EE 435

Lecture 26

Data Converters
Common Centroid Layouts

Almost Theorem:

If \( p(x,y) \) varies linearly throughout a two-dimensional region, then 
\[ p_{EQ} = p(x_0, y_0) \]
where \( x_0, y_0 \) is the geometric centroid to the region.

Parameters such at \( V_T, \mu \) and \( C_{OX} \) vary throughout a two-dimensional region.

If a parameter varies linearly throughout a two-dimensional region, it is said to have a linear gradient.

Many parameters have a dominantly linear gradient over rather small regions.
If $\rho(x,y)$ varies linearly in any direction, then the theorem states

$$p_{EQ} = \frac{1}{A} \int_A p(x,y) \, dx \, dy$$

$(x_0,y_0)$ is geometric centroid

$$p_{EQ} = \frac{1}{A} \int_A p(x,y) \, dx \, dy = p(x_0,y_0)$$
Common Centroid of Multiple Segmented Geometries
Common Centroid Layout Surrounded by Dummy Devices
Data Converters

Types:

A/D (Analog to Digital)
Converting Analog Input to a Digital Output

D/A (Digital to Analog)
Converting a Digital Input to an Analog Output

A/D is the world’s most widely used mixed-signal component

D/A is often included in a FB path of an A/D

A/D and D/A fields will remain hot indefinitely
technology advances make data converter design more challenging
embedded applications
designs often very application dependent
D/A Converters

Basic structure:

\[ \bar{x}_{\text{IN}} \rightarrow \bar{x}_{\text{OUT}} \]

\[ x_{\text{REF}} \]

Basic structure with differential outputs:

\[ \bar{x}_{\text{IN}} \rightarrow \bar{x}_{\text{OUT}} \]

\[ x_{\text{REF}} \]

\[ x_{\text{REF}} \]

\[ x_{\text{REF}} \]
D/A Converters

Notation:
D/A Converters

$$\tilde{X}_{IN} = \langle b_{n-1}, b_{n-1}, \ldots, b_1, b_0 \rangle$$

- $b_0$ is the Least Significant Bit (LSB)
- $b_{n-1}$ is the Most Significant Bit (MSB)

Note: some authors use different index notation

An Ideal DAC is characterized at low frequencies by its static performance
D/A Converters

\[ \tilde{X}_{IN} = \langle b_{n-1}, b_{n-1}, \ldots b_1, b_0 \rangle \]

An Ideal DAC transfer characteristic (3-bits)

Code \( C_k \) is used to represent the decimal equivalent of the binary number \( \langle b_{n-1} \ldots b_0 \rangle \)
D/A Converters

\[ X_{IN} = \langle b_{n-1}, b_{n-1}, \ldots b_1, b_0 \rangle \]

An Ideal DAC transfer characteristic (3-bits)
D/A Converters

\[ \bar{X}_{IN} = \langle b_{n-1}, b_{n-1}, \ldots, b_1, b_0 \rangle \]

An Ideal DAC transfer characteristic (3-bits)

All points of this ideal DAC lie on a straight line
Most D/A ideally have a linear relationship between binary input and analog output.
- Output represents a discrete set of continuous variables.
- Typically this number is an integral power of 2, i.e. $2^n$.
- $\tilde{X}_{IN}$ is always dimensionless.

$\tilde{X}_{OUT}$ could have many different dimensions.
- An ideal nonlinear characteristic is also possible (waveform generation and companding).
- Will assume a linear transfer characteristic is desired unless specifically stated to the contrary.
D/A Converters

For this ideal DAC

\[ X_{\text{OUT}} = X_{\text{REF}} \left( \sum_{j=1}^{n} \frac{b_{n-j}}{2^j} \right) \]

\[ X_{\text{OUT}} = X_{\text{REF}} \sum_{j=1}^{n} \frac{b_{n-j}}{2^j} \]

- Number of outputs gets very large for \( n \) large
- Spacing between outputs is \( X_{\text{REF}}/2^n \) and gets very small for \( n \) large
D/A Converters

• Ideal steps all equal and termed the LSB
• $X_{LSB}$ gets very small for small $X_{REF}$ and large n

**Example**

If $X_{REF} = 1V$ and $n=16$, then $N = 2^{16} = 65,536$, $X_{LSB} = 15.25\mu V$
D/A Converters

An alternate ideal 3-bit DAC

Irrespective of which form is considered, the increment in the output for one Boolean bit change in the input is $\Delta X_{\text{LSB}}$ and the total range is 1LSB less than $X_{\text{REF}}$.
Applications of DACs

- Waveform Generation
- Voltage Generation
- Analog Trim or Calibration
- Industrial Control Systems
- Feedback Element in ADCs
- ....
Waveform Generation with DACs

Ramp (Saw-tooth) Generator

Example: For $n=3$

Example: For large $n$
Waveform Generation with DACs

Sine Wave Generator

Distortion of the desired waveforms occurs due to both time and amplitude quantization.

Often a filter precedes or follows the buffer amplifier to smooth the output waveform.
A/D Converters

Basic structure:

Input range is \( X_{\text{REF}} \)

Basic structure with differential inputs/references:

Input range is \( X^+_{\text{REF}} - X^-_{\text{REF}} \)

Input range is \( 2(X^+_{\text{REF}} - X^-_{\text{REF}}) \)
A/D Converters

Notation:
A/D Converters

\[ \tilde{X}_{\text{OUT}} = <d_{n-1}, d_{n-2}, \ldots, d_0> \]

- \( d_0 \) is the Least Significant Bit (LSB)
- \( d_{n-1} \) is the Most Significant Bit (MSB)

Notes:
Indexing notation reversed from what we used for DAC
some authors use different index notation

An Ideal ADC is characterized at low frequencies by its static performance
A/D Converters

An Ideal ADC transfer characteristic (3-bits)

\[ \bar{x}_{\text{OUT}} = <d_{n-1}, d_{n-2}, \ldots, d_0> \]

\[ x_{\text{LSB}} = \frac{x_{\text{REF}}}{2^n} \]

\[ x_{\text{REF}} - x_{\text{LSB}} \]

\[ x_{\text{IN}} \quad x_{\text{OUT}} \]
A/D Converters

An Ideal ADC transfer characteristic (3-bits)

\[ \tilde{x}_{IN} \leftrightarrow \tilde{x}_{OUT} \]

\[ x_{LSB} = \frac{x_{REF}}{2^n} \]

The second vertical axis, labeled \( \tilde{x}_{IN} \) is the interpreted value of \( x_{IN} \)
A/D Converters

For this ideal ADC

\[ x_{IN} = x_{REF} \left( \frac{d_{n-1}}{2} + \frac{d_{n-2}}{4} + \frac{d_{n-3}}{8} + \ldots + \frac{d_1}{2^{n-1}} + \frac{d_0}{2^n} \right) + \epsilon \]

where \( \epsilon \) is small (typically less than 1LSB)

\[ x_{IN} = x_{REF} \sum_{j=1}^{n} \frac{d_{n-j}}{2^j} + \epsilon \]

- Number of bins gets very large for \( n \) large
- Spacing between break points is \( x_{REF}/2^n \) and gets very small for \( n \) large

\( \epsilon \) is the quantization error and is inherent in any ADC
A/D Converters

Transition Points

- Actual values of $x_{IN}$ where transitions occur are termed transition points or break points.
- For an ideal n-bit ADC, there are $2^n-1$ transition points.
- Ideally the transition points are all separated by 1 LSB -- $X_{LSB} = X_{REF}/2^n$.
- Ideally the transition points are uniformly spaced.
- In an actual ADC, the transition points will deviate a little from their ideal location.

Labeling Convention:

We will define the transition point $X_{Tk}$ to be the break point where the transition in the code output to code $C_k$ occurs. This seemingly obvious ordering of break points becomes ambiguous, though, when more than one break points cause a transition to code $C_k$ which can occur in some nonideal ADCs.
A/D Converters

Quantization Errors

$$x_{T1} = x_{LSB}$$

$$\varepsilon_Q = \tilde{X}_{OUT} - X_{IN}$$

Magnitude of $$\varepsilon_Q$$ bounded by $$X_{LSB}$$
A/D Converters

Quantization Errors

Another Ideal ADC

\(x_{T1} = \frac{x_{\text{LSB}}}{2}\)

\(\varepsilon_Q = \tilde{x}_{\text{OUT}} - x_{\text{IN}}\)

Magnitude of \(\varepsilon_Q\) bounded by \(\frac{1}{2} x_{\text{LSB}}\)

Is the performance of this ideal ADC really better than that of the previous ideal ADC?