Common-Mode Feedback
Offset Voltage

Can be modeled as a dc voltage source in series with the input
Random Offset Voltages

- Correspondingly:

\[
\sigma_{V_{OS}}^2 = 2 \left[ \frac{A_{V_{TON}}^2}{W_n L_n} + \frac{\mu_p}{\mu_n} \frac{L_n}{W_n L_p^2} A_{V_{TOP}}^2 + \frac{V_{EBn}^2}{4} \right] + 2A_L^2 \left[ \frac{1}{W_n L_n^2} + \frac{1}{W_p L_p^2} \right] + A_w^2 \left[ \frac{1}{L_n W_n^2} + \frac{1}{L_p W_p^2} \right]
\]

which again simplifies to

\[
\sigma_{V_{OS}}^2 \approx 2 \left[ \frac{A_{V_{TON}}^2}{W_n L_n} + \frac{\mu_p}{\mu_n} \frac{L_n}{W_n L_p^2} A_{V_{TOP}}^2 \right]
\]

Note these offset voltage expressions are identical!
Random Offset Voltages

It can be shown that

$$\sigma_{V_{OS}}^2 \approx 2V_t^2 \left[ \frac{A_{Jn}^2}{A_{En}} + \frac{A_{Jp}^2}{A_{Ep}} \right]$$

where very approximately

$$A_{Jn} = A_{Jp} = 0.1\mu$$
Random Offset Voltages

Typical offset voltages:

MOS - 5mV to 50MV
BJT - 0.5mV to 5mV

These can be scaled with extreme device dimensions

Often more practical to include offset-compensation circuitry
Common Centroid Layouts

Define \( p \) to be a process parameter that varies with lateral position throughout the region defined by the channel of the transistor.

**Almost Theorem:**

If \( p(x,y) \) varies linearly throughout a two-dimensional region, then

\[
p_{EQ} = \frac{1}{A} \int_A p(x,y) dxdy
\]

Parameters such as \( V_T \), \( \mu \) and \( C_{OX} \) vary throughout a two-dimensional region.

If a parameter varies linearly throughout a two-dimensional region, it is said to have a linear gradient.
Common Centroid Layouts

Almost Theorem:

If \( p(x,y) \) varies linearly throughout a two-dimensional region, then
\[ p_{EQ} = p(x_0,y_0) \]
where \( x_0,y_0 \) is the geometric centroid to the region.

Parameters such at \( V_T \), \( \mu \) and \( C_{OX} \) vary throughout a two-dimensional region.

If a parameter varies linearly throughout a two-dimensional region, it is said to have a linear gradient.

Many parameters have a dominantly linear gradient over rather small regions.
If \( \rho(x,y) \) varies linearly in any direction, then the theorem states

\[
\rho_{\text{EQ}} = \frac{1}{A} \int_A \rho(x,y) \, dx \, dy
\]

\((x_0, y_0)\) is geometric centroid

\[
\rho_{\text{EQ}} = \frac{1}{A} \int_A \rho(x,y) \, dx \, dy = \rho(x_0, y_0)
\]
Common Centroid Layouts

A layout of two devices is termed a common-centroid layout if both devices have the same geometric centroid.

Almost Theorem:

If $p(x,y)$ varies linearly throughout a two-dimensional region, then if two have the same centroid, the parameters are matched!

Note: This is true independent of the magnitude and direction of the gradient!
Common Centroid of Multiple Segmented Geometries
Common Centroid Layout Surrounded by Dummy Devices
Repeatedly throughout the course, we have added a footnote on fully-differential circuits that a common-mode feedback circuit (CMFB) is needed.

The CMFB circuit is needed to establish or stabilize the operating point or operating points of the op amp.
Common-Mode Feedback

On the reference op amp, the CMFB signal can be applied to either the p-channel biasing transistors or to the tail current transistor.

It is usually applied only to a small portion of the biasing transistors though often depicted as shown.

There is often considerable effort devoted to the design of the CMFB though little details are provided in most books and the basic concepts of the CMFB are seldom rigorously developed and often misunderstood.
Common-Mode Feedback

Partitioning biasing transistors for $V_{FB}$ insertion

(Nominal device matching assumed, all L’s equal)

$V_{DD}$

$M_3 \quad V_{B1} \quad M_4$

Ideal ( Desired) biasing

$V_{DD}$

$M_3 \quad V_{FB} \quad M_4$

$V_{FB}$ insertion

$W_{3A} + W_{3B} = W_3$

$W_{3B} \ll W_{3A}$

Partitioned $V_{FB}$ insertion
Basic Operation of CMFB Block

$V_{O1}$  \hspace{1cm} $V_{O2}$  \hspace{1cm} $V_{OXX}$

$V_{FB}$

$V_{OXX}$ is the desired quiescent voltage at the stabilization node (irrespective of where $V_{FB}$ goes)

$V_{FB} = \left( \frac{V_{O1} + V_{O2}}{2} \right) A(s)$
Basic Operation of CMFB Block

- Comprised of two fundamental blocks
  - Averager
  - Differential amplifier
- Sometimes combined into single circuit block
- Compensation of the CMFB path often required!!

\[ V_{FB} = \left( \frac{V_{O1} + V_{O2}}{2} \right) A(s) \]
Mathematics behind CMFB

(consider an example that needs a CMFB)

Notice there are two capacitors and thus two poles in this circuit
Mathematics behind CMFB

(consider an example that needs a CMFB)

Small-signal model showing axis of symmetry

$2W'_{3} = W_{3}$
Mathematics behind CMFB

(consider an example that needs a CMFB)

Small-signal difference-mode half circuit

\[
\frac{V_d}{2} = V_{OD}
\]

\[
\frac{g_m}{2} \cdot \frac{V_d}{2} = 0
\]

\[
A_{DIFF} = \frac{-g_m}{2(sC+g_{01}+g_{05})}
\]

\[
p_{DIFF} = \frac{-g_{01}+g_{05}}{C}
\]

Note there is a single-pole in this circuit

What happened to the other pole?
Mathematics behind CMFB

(consider an example that needs a CMFB)

Standard small-signal common-mode half circuit

$$V_{OC} \left( sC + g_{01} + g_{05} \right) + g_{m1} \left( V_{COM} - V_S \right) = 0$$

$$V_S \left( g_{01} + g_{03}/2 \right) - g_{m1} \left( V_{COM} - V_S \right) = V_{OC} g_{01}$$

$$A_{COM} = \frac{-g_{m1} \left( g_{01} + g_{03}/2 \right)}{\left( sC + g_{01} + g_{05} \right) \left( g_{m1} + g_{01} + g_{03}/2 \right) - g_{m1} g_{01}} = \frac{-g_{01} + g_{03}/2}{sC + g_{05}}$$

$$p_{COM} = -\frac{g_{05}}{C}$$

Note there is a single-pole in this circuit

And this is different from the difference-mode pole

But the common-mode gain tells little, if anything, about the CMFB
Mathematics behind CMFB

(consider an example that needs a CMFB)

\[ A_{\text{COM}} = -\frac{g_{01} + g_{03}/2}{sC + g_{05}} \]
\[ \rho_{\text{COM}} = -\frac{g_{05}}{C} \]

\[ A_{\text{DIFF}} = -\frac{g_{m1}}{2 sC + g_{01} + g_{05}} \]
\[ \rho_{\text{DIFF}} = -\frac{g_{01} + g_{05}}{C} \]

- Difference-mode analysis completely hides all information about common-mode
- This also happens in simulations
- Common-mode analysis completely hides all information about difference-mode
- This also happens in simulations
- Difference-mode poles may move into RHP with FB so compensation is required for stabilization (or proper operation)
- Common-mode poles may move into RHP with FB so compensation is required for stabilization (or proper operation)
- Difference-mode simulations tell nothing about compensation requirements for common-mode feedback
- Common-mode simulations tell nothing about compensation requirements for difference-mode feedback
Mathematics behind CMFB

(consider an example that needs a CMFB)

\[
A_{\text{COM}} \approx -\frac{g_{01} + g_{03}/2}{sC + g_{05}} \quad p_{\text{COM}} = -\frac{g_{05}}{C}
\]

\[
A_{\text{DIFF}} = \frac{-g_m}{2} \quad p_{\text{DIFF}} = -\frac{g_{01} + g_{05}}{C}
\]

- Common-mode and difference-mode gain expressions often include same components though some may be completely absent in one or the other mode
- Compensation capacitors can be large for compensating either the common-mode or difference-mode circuits
- Highly desirable to have the same compensation capacitor serve as the compensation capacitor for both difference-mode and common-mode operation
  - But tradeoffs may need to be made in phase margin for both modes if this is done
- Better understanding of common-mode feedback is needed to provide good solutions to the problem
Mathematics behind CMFB

(consider an example that needs a CMFB)

Standard small-signal common-mode half circuit

\[ V_{OC} (sC+g_{01}+g_{05}) + g_{m1} (V_{COM} - V_S) = 0 \]

\[ V_S (g_{01} + g_{03}/2) - g_{m1} (V_{COM} - V_S) = V_{OC} g_{01} \]

Note there is a single-pole in this circuit

And this is different from the difference-mode pole

But the common-mode gain tells little, if anything, about the CMFB
Common-Mode and Difference-Mode Issues

Overall poles are the union of the common-mode and difference mode poles

Separate analysis generally require to determine common-mode and difference-mode performance

Some amplifiers will need more than one CMFB
Definition: The common-mode offset voltage is the voltage that must be applied to the biasing node at the CMFB point to obtain the desired operating point at the stabilization node.
Common-mode offset voltage

Consider again the Common-mode half circuit

There are three common-mode inputs to this circuit!
The common-mode signal input is distinct from the input that is affected by $V_{\text{COFF}}$
The gain from the common-mode input where $V_{\text{FB}}$ is applied may be critical!
Common-mode gains

\[ A_{\text{COM}0} = \frac{g_{m5}}{g_{05}} = \frac{2I_T / V_{EB5}}{\lambda I_T / 2} = - \frac{4}{V_{EB5}} \lambda \]

\[ A_{\text{COM}20} = \frac{g_{m5}}{g_{05}} = \frac{2I_T / V_{EB5}}{\lambda I_T / 2} = - \frac{2}{V_{EB5}} \lambda \]

Although the common-mode gain \( A_{\text{COM}0} \) is very small, \( A_{\text{COM}20} \) is very large!

Shift in \( V_{02Q} \) from \( V_{OXX} \) is the product of the common-mode offset voltage and \( A_{\text{COM}20} \)
Effect of common-mode offset voltage

How much change in $V_{02}$ is acceptable?

How big is $V_{COFF}$?

How big is $A_{COM20}$?

If change in $V_{02}$ is too large, CMFB is needed

$$A_{COM20} \approx -\frac{4}{V_{EB5} \lambda}$$

$$\Delta V_{02} = A_{COM20} V_{COFF}$$
Effect of common-mode offset voltage

How much change in $V_{02}$ is acceptable? (assume e.g. 50mV)

How big if $V_{COFF}$? (similar expressions for $V_{OS}$, assume, e.g. 25mV)

How big is $A_{COM20}$? (if $\lambda=0.01$, $V_{EB}=0.2$, $A_{COM20}=2000$)

If change in $V_{02}$ is too large, CMFB is needed

$50\text{mV} < 2000 \times 25\text{mV}$
How much gain is needed in the CMFB amplifier?

CMFB must compensate for $V_{COFF}$

Want to guarantee $|V_{02Q} - V_{0XX}| < \Delta V_{OUT-ACCEPTABLE}$

This is essentially the small-signal output with a small-signal input of $V_{COFF}$
How much gain is needed in the CMFB amplifier?

Do a small-signal analysis, only input is \( V_{\text{COFF}} \)

\[
V_{\text{O2}} = (V_{\text{O2A}} + V_{\text{COFF}}) A_{\text{COM2}}
\]

\[
V_{\text{O2}} = V_{\text{COFF}} \frac{A_{\text{COM2}}}{1 - AA_{\text{COM2}}}
\]

\[
\Delta V_{\text{OUT-ACCEPTABLE}} = V_{\text{COFF}} \frac{A_{\text{COM2}}}{1 - AA_{\text{COM2}}}
\]

Want to guarantee

\[
|V_{\text{O2Q}} - V_{\text{OXX}}| < \Delta V_{\text{OUT-ACCEPTABLE}}
\]
How much gain is needed in the CMFB amplifier?

- This does not require a particularly large gain
- This is the loop that must be compensated since A will be frequency dependent
- Miller compensation capacitor for compensation of differential loop will often appear in shunt with $C_2$
- Can create this loop without CM inputs on fully differential structure for simulations
- Results extend readily to two-stage structures with no big surprises

\[ \Delta V_{\text{OUT-ACCEPTABLE}} = V_{\text{COFF}} \frac{A_{\text{COM2}}}{1 - AA_{\text{COM2}}} \]
Several (but not too many) CMFB circuits exist
Can be classified as either continuous-time or discrete-time
End of Lecture 23