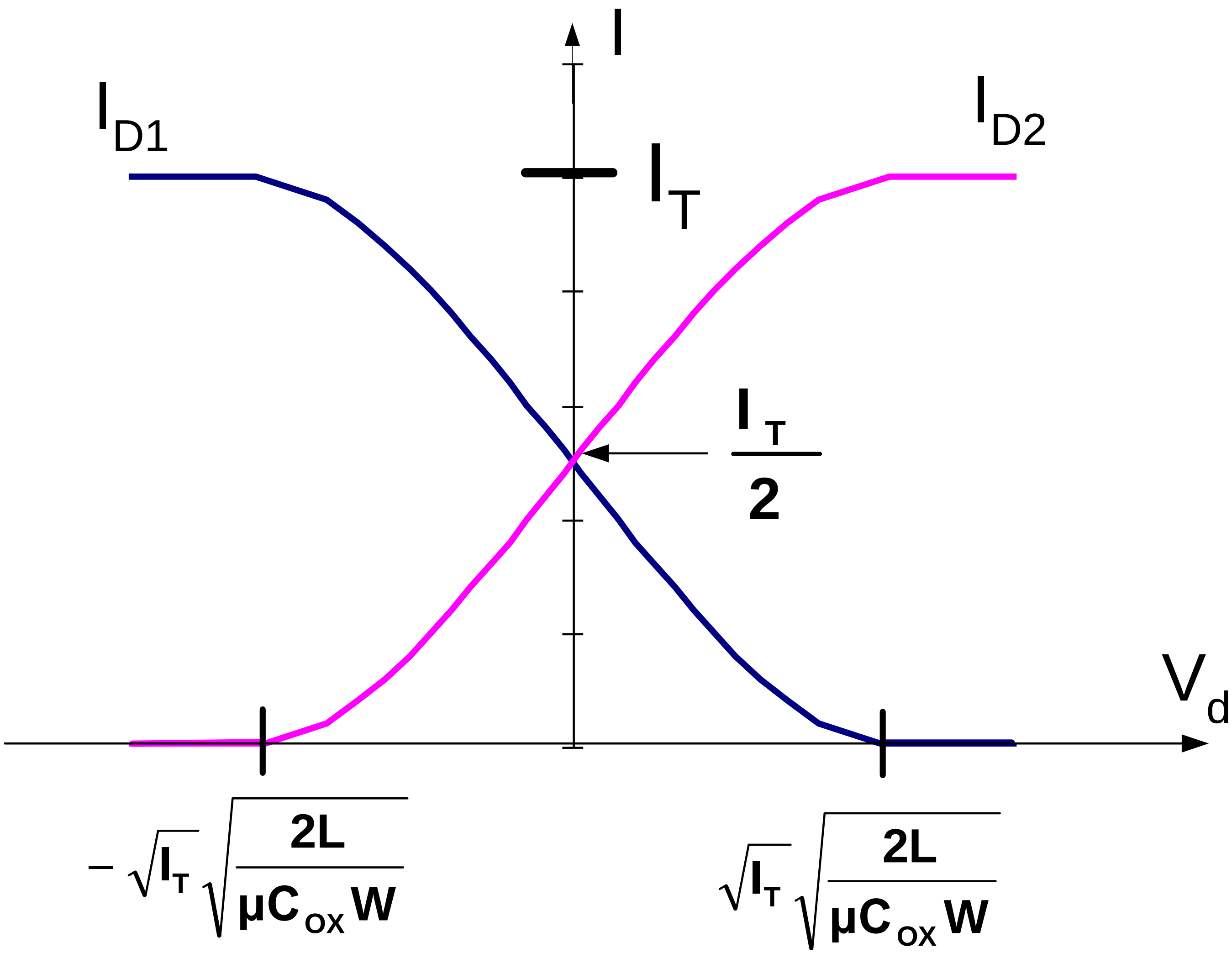
$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right)$$

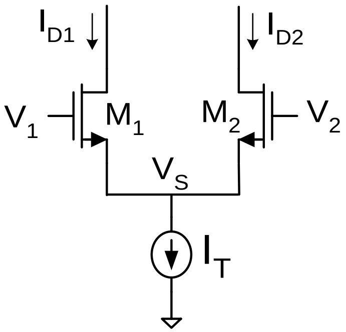
$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$

What values of V_d will cause all of the current to be steered to the left or the right ?

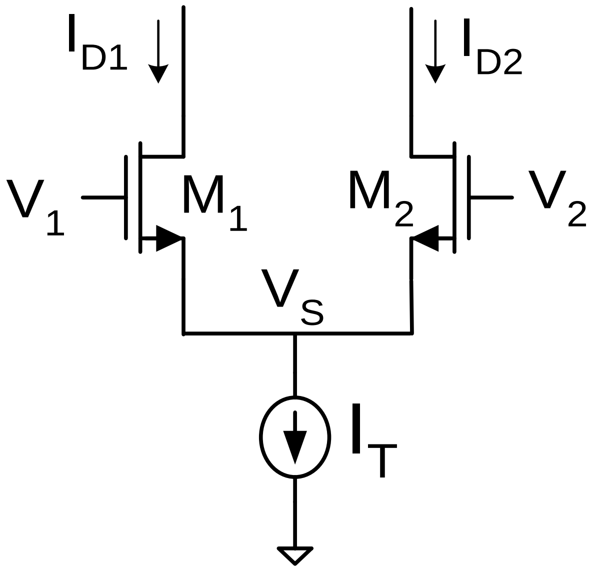
$$V_{dx} = \pm\sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T}\right)$$

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$





Q-point Calculations



$$\frac{I_T}{2} = \frac{\mu C_{OX} W}{2L}\left(V_{EB}\right)^2$$

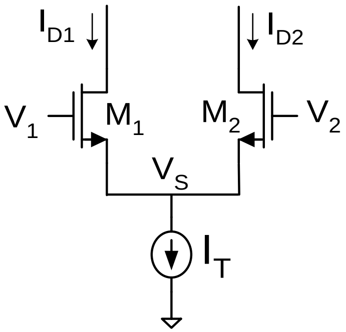
$$V_{EB} = \sqrt{I_T}\sqrt{\frac{L}{\mu C_{OX} W}}$$

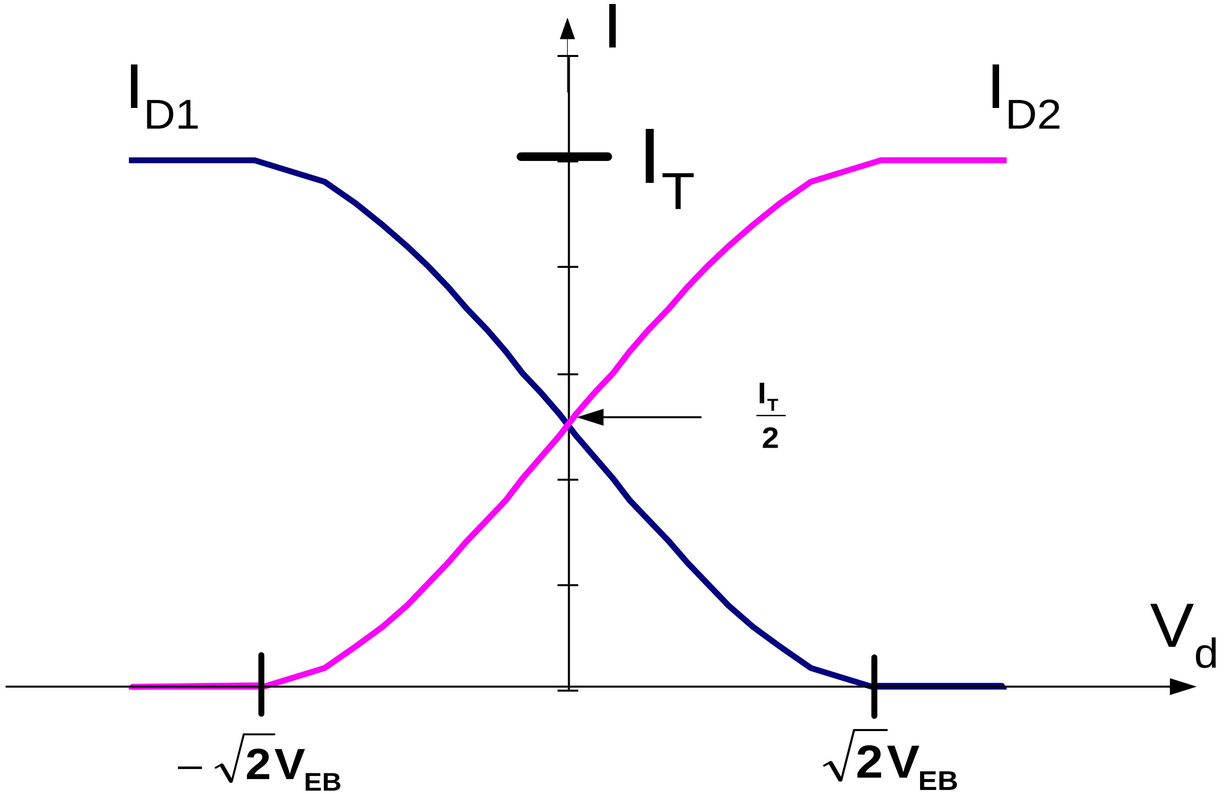
$$V_{dx} = \pm\sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T}\right)$$

Observe !!

$$V_{dx} = \pm\sqrt{2}V_{EB}$$

$$V_d = \sqrt{\frac{2L}{\mu C_{ox} W}} \left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}} \right)$$

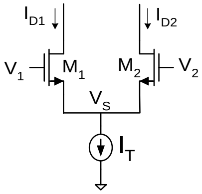


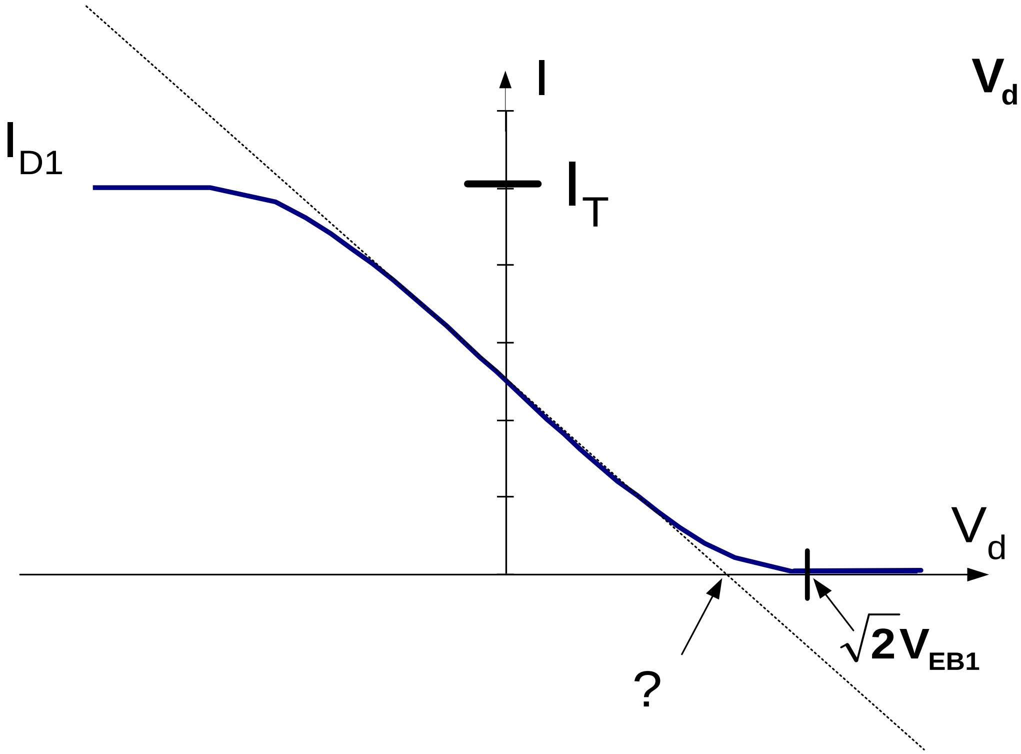


V_{EB} affects linearity

How linear is the amplifier ?

How linear is the amplifier ?





$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right)$$

Consider the fit line:

$$I = mV_d + h$$

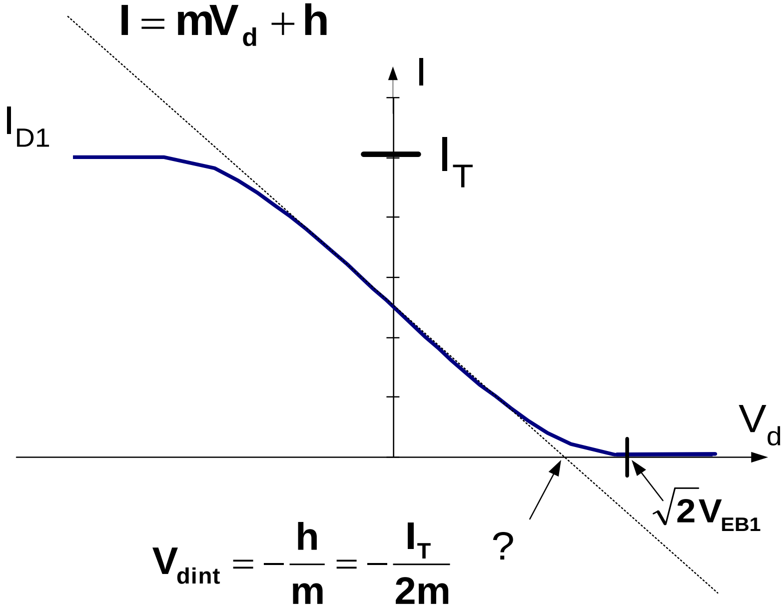
When V_d=0, I=I_T/2, thus

$$h = \frac{I_T}{2}$$

$$V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m}$$

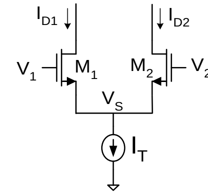
$$m = \left.\frac{\partial I_{D1}}{\partial V_d}\right|_{Q-pt}$$

How linear is the amplifier ?



$$I = mV_d + h$$

$$V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m}$$



$$m = \left.\frac{\partial I_{D1}}{\partial V_d}\right|_{Q-pt}$$

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_T - I_{D1}} - \sqrt{I_{D1}}\right)$$

$$\frac{\partial V_d}{\partial I_{D1}} = \left.\sqrt{\frac{2L}{\mu C_{OX} W}}\left(\frac{1}{2}(I_T - I_{D1})^{-1/2}(-1) - \frac{1}{2}(I_{D1})^{-1/2}\right)\right|_{Q-point}$$

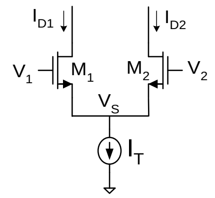
$$\frac{\partial V_d}{\partial I_{D1}} = -2\sqrt{\frac{L}{\mu C_{OX} W}}\sqrt{\frac{1}{I_T}}$$

$$\sqrt{\frac{L}{\mu C_{OX} W}} = \frac{V_{EB1}}{\sqrt{I_T}}$$

$$\frac{\partial V_d}{\partial I_{D1}} = -2\frac{V_{EB1}}{I_T}$$

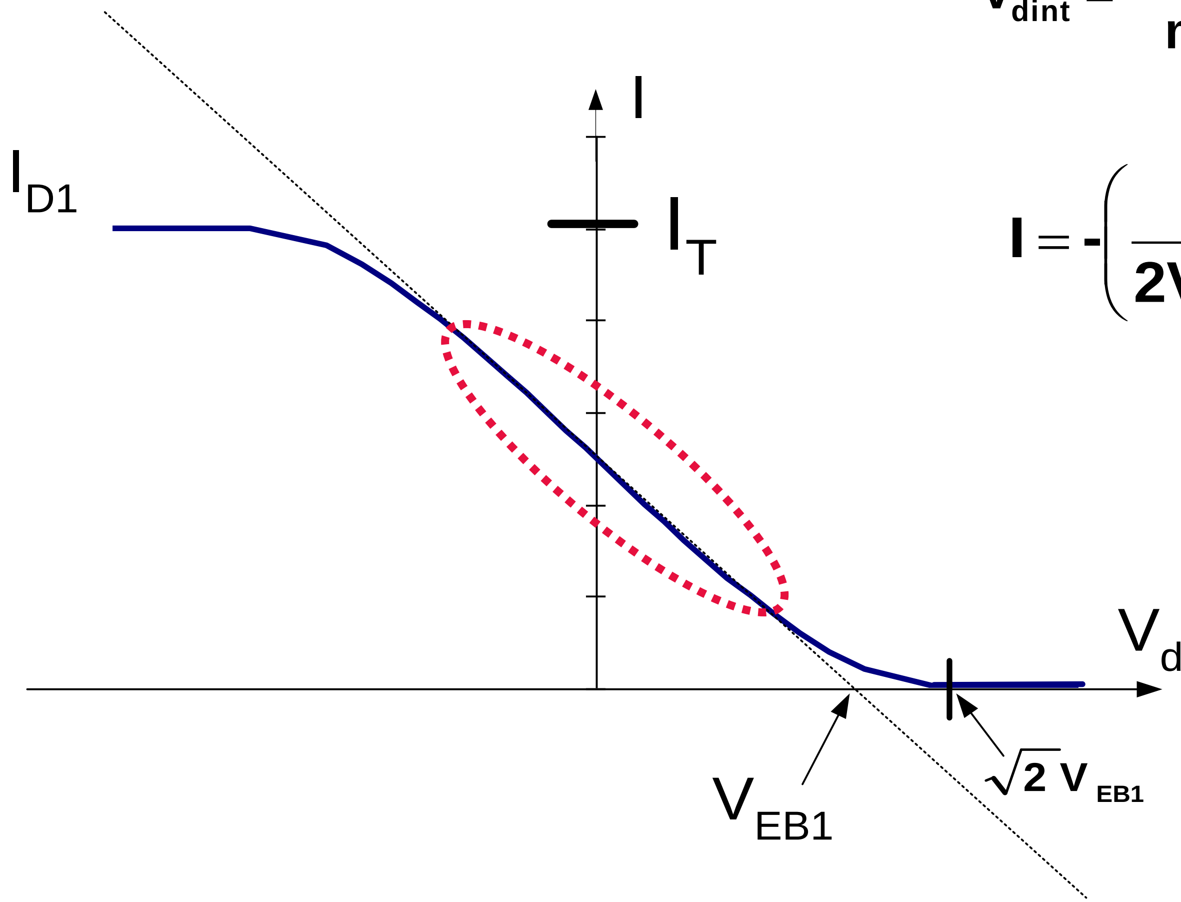
$$m = \left.\frac{\partial I_{D1}}{\partial V_d}\right|_{Q-pt} = -\frac{I_T}{2V_{EB1}}$$

How linear is the amplifier ?

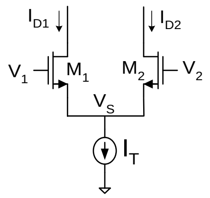


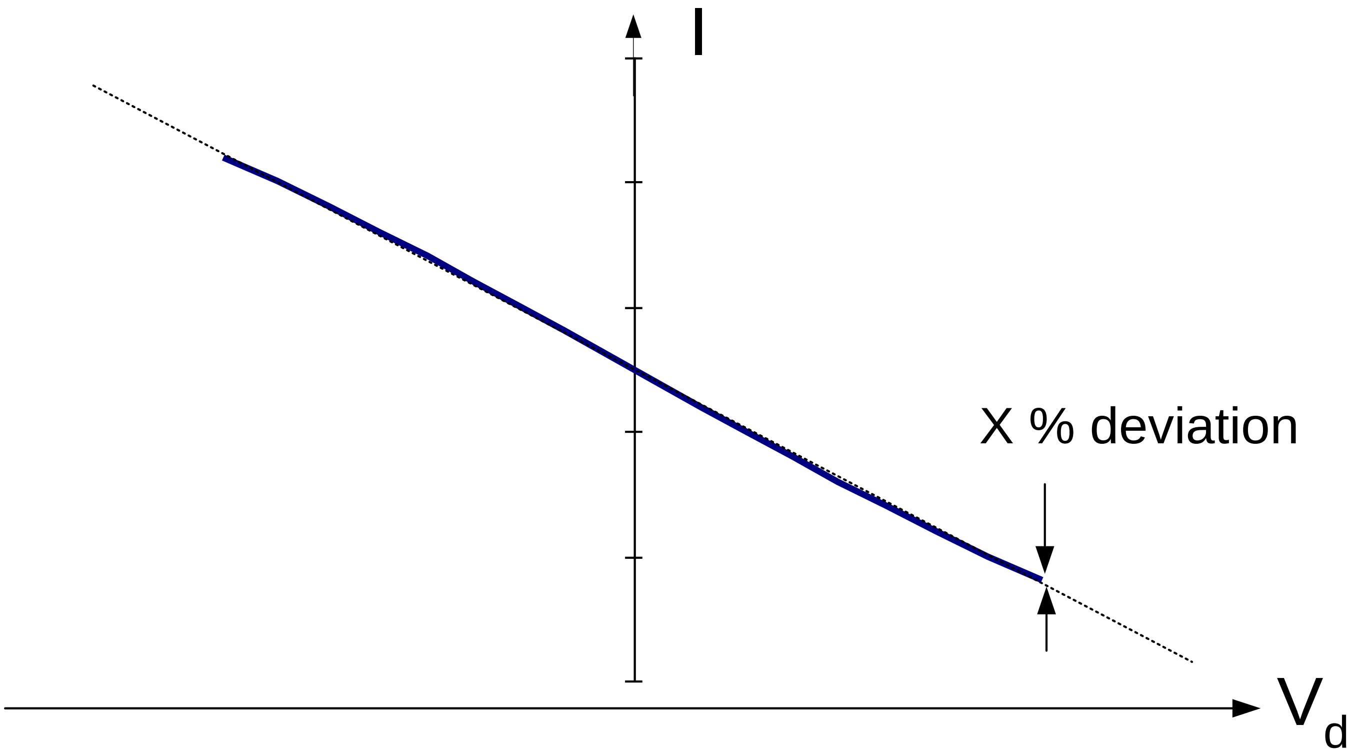
$$V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m} = V_{EB1}$$

$$I = -\left(\frac{I_T}{2V_{EB1}}\right)V_d + \frac{I_T}{2}$$



How linear is the amplifier ?

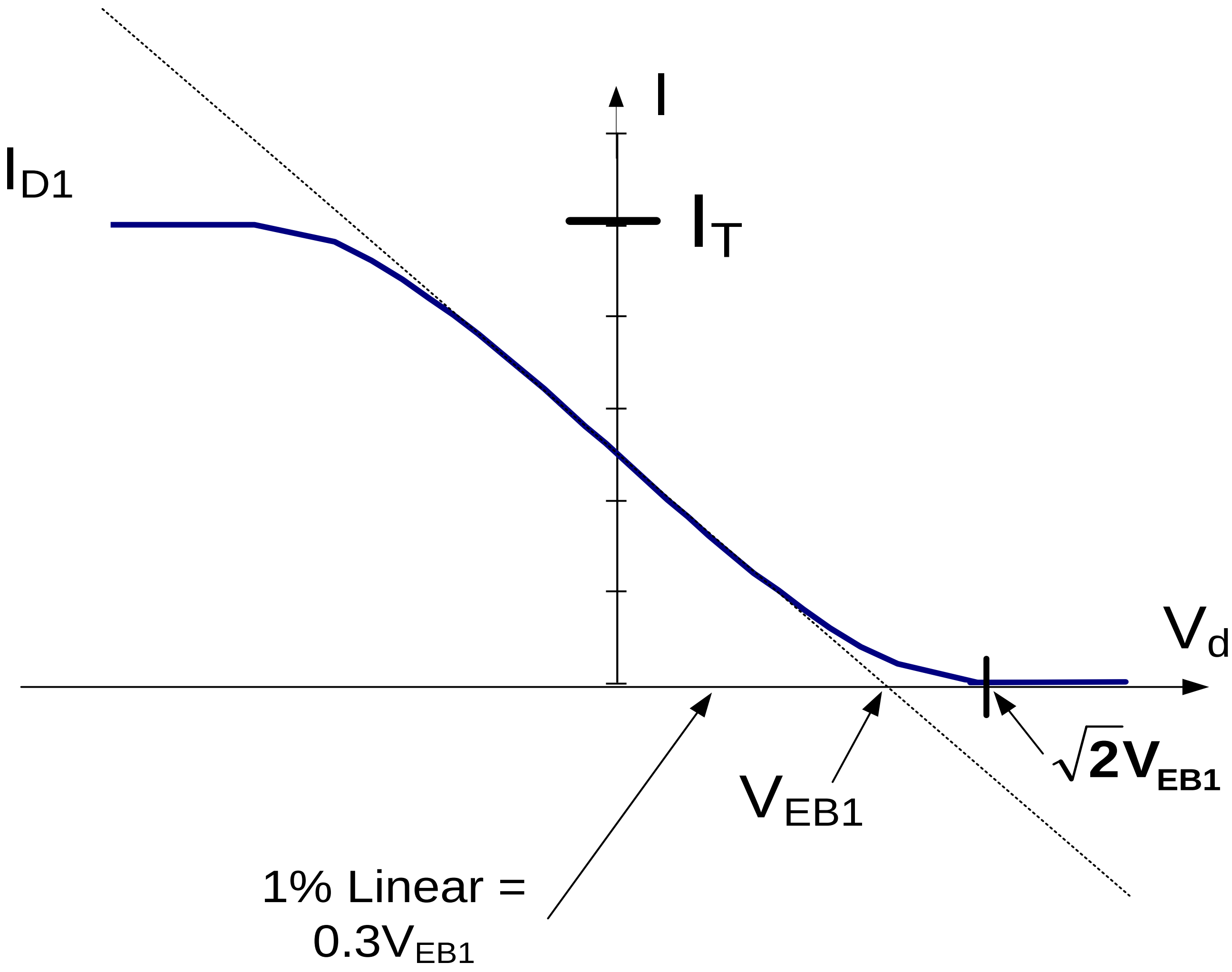


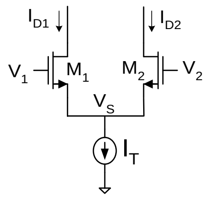


It can be shown that a 1% deviation from the straight line occurs at

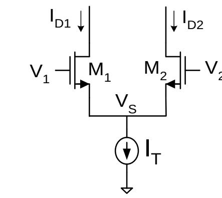
$V_d \cong \frac{V_{EB}}{3}$ and a 0.1% variation occurs at $V_d \cong \frac{V_{EB}}{10}$

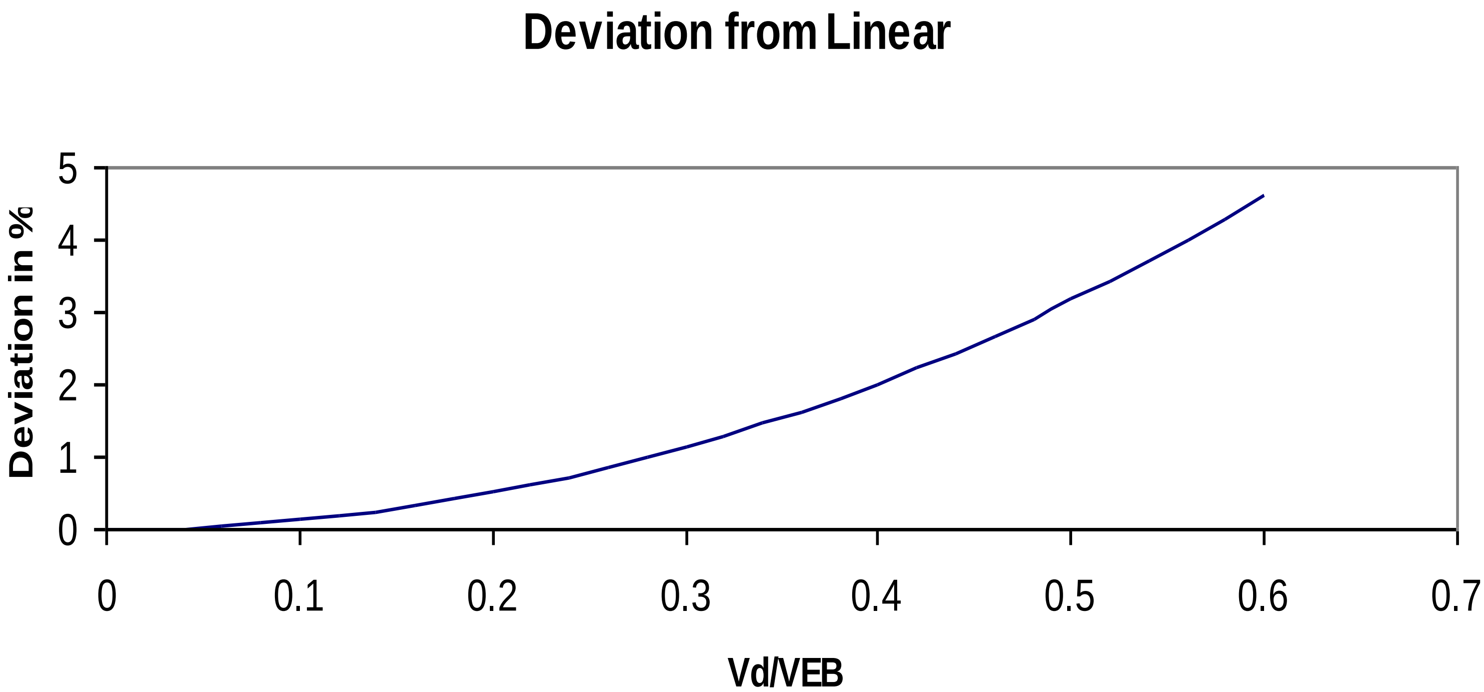
How linear is the amplifier ?





How linear is the amplifier ?





Vd/VEB	θ	Vd/VEB	θ	Vd/VEB	θ
0.02	0.005	0.22	0.607	0.42	2.23
0.04	0.020	0.24	0.723	0.44	2.45
0.06	0.045	0.26	0.849	0.46	2.68
0.08	0.080	0.28	0.985	0.48	2.92
0.1	0.125	0.3	1.13	0.5	3.18
0.12	0.180	0.32	1.29	0.52	3.44
0.14	0.245	0.34	1.46	0.54	3.71
0.16	0.321	0.36	1.63	0.56	4.00
0.18	0.406	0.38	1.82	0.58	4.30
0.2	0.501	0.4	2.02	0.6	4.61

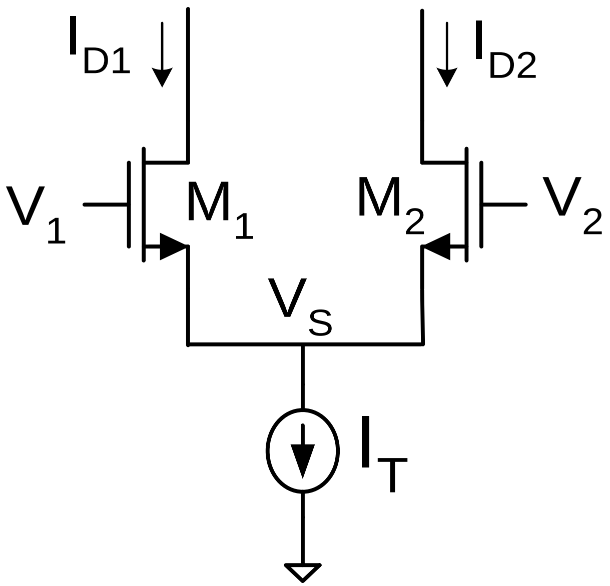
How linear is the amplifier ?

Distortion in the differential pair is another useful metric for characterizing linearity of I_{D1} and I_{D2} with sinusoidal differential excitation

Consider again the differential pair and assume excited differentially with

$$V_2 = \frac{V_d}{2} \qquad V_1 = -\frac{V_d}{2}$$

and assume $V_d = V_m \sin(\omega t)$



$$\mathbf{V_d = V_2 - V_1}$$

Recall:

$$V_d = \sqrt{\frac{2L}{\mu C_{OX} W}}\left(\sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}\right)$$

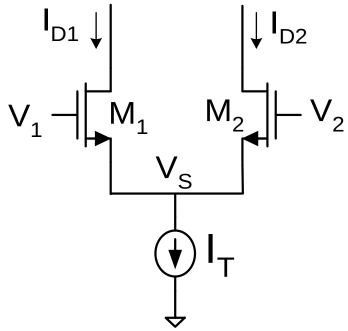
Define

$$\theta = \frac{\mu C_{OX} W}{2L}$$

Thus can express as

$$\sqrt{\theta} V_d = \sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}$$

How linear is the amplifier ?



and assume $V_d = V_m \sin(\omega t)$ $\qquad \theta = \dfrac{\mu C_{OX} W}{2L}$

$$\sqrt{\theta} V_d = \sqrt{I_{D2}} - \sqrt{I_T - I_{D2}}$$

Squaring, regrouping, and squaring we obtain

$$\theta V_d^2 = I_{D2} + (I_T - I_{D2}) - 2\sqrt{I_{D2}}\sqrt{I_T - I_{D2}}$$

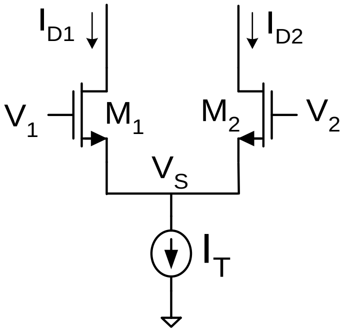
$$\theta V_d^2 = I_T - 2\sqrt{I_{D2}}\sqrt{I_T - I_{D2}}$$

$$\left(\theta V_d^2 - I_T\right)^2 = 4 I_{D2}\left(I_T - I_{D2}\right)$$

This latter equation can be expressed as a second-order polynomial in I_{D2} as

$$I_{D2}^2 - I_{D2} I_T + \left(\frac{\theta V_d^2 - I_T}{2}\right)^2 = 0$$

How linear is the amplifier ?



and assume $V_d = V_m \sin(\omega t)$ $\qquad \theta = \dfrac{\mu C_{OX} W}{2L}$

$$I_{D2}^2 - I_{D2} I_T + \left(\frac{\theta V_d^2 - I_T}{2} \right)^2 = 0$$

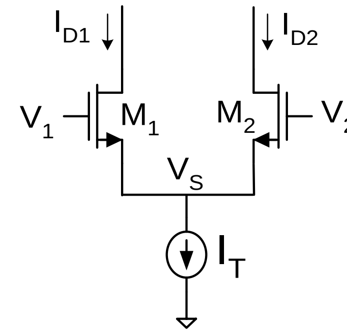
Solving, we obtain

$$I_{D2} = \frac{I_T}{2} + \sqrt{\left(\frac{I_T}{2}\right)^2 - \left(\frac{\theta V_d^2 - I_T}{2}\right)^2}$$

$$I_{D2} = \frac{I_T}{2} + \sqrt{\left(\frac{I_T}{2}\right)^2 - \left(\frac{\theta V_d^2}{2}\right)^2 - \left(\frac{I_T}{2}\right)^2 + \frac{\theta I_T}{2} V_d^2}$$

$$I_{D2} = \frac{I_T}{2} + \sqrt{\frac{\theta I_T}{2} V_d^2 - \left(\frac{\theta V_d^2}{2}\right)^2}$$

How linear is the amplifier ?



and assume $V_d = V_m \sin(\omega t)$ $\theta = \dfrac{\mu C_{OX} W}{2L}$

$$I_{D2} = \frac{I_T}{2} + \sqrt{\frac{\theta I_T}{2} V_d^2 - \left(\frac{\theta V_d^2}{2} \right)^2}$$

This can be expressed as

$$I_{D2} = \frac{I_T}{2} + V_d \sqrt{\frac{\theta I_T}{2}} \sqrt{1 - V_d^2 \frac{\theta}{2 I_T}}$$

$$\sqrt{1-x} \cong 1 - \frac{x}{2} - \frac{x^2}{8} + \ldots$$

Using a Truncated Taylor's series, we obtain:

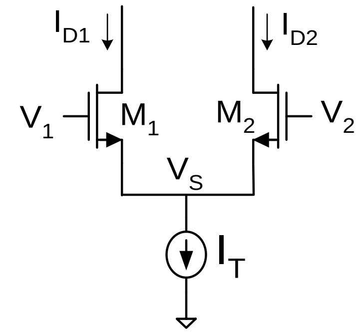
$$I_{D2} \simeq \frac{I_T}{2} + V_d \sqrt{\frac{\theta I_T}{2}} \left(1 - V_d^2 \frac{\theta}{4 I_T} \right)$$

Note this has no second-order term thus the dominant distortion when $V_d = V_m \sin(\omega t)$ will be a third-order harmonic

How linear is the amplifier ?

$$I_{D2} \simeq \frac{I_T}{2} + V_d \sqrt{\frac{\theta I_T}{2}} \left(1 - V_d^2 \frac{\theta}{4 I_T} \right)$$

$$\theta = \frac{\mu C_{OX} W}{2L}$$



Substituting in $V_d = V_m \sin(\omega t)$

$$I_{D2} \simeq \frac{I_T}{2} + V_m \sin(\omega t) \sqrt{\frac{\theta I_T}{2}} \left(1 - V_m^2 \sin^2(\omega t) \frac{\theta}{4 I_T} \right)$$

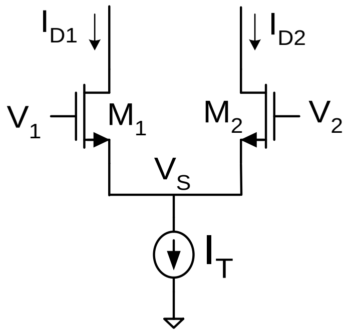
$$I_{D2} \simeq \frac{I_T}{2} + \left[V_m \sqrt{\frac{\theta I_T}{2}} \right] \sin(\omega t) - \left[V_m^3 \frac{\theta^{\frac{3}{2}}}{4\sqrt{2}\sqrt{I_T}} \right] \sin^3(\omega t)$$

$$\sin^3(\omega t) = \frac{3}{4} \sin(\omega t) - \frac{1}{4} \sin(3\omega t)$$

$$I_{D2} \simeq \frac{I_T}{2} + \left[V_m \sqrt{\frac{\theta I_T}{2}} \right] \sin(\omega t) - \left[V_m^3 \frac{\theta^{\frac{3}{2}}}{4\sqrt{2}\sqrt{I_T}} \right] \left[\frac{3}{4} \sin(\omega t) - \frac{1}{4} \sin(3\omega t) \right]$$

$$I_{D2} \simeq \frac{I_T}{2} + \left[V_m \sqrt{\frac{\theta I_T}{2}} - V_m^3 \frac{3\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} \right] \sin(\omega t) + \left[V_m^3 \frac{\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} \right] \left[\sin(3\omega t) \right]$$

How linear is the amplifier ?



$$I_{D2} \simeq \frac{I_T}{2} + V_d \sqrt{\frac{\theta I_T}{2}} \left(1 - V_d^2 \frac{\theta}{4 I_T} \right)$$

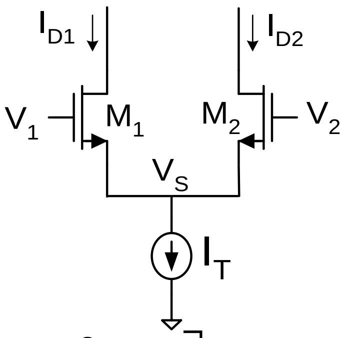
$$\theta = \frac{\mu C_{OX} W}{2L}$$

$$I_{D2} \simeq \frac{I_T}{2} + \left[V_m \sqrt{\frac{\theta I_T}{2}} - V_m^3 \frac{3\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} \right] \sin(\omega t) + \left[V_m^3 \frac{\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} \right] \left[\sin(3\omega t) \right]$$

$$I_{D2} \simeq a_0 + a_1 \sin(\omega t) + a_3 (3\omega t)$$

$$a_1 = \left[V_m \sqrt{\frac{\theta I_T}{2}} - V_m^3 \frac{3\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} \right] \qquad a_3 = \left[\frac{\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} \right] V_m^3$$

How linear is the amplifier ?



$$I_{D2} \simeq a_0 + a_1 \sin(\omega t) + a_3 \sin(3\omega t)$$

$$a_1 = \left[V_m \sqrt{\frac{\theta I_T}{2}} - V_m^3 \frac{3\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} \right] \qquad a_3 = \left[\frac{\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} \right] V_m^3$$

$$\text{THD} = 20\log\left(\frac{\sqrt{\sum_{k=2}^{\infty} a_k^2}}{a_1} \right)$$

Substituting in we obtain

$$\text{THD} = 20\log\left(\frac{\dfrac{\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}} V_m^3}{V_m \sqrt{\dfrac{\theta I_T}{2}} - V_m^3 \dfrac{3\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}}} \right)$$

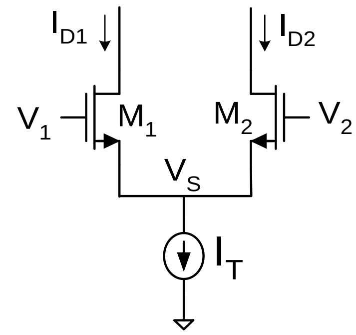
where $\theta = \dfrac{\mu C_{OX} W}{2L}$

This expression gives little insight.

Consider expression in the practical parameter domain:

$$I_T = \frac{\mu C_{OX} W}{L} V_{EB1}^2$$

How linear is the amplifier ?



$$I_{D2} \simeq a_0 + a_1 \sin(\omega t) + a_3 \sin(3\omega t)$$

$$THD = 20\log\left(\frac{\frac{\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}}V_m^3}{V_m\sqrt{\frac{\theta I_T}{2}} - V_m^3\frac{3\theta^{\frac{3}{2}}}{16\sqrt{2}\sqrt{I_T}}}\right)$$

$$\theta = \frac{\mu C_{OX} W}{2L}$$

$$I_T = \frac{\mu C_{OX} W}{L} V_{EB1}^2$$

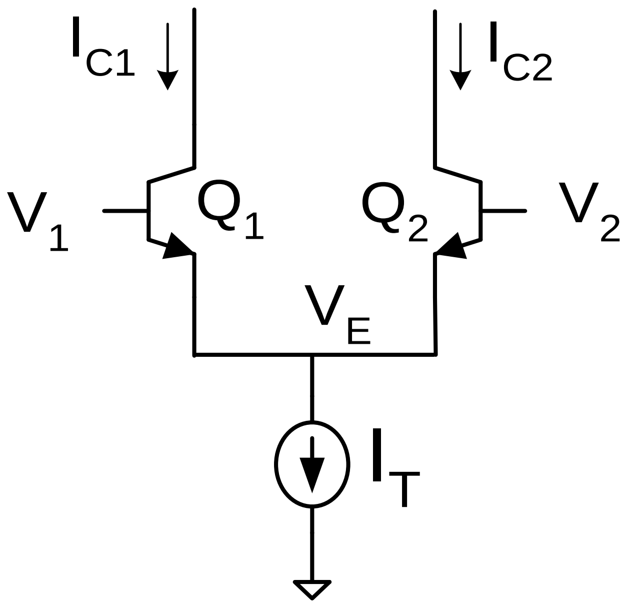
Eliminating I_T and θ, we obtain

$$THD = -20\log\left(32\left(\frac{V_{EB1}}{V_m}\right)^2 - 3\right)$$

V_m / V_{EB1}	THD (dB)
2.5	-6.52672
1	-29.248
0.5	-41.9382
0.25	-54.1344
0.1	-70.0949
0.05	-82.1422
0.025	-94.1849
0.01	-110.103

Thus to minimize THD, want V_{EB} large and V_M small

Bipolar Differential Pair



$$\left.\begin{aligned} I_{C1} &= J_S A_{E1} e^{\frac{V_1 - V_E}{V_t}} \\ I_{C2} &= J_S A_{E2} e^{\frac{V_2 - V_E}{V_t}} \\ I_{C1} + I_{C2} &= I_T \end{aligned}\right\}$$

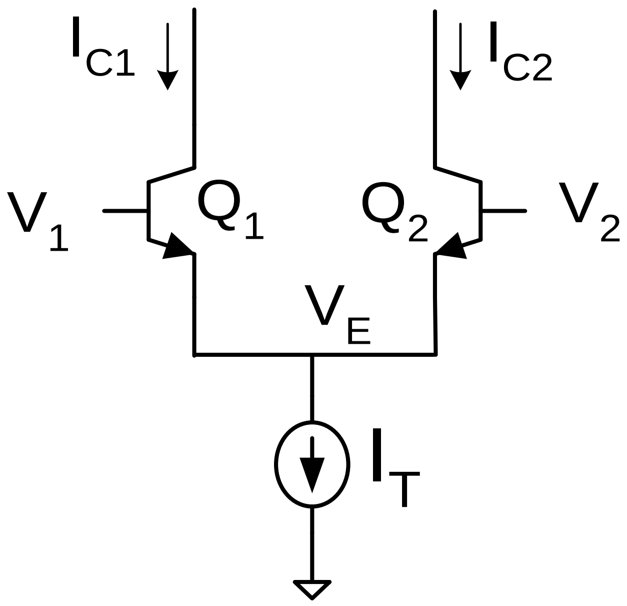
$$V_1 = V_E + V_t \ln\left(\frac{I_{C1}}{J_S A_{E1}}\right)$$

$$V_2 = V_E + V_t \ln\left(\frac{I_{C2}}{J_S A_{E2}}\right)$$

$$V_d = V_2 - V_1$$

$$V_d = V_t\left(\ln\left(\frac{I_{C2}}{J_S A_{E2}}\right) - \ln\left(\frac{I_{C1}}{J_S A_{E1}}\right)\right) \overset{A_{E1}=A_{E2}}{=} V_t \ln\left(\frac{I_{C2}}{I_{C1}}\right)$$

Bipolar Differential Pair



$$V_d = V_2 - V_1$$

$$V_d = V_t\left(\ln\left(\frac{I_{C2}}{J_S A_{E2}}\right) - \ln\left(\frac{I_{C1}}{J_S A_{E1}}\right)\right) \overset{A_{E1}=A_{E2}}{=} V_t \ln\left(\frac{I_{C2}}{I_{C1}}\right)$$

$$V_d = V_t \ln\left(\frac{I_T - I_{C1}}{I_{C1}}\right)$$

$$V_d = V_t \ln\left(\frac{I_{C2}}{I_T - I_{C2}}\right)$$

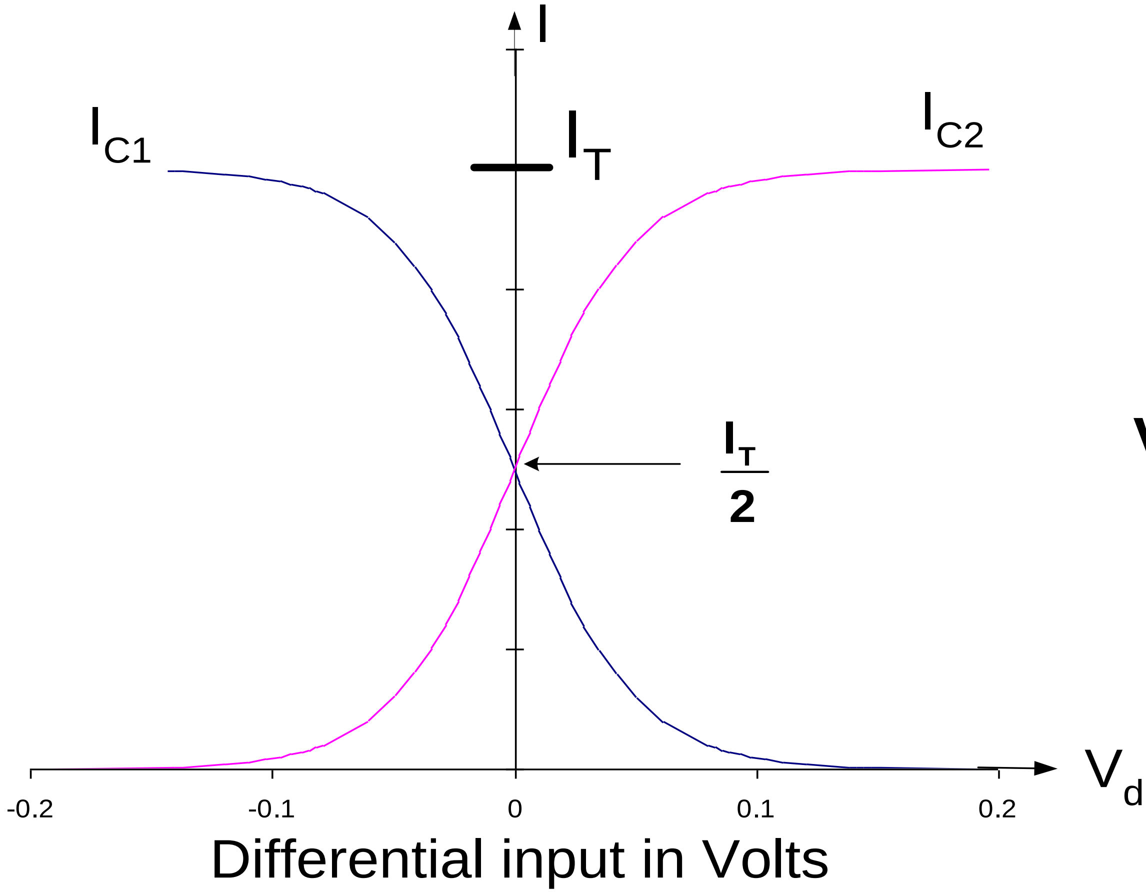
At $I_{C1}=I_{C2}=I_T/2$, $V_d=0$

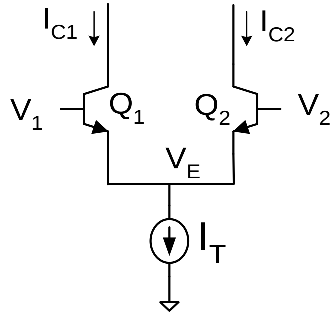
As I_{C1} approaches 0, V_d approaches infinity

As I_{C1} approaches I_T, V_d approaches minus infinity

Transition much steeper than for MOS case

Transfer Characteristics of Bipolar Differential Pair



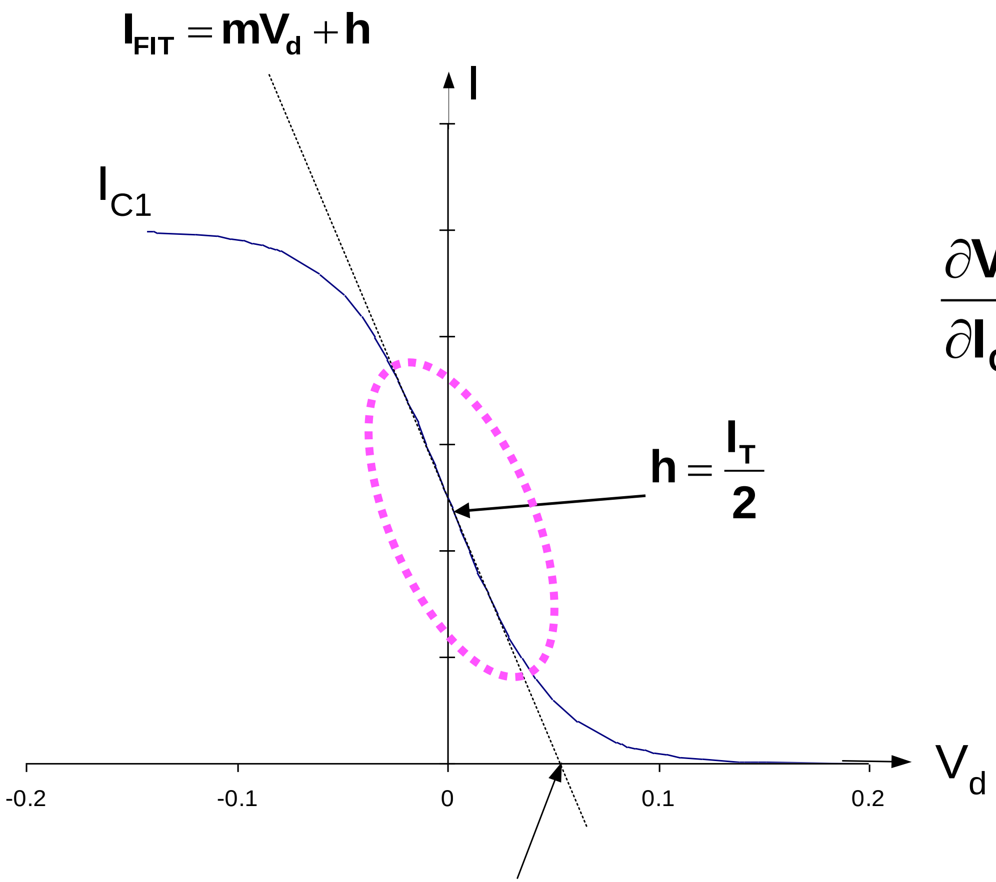


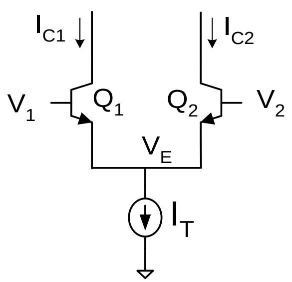
$$V_d = V_t \ln\left(\frac{I_T - I_{C1}}{I_{C1}}\right)$$

Transition much steeper than for MOS case

Asymptotic Convergence to 0 and I_T

Signal Swing and Linearity of Bipolar Differential Pair





$$V_{dint} = -\frac{h}{m} = ?$$

$$m = \left.\frac{\partial I_{C1}}{\partial V_d}\right|_{Q-point}$$

$$\left.\frac{\partial V_d}{\partial I_{C1}}\right|_{Q-point} = -V_t \left.\frac{I_T}{I_{C1}(I_T - I_{C1})}\right|_{I_{C1}=\frac{I_T}{2}}$$

$$\left.\frac{\partial V_d}{\partial I_{C1}}\right|_{Q-point} = -\frac{4V_t}{I_T}$$

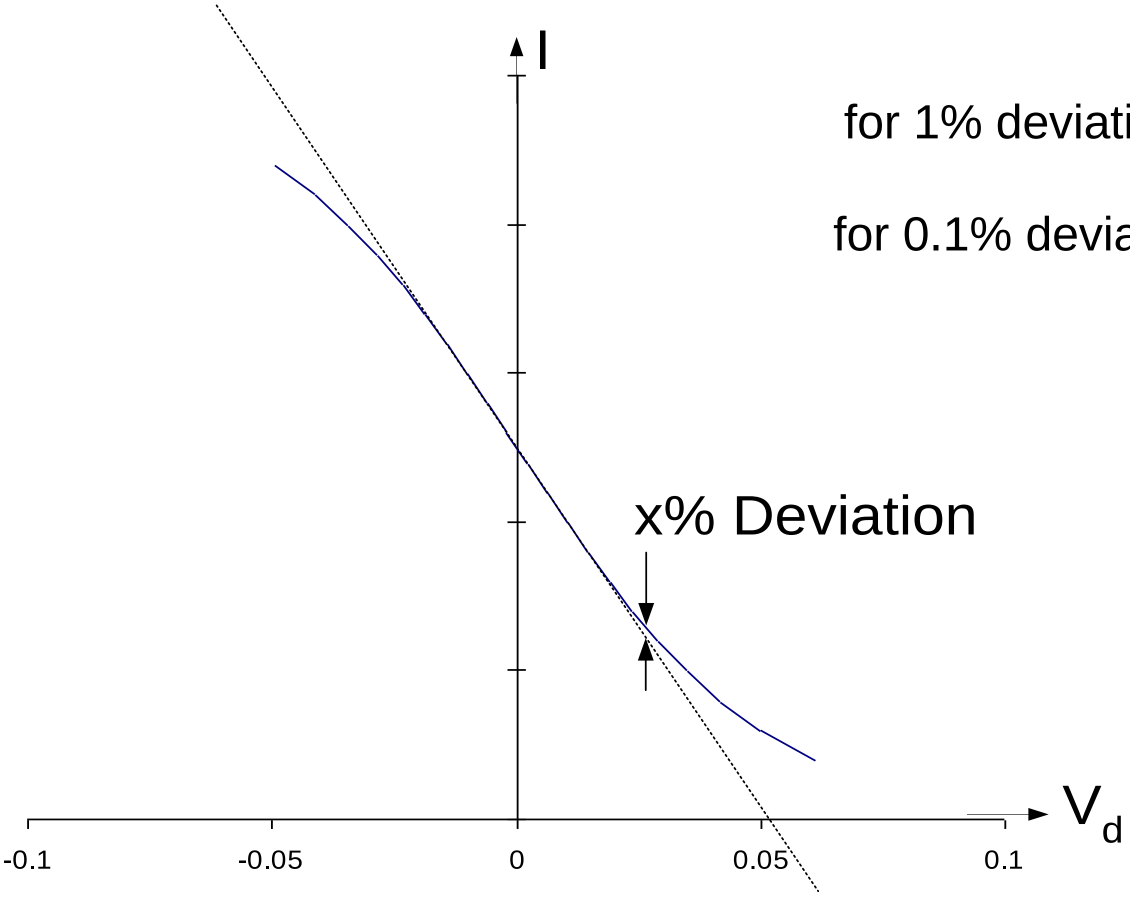
$$I_{FIT} = -\frac{I_T}{4V_t} V_d + \frac{I_T}{2}$$

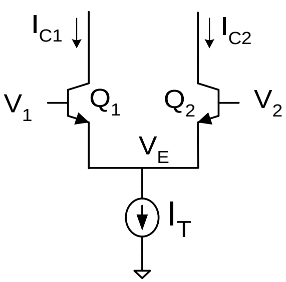
$$V_{dint} = -\frac{h}{m} = 2V_t$$

Signal Swing and Linearity of Bipolar Differential Pair

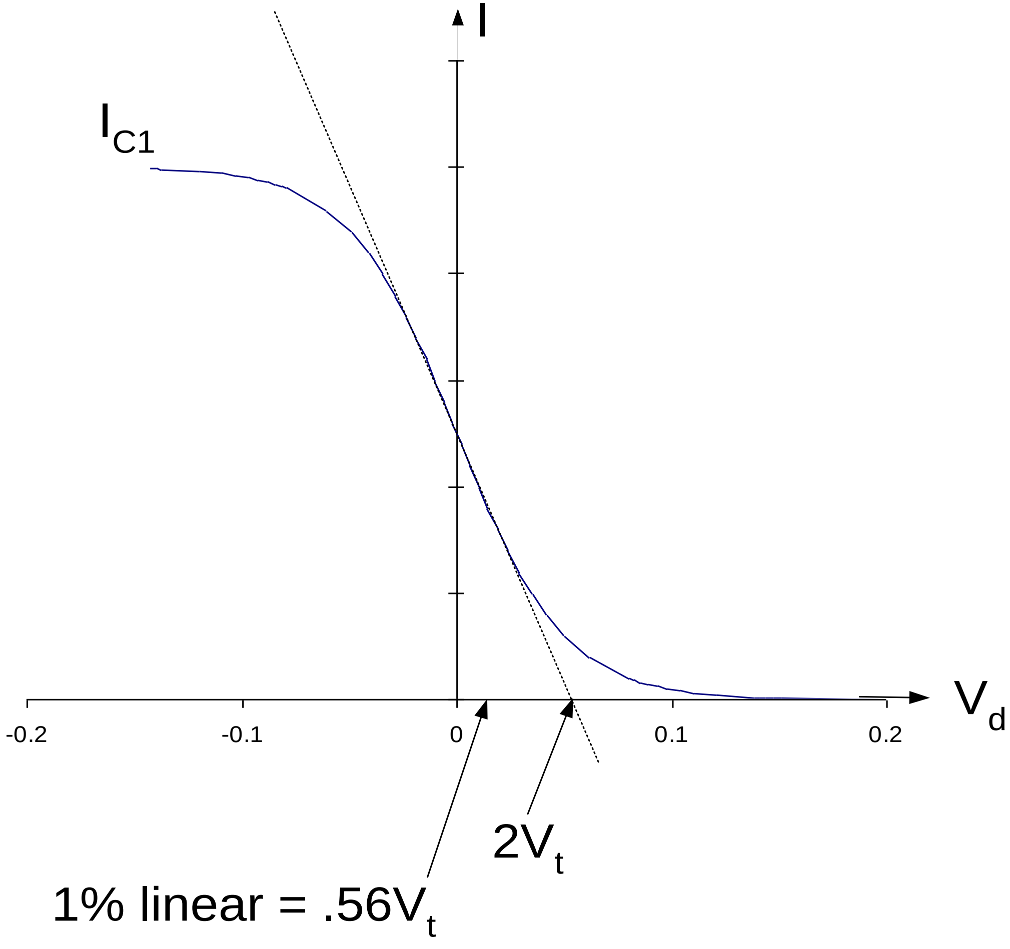
for 1% deviation, V_d=.56V_t

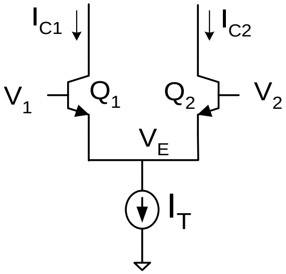
for 0.1% deviation, V_d=.27V_t





Signal Swing and Linearity of Bipolar Differential Pair





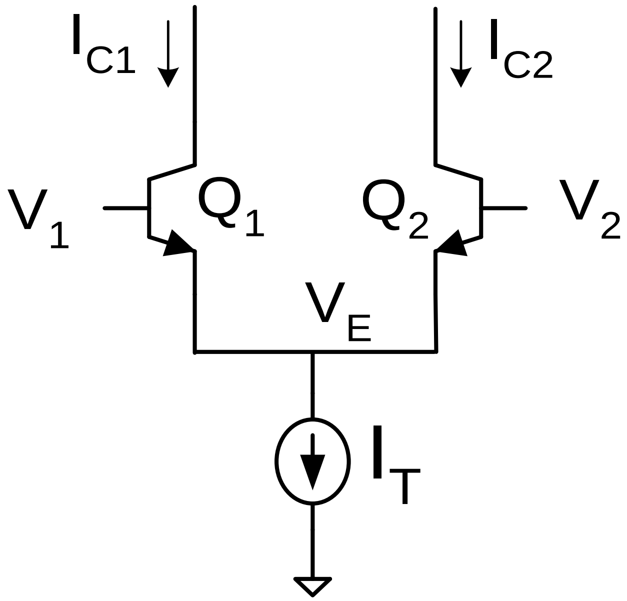
How linear is the amplifier ?

Distortion in the differential pair is another useful metric for characterizing linearity of I_{C1} and I_{c2} with sinusoidal differential excitation

Consider again the differential pair and assume excited differentially with

$$V_2 = \frac{V_d}{2} \qquad V_1 = -\frac{V_d}{2}$$

and assume $V_d = V_m \sin(\omega t)$



$$\mathbf{V_d = V_2 - V_1}$$

Recall:

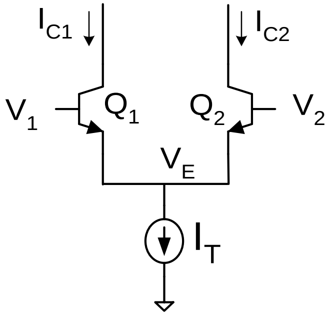
$$V_d = V_t \ln\left(\frac{I_T - I_{C1}}{I_{C1}}\right)$$

Thus can express as

$$e^{\frac{V_d}{V_t}} = \frac{I_T - I_{C1}}{I_{C1}}$$

$$I_{C1} = I_T\left(1 + e^{\frac{V_d}{V_t}}\right)^{-1}$$

How linear is the amplifier ?



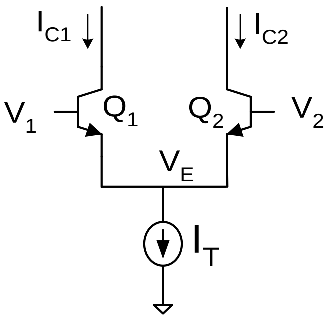
$$I_{C1} = I_T \left(1 + e^{\frac{V_d}{V_t}} \right)^{-1}$$

$V_d = V_m \sin(\omega t)$

Consider a Taylor's Series Expansion

$$I_{C1} = I_{C1}\Big|_{V_d=0} + \frac{\partial I_{C1}}{\partial V_d}\Bigg|_{V_d=0} V_d + \frac{1}{2!}\frac{\partial^2 I_{C1}}{\partial V_d^2}\Bigg|_{V_d=0} V_d^2 + \frac{1}{3!}\frac{\partial^3 I_{C1}}{\partial V_d^3}\Bigg|_{V_d=0} V_d^3 \quad + H.O.T$$

How linear is the amplifier ?



$V_d = V_m \sin(\omega t)$

$$I_{C1} = I_{C1}\Big|_{V_d=0} + \frac{\partial I_{C1}}{\partial V_d}\Big|_{V_d=0} V_d + \frac{1}{2!}\frac{\partial^2 I_{C1}}{\partial V_d^2}\Big|_{V_d=0} V_d^2 + \frac{1}{3!}\frac{\partial^3 I_{C1}}{\partial V_d^3}\Big|_{V_d=0} V_d^3 \quad + H.O.T$$

$$\frac{\partial I_{C1}}{\partial V_d} = -\frac{I_T}{V_t}\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}}$$

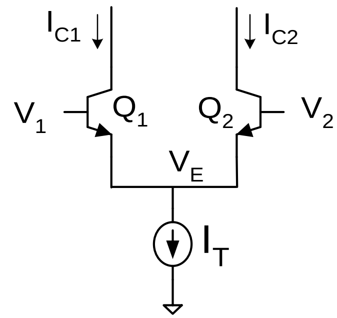
$$\frac{\partial^2 I_{C1}}{\partial V_d^2} = -\frac{I_T}{V_t}\left[\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}}\frac{1}{V_t} - 2\, e^{\frac{V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3} e^{\frac{V_d}{V_t}}\frac{1}{V_t}\right]$$

$$\frac{\partial^2 I_{C1}}{\partial V_d^2} = -\frac{I_T}{V_t^2}\left[\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}} - 2\, e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3}\right]$$

$$\frac{\partial^3 I_{C1}}{\partial V_d^3} = -\frac{I_T}{V_t^2}\left[\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}}\frac{1}{V_t} - 2e^{\frac{V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3} e^{\frac{V_d}{V_t}}\frac{1}{V_t} + 6\, e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-4} e^{\frac{V_d}{V_t}}\frac{1}{V_t} - 2e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3}\frac{2}{V_t}\right]$$

$$\frac{\partial^3 I_{C1}}{\partial V_d^3} = -\frac{I_T}{V_t^3}\left[\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}} - 2e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3} + 6e^{\frac{3V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-4} - 4e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3}\right]$$

How linear is the amplifier ?



$V_d = V_m \sin(\omega t)$

$$I_{C1} = I_{C1}\Big|_{V_d=0} + \frac{\partial I_{C1}}{\partial V_d}\Big|_{V_d=0} V_d + \frac{1}{2!}\frac{\partial^2 I_{C1}}{\partial V_d^2}\Big|_{V_d=0} V_d^2 + \frac{1}{3!}\frac{\partial^3 I_{C1}}{\partial V_d^3}\Big|_{V_d=0} V_d^3 + H.O.T$$

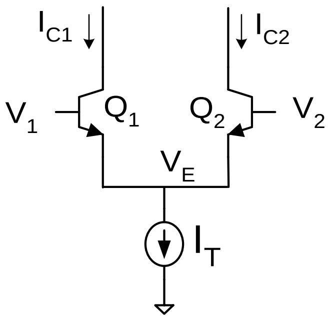
$$\frac{\partial I_{C1}}{\partial V_d}\Big|_{V_d=0} = -\frac{I_T}{V_t}\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}}\Big|_{V_d==0} = -\frac{I_T}{V_t}(2)^{-2} = -\frac{I_T}{4V_t}$$

$$\frac{\partial^2 I_{C1}}{\partial V_d^2}\Big|_{V_d=0} = -\frac{I_T}{V_t^2}\left[\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}} - 2\, e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3}\right]\Big|_{V_d=0} = -\frac{I_T}{V_t^2}\left[(2)^{-2} - 2(2)^{-3}\right] = 0$$

$$\frac{\partial^3 I_{C1}}{\partial V_d^3}\Big|_{V_d=0} = -\frac{I_T}{V_t^3}\left[\left(1+e^{\frac{V_d}{V_t}}\right)^{-2} e^{\frac{V_d}{V_t}} - 2e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3} + 6e^{\frac{3V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-4} - 4e^{\frac{2V_d}{V_t}}\left(1+e^{\frac{V_d}{V_t}}\right)^{-3}\right]\Big|_{V_d=0} = -\frac{I_T}{V_t^3}\left[(2)^{-2} - 2(2)^{-3} + 6(2)^{-4} - 4(2)^{-3}\right] = \frac{I_T}{8V_t^3}$$

$$\frac{\partial I_{C1}}{\partial V_d}\Big|_{V_d=0} = -\frac{I_T}{4V_t} \qquad \frac{\partial^2 I_{C1}}{\partial V_d^2}\Big|_{V_d=0} = 0 \qquad \frac{\partial^3 I_{C1}}{\partial V_d^3}\Big|_{V_d=0} = \frac{I_T}{8V_t^3}$$

How linear is the amplifier ?



$V_d = V_m \sin(\omega t)$

$$I_{C1} = I_{C1}\Big|_{V_d=0} + \frac{\partial I_{C1}}{\partial V_d}\Big|_{V_d=0} V_d + \frac{1}{2!}\frac{\partial^2 I_{C1}}{\partial V_d^2}\Big|_{V_d=0} V_d^2 + \frac{1}{3!}\frac{\partial^3 I_{C1}}{\partial V_d^3}\Big|_{V_d=0} V_d^3 + H.O.T$$

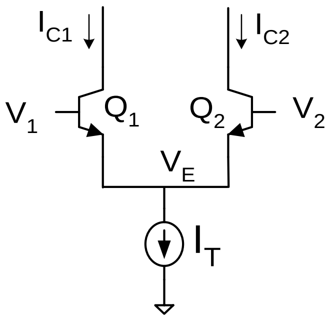
$$\frac{\partial I_{C1}}{\partial V_d}\Big|_{V_d=0} = -\frac{I_T}{4V_t} \qquad \frac{\partial^2 I_{C1}}{\partial V_d^2}\Big|_{V_d=0} = 0 \qquad \frac{\partial^3 I_{C1}}{\partial V_d^3}\Big|_{V_d=0} = \frac{I_T}{8V_t^3}$$

$$I_{C1} \cong \frac{I_T}{2} - \frac{I_T}{4V_t} V_d + \frac{I_T}{48V_t^3}\Big| V_d^3$$

$$I_{C1} \cong \frac{I_T}{2} - \frac{I_T}{4V_t} V_m \sin(\omega t) + \frac{I_T}{48V_t^3} V_m^3 \sin^3(\omega t)$$

$$\sin^3(\omega t) = \frac{3}{4}\sin(\omega t) - \frac{1}{4}\sin(3\omega t)$$

How linear is the amplifier ?



$$V_d = V_m \sin(\omega t)$$

$$I_{C1} = I_{C1}\Big|_{V_d=0} + \frac{\partial I_{C1}}{\partial V_d}\Big|_{V_d=0} V_d + \frac{1}{2!}\frac{\partial^2 I_{C1}}{\partial V_d^2}\Big|_{V_d=0} V_d^2 + \frac{1}{3!}\frac{\partial^3 I_{C1}}{\partial V_d^3}\Big|_{V_d=0} V_d^3 + H.O.T$$

$$I_{C1} \cong \frac{I_T}{2} - \frac{I_T}{4V_t} V_m \sin(\omega t) + \frac{I_T}{48V_t^3} V_m^3 \left[\frac{3}{4}\sin(\omega t) - \frac{1}{4}\sin(3\omega t)\right]$$

$$I_{C1} \cong \frac{I_T}{2} + \left[\frac{3I_T}{4 \bullet 48V_t^3} V_m^3 - \frac{I_T}{4V_t} V_m\right]\sin(\omega t) - \frac{I_T}{4 \bullet 48V_t^3} V_m^3 \sin(3\omega t)$$

Thus:

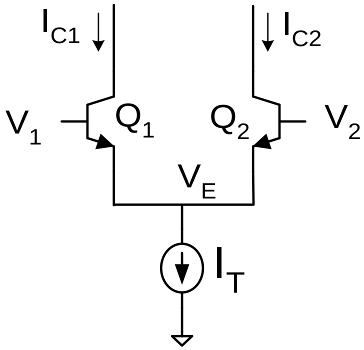
$$THD = 20\log\left(\frac{V_m^2}{\left[48V_t^2 - 3V_m^2\right]}\right)$$

or, equivalently

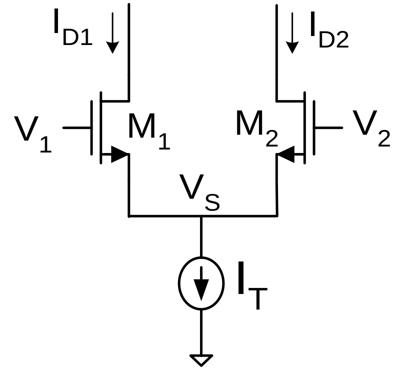
$$THD = -20\log\left(48\left(\frac{V_t}{V_m}\right)^2 - 3\right)$$

V_m/V_t	THD (dB)
2.5	-13.4049
1	-33.0643
0.5	-45.5292
0.25	-57.6732
0.1	-73.6194
0.05	-85.6647
0.025	-97.7069
0.01	-113.625

Comparison of Distortion in BJT and MOSFET Pairs



$V_d = V_m \sin(\omega t)$



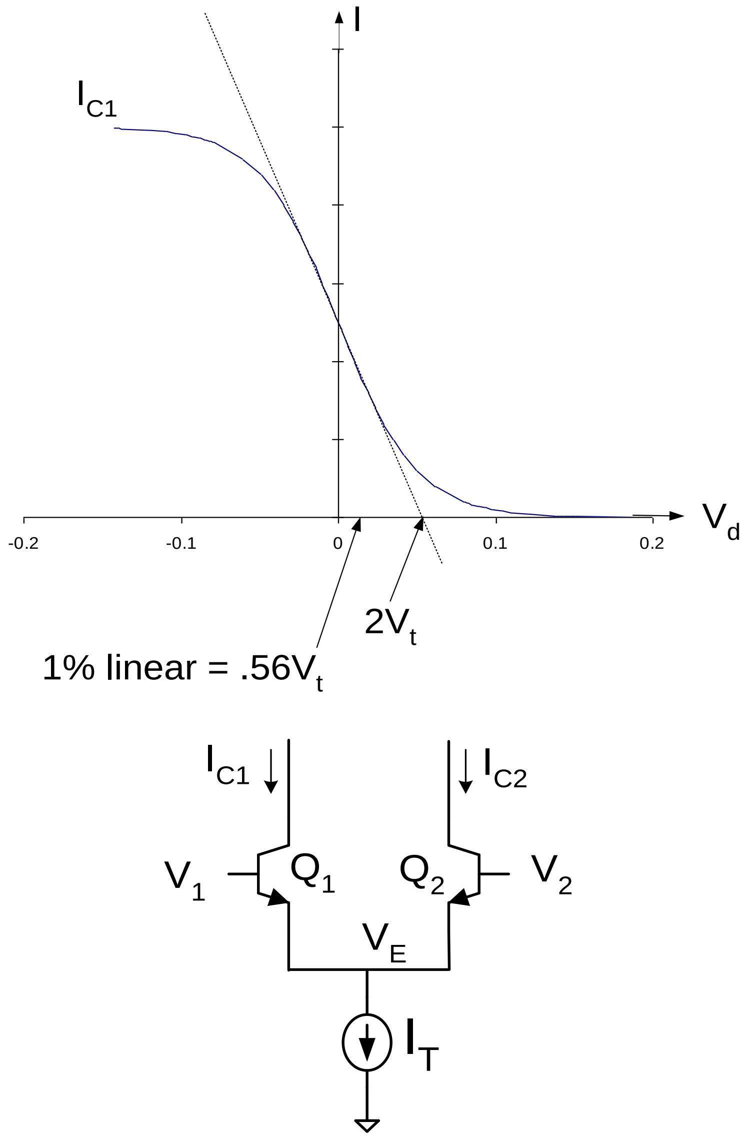
$$THD = -20\log\left(48\left(\frac{V_t}{V_m}\right)^2 - 3\right)$$

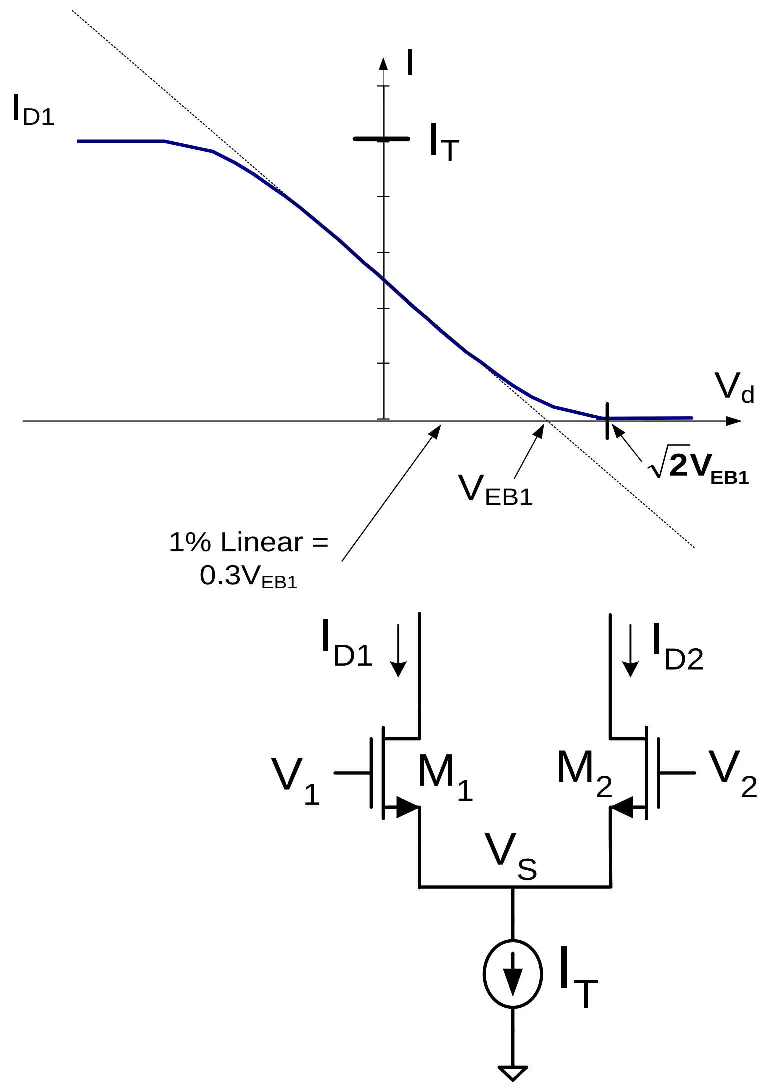
$$THD = -20\log\left(32\left(\frac{V_{EB1}}{V_m}\right)^2 - 3\right)$$

V_m/V_t	THD (dB)
2.5	-13.4049
1	-33.0643
0.5	-45.5292
0.25	-57.6732
0.1	-73.6194
0.05	-85.6647
0.025	-97.7069
0.01	-113.625

V_m/V_{EB1}	THD (dB)
2.5	-6.52672
1	-29.248
0.5	-41.9382
0.25	-54.1344
0.1	-70.0949
0.05	-82.1422
0.025	-94.1849
0.01	-110.103

Linearity and Signal Swing Comparison of Bipolar/MOS Differential Pair





End of Lecture 21