Lecture 27

Data Converters

- INL of ADC
- Differential Nonlinearity
- Spectral Performance
Integral Nonlinearity (DAC)

Nonideal DAC

INL often expressed in LSB

\[
\text{INL}_k = \frac{x_{\text{OUT}}(k) - x_{\text{OF}}(k)}{x_{\text{LSB}}}
\]

\[
\text{INL} = \max_{0 \leq k \leq N-1} \{ |\text{INL}_k| \}
\]

- INL is often the most important parameter of a DAC
- INL\(_0\) and INL\(_{N-1}\) are 0 (by definition)
- There are N-2 elements in the set of INL\(_k\) that are of concern
- INL is almost always nominally 0 (i.e. designers try to make it 0)
- INL is a random variable at the design stage
- INL\(_k\) is a random variable for 0 < k < N-1
- INL\(_k\) and INL\(_{k+j}\) are almost always correlated for all k, j (not incl 0, N-1)
- Fit Line is a random variable
- INL is the N-2 order statistic of a set of N-2 correlated random variables
Integral Nonlinearity (ADC)

Integral Non-Linearity (INL)

Integral Non-Linearity (INL) is defined as the sum from the first to the current conversion (integral) of the non-linearity at each code (Code DNL). For example, if the sum of the DNL up to a particular point is 1 LSB, it means the total of the code widths to that point is 1 LSB greater than the sum of the ideal code widths. Therefore, the current point will convert one code lower than the ideal conversion.

In more fundamental terms, INL represents the curvature in the Actual Transfer Function relative to a baseline transfer function, or the difference between the current and the ideal transition voltages. There are three primary definitions of INL in common use. They all have the same fundamental definition except they are measured against different transfer functions. This fundamental definition is:

$$\text{Code INL} = V(\text{Current Transition}) - V(\text{Baseline Transition})$$

$$\text{INL} = \max(\text{Code INL})$$

Actually probably more than 3
Integral Nonlinearity (ADC)

Nonideal ADC

Transition points are not uniformly spaced!
More than one definition for INL exists!
Will give two definitions here
Integral Nonlinearity (ADC)

Consider end-point fit line with interpreted output axis

\[ x_{\text{INF}}(x_{\text{IN}}) = \bar{m} x_{\text{IN}} + \left( \frac{x_{\text{LSB}}}{2} - m x_{T1} \right) \]

\[ m = \frac{(N-2) x_{\text{LSB}}}{x_{T7} - x_{T1}} \]
Integral Nonlinearity (ADC)

Nonideal ADC

Continuous-input based INL definition

\[ \text{INL} = \max_{0 \leq x_{\text{IN}} \leq x_{\text{REF}}} \left\{ \| \text{INL}(x_{\text{IN}}) \| \right\} \]
Integral Nonlinearity (ADC)

Nonideal ADC
Continuous-input based INL definition

\[
\tilde{\chi}_{IN} = \frac{\chi_{IN} - \chi_{INF}(\chi_{IN})}{X_{LSB}}
\]

\[
INL = \max_{0 \leq \chi_{IN} \leq \chi_{REF}} \left\{ |INL(\chi_{IN})| \right\}
\]
With this definition of INL, the INL of an ideal ADC is \( x_{\text{LSB}}/2 \) (for \( x_{T1} = x_{\text{LSB}} \))

This is effective at characterizing the overall nonlinearity of the ADC but does not vanish when the ADC is ideal and the effects of the breakpoints are not explicit.
Integral Nonlinearity (ADC)

Nonideal ADC

Break-point INL definition (most popular)

Place \( N-3 \) uniformly spaced points between \( X_{T1} \) and \( X_{T(N-1)} \) designated \( X_{FTk} \)

\[
\text{INL}_k = X_{Tk} - X_{FTk} \quad 1 \leq k \leq N-2
\]

\[
\text{INL} = \max_{2 \leq k \leq N-2} \{ |\text{INL}_k| \} 
\]
Integral Nonlinearity (ADC)

Nonideal ADC

Break-point INL definition

Often expressed in LSB

\[ \text{INL}_k = \frac{X_{T_k} - X_{FT_k}}{X_{\text{LSB}}} \quad 1 \leq k \leq N-2 \]

\[ \text{INL} = \max_{2 \leq k \leq N-2} \left\{ \| \text{INL}_k \| \right\} \]

For an ideal ADC, INL is ideally 0
Integral Nonlinearity (ADC)

Nonideal ADC

Break-point INL definition

\[ \text{INL}_k = \frac{x_{Tk} - x_{FTl}}{x_{LSB}} \quad 1 \leq k \leq N-2 \]

\[ \text{INL} = \max_{2 \leq k \leq N-2} \{|\text{INL}_k|\} \]

- INL is often the most important parameter of an ADC
- \( \text{INL}_1 \) and \( \text{INL}_{N-1} \) are 0 (by definition)
- There are \( N-3 \) elements in the set of \( \text{INL}_k \) that are of concern
- INL is a random variable at the design stage
- \( \text{INL}_k \) is a random variable for \( 0 < k < N-1 \)
- \( \text{INL}_k \) and \( \text{INL}_{k+j} \) are correlated for all \( k,j \) (not incl 0, N-1) for most architectures
- Fit Line (for cont INL) and uniformly spaced break pts (breakpoint INL) are random variables
- INL is the \( N-3 \) order statistic of a set of \( N-3 \) correlated random variables (breakpoint INL)
Integral Nonlinearity (ADC)

Nonideal ADC

Break-point INL definition

\[
\text{INL}_k = \frac{x_{T_k} - x_{F1}}{x_{\text{LSB}}} \quad 1 \leq k \leq N-2
\]

\[
\text{INL} = \max_{2 \leq k \leq N-2} \{ |\text{INL}_k| \}
\]

- At design stage, INL characterized by standard deviation of the random variable
- Closed-form expressions for INL almost never exist because PDF of order statistics of correlated random variables is extremely complicated
- Simulation of INL very time consuming if \( n \) is very large (large sample size required to establish reasonable level of confidence)
  - Model parameters become random variables
  - Process parameters affect multiple model parameters causing model parameter correlation
  - Simulation times can become very large
Integral Nonlinearity (ADC)

Nonideal ADC

Break-point INL definition

$$\text{INL}_k = \frac{X_{Tk} - X_{FTk}}{X_{LSB}} \quad 1 \leq k \leq N-2$$

$$\text{INL} = \max_{2 \leq k \leq N-2} \{ |\text{INL}_k| \}$$

• INL can be readily measured in laboratory but often dominates test costs because of number of measurements needed when n is large
• INL is a random variable and is a major contributor to yield loss in many designs
• Expected value of $\text{INL}_k$ at $k=(N-1)/2$ is largest for many architectures
• This definition does not account for missing transitions
• Major effort in ADC design is in obtaining an acceptable yield
INL-based ENOB

Consider initially the continuous INL definition for an ADC where the INL of an ideal ADC is $X_{LSB}/2$

Assume

$$\text{INL} = \theta X_{\text{REF}} = \nu X_{\text{LSBR}}$$

where $X_{\text{LSBR}}$ is the LSB based upon the defined resolution

Define the effective LSB by

$$X_{\text{LSBEFF}} = \frac{X_{\text{REF}}}{2^n_{\text{EQ}}}$$

Thus

$$\text{INL} = \theta 2^{n_{\text{EQ}}} X_{\text{LSBEFF}}$$

Since an ideal ADC has an INL of $X_{LSB}/2$, express INL in terms of ideal ADC

$$\text{INL} = \left[ \theta 2^{(n_{\text{EQ}}+1)} \right] \left( \frac{X_{\text{LSBEFF}}}{2} \right)$$

Setting term in [ ] to 1, can solve for $n_{\text{EQ}}$ to obtain

$$\text{ENOB} = n_{\text{EQ}} = \log_2 \left( \frac{1}{2\theta} \right) = n_R - 1 - \log_2 (\nu)$$

where $n_R$ is the defined resolution
INL-based ENOB

$$\text{ENOB} = n_R - 1 - \log_2(\nu)$$

Consider an ADC with specified resolution of $n_R$ and INL of $\nu$ LSB

<table>
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<tr>
<th>$\nu$</th>
<th>ENOB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>$n$</td>
</tr>
<tr>
<td>1</td>
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<tr>
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<td>8</td>
<td>$n-4$</td>
</tr>
<tr>
<td>16</td>
<td>$n-5$</td>
</tr>
</tbody>
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Performance Characterization of Data Converters

• Static characteristics
  – Resolution
  – Least Significant Bit (LSB)
  – Offset and Gain Errors
  – Absolute Accuracy
  – Relative Accuracy
  – Integral Nonlinearity (INL)
  – Differential Nonlinearity (DNL)
  – Monotonicity (DAC)
  – Missing Codes (ADC)
  – Low-f Spurious Free Dynamic Range (SFDR)
  – Low-f Total Harmonic Distortion (THD)
  – Effective Number of Bits (ENOB)
  – Power Dissipation
Differential Nonlinearity (DAC)

Nonideal DAC

\[ DNL(k) = \frac{X_{\text{OUT}(k)} - X_{\text{OUT}(k-1)} - X_{\text{LSB}}}{X_{\text{LSB}}} \]

DNL(k) is the actual increment from code (k-1) to code k, minus the ideal increment normalized to \( X_{\text{LSB}} \).
Differential Nonlinearity (DAC)

Nonideal DAC

Increment at code $k$ is a signed quantity and will be negative if $X_{\text{OUT}}(k) < X_{\text{OUT}}(k-1)$

$$\text{DNL}(k) = \frac{X_{\text{OUT}}(k) - X_{\text{OUT}}(k-1) - X_{\text{LSB}}}{X_{\text{LSB}}}$$

$$\text{DNL} = \max \left\{ \{\text{DNL}(k)\} \right\}_{1 \leq k \leq N-1}$$

DNL=0 for an ideal DAC
Monotonicity (DAC)

Nonideal DAC

Monotone DAC

Non-monotone DAC

Definition:
A DAC is monotone if $X_{\text{OUT}}(k) > X_{\text{OUT}}(k-1)$ for all $k$

Theorem:
A DAC is monotone if DNL(k) > -1 for all $k$
**Theorem:** The \( \text{INL}_k \) of a DAC can be obtained from the DNL by the expression

\[
\text{INL}_k = \sum_{i=1}^{k} \text{DNL}(i)
\]

**Caution:** Be careful about using this theorem to measure the INL since errors in DNL measurement (or simulation) can accumulate

**Corollary:** \( \text{DNL}(k) = \text{INL}_k - \text{INL}_{k-1} \)
Theorem: If the INL of a DAC satisfies the relationship

\[ \text{INL} < \frac{1}{2} \cdot X_{\text{LSB}} \]

then the DAC is monotone

Note: This is a necessary but not sufficient condition for monotonicity
Differential Nonlinearity (ADC)

**DNL(k)** is the code width for code k – ideal code width normalized to $X_{LSB}$

$$DNL(k) = \frac{X_{T(k+1)} - X_{Tk} - X_{LSB}}{X_{LSB}}$$
Differential Nonlinearity (ADC)

Nonideal ADC

\[
DNL(k) = \frac{X_{T(k+1)} - X_{Tk} - X_{LSB}}{X_{LSB}}
\]

\[DNL = \max_{2 \leq k \leq N-1} \left\{ \left| DNL(k) \right| \right\}\]

DNL=0 for an ideal ADC

Note: In some nonideal ADCs, two or more break points could cause transitions to the same code \( C_k \) making the definition of DNL ambiguous.
Monotonicity in an ADC

Nonideal ADCs

**Monotone ADC**

Definition: An ADC is monotone if the

$$\bar{X}_{\text{OUT}}(x_k) \geq \bar{X}_{\text{OUT}}(x_m)$$

whenever $$x_k \geq x_m$$

Note: Have used $$x_{B_k}$$ instead of $$x_{T_k}$$ since more than one transition point to a given code

Note: Some authors do not define monotonicity in an ADC.
No missing codes

Definition: An ADC has no missing codes if there are N-1 transition points and a single LSB code increment occurs at each transition point. If these criteria are not satisfied, we say the ADC has missing code(s).

Note: With this definition, all codes can be present but we still say it has “missing codes”

Note: Some authors claim that missing codes in an ADC are the counterpart to nonmonotonicity in a DAC. This association is questionable.
Missing Codes (ADC)

Nonideal ADCs

Missing codes

Missing code with all codes present
Weird Things Can Happen

Nonideal ADCs

- Multiple outputs for given inputs
- All codes present but missing codes

Be careful on definition and measurement of linearity parameters to avoid having weird behavior convolute analysis, simulation or measurements

Most authors (including manufacturers) are sloppy with their definitions of data converter performance parameters and are not robust to some weird operation
Linearity Measurements (testing)

Consider ADC

Linearity testing often based upon code density testing

Code density testing:

Ramp or multiple ramps often used for excitation
Linearity of test signal is critical (typically 3 or 4 bits more linear than DUT)
Linearity Measurements (testing)

Code density testing:

- First and last bins generally have many extra counts (and thus no useful information)
- Typically average 16 or 32 hits per code
Linearity Measurements (testing)

Code density testing:

\[ \bar{C} = \frac{\sum_{i=1}^{N-2} \hat{C}_i}{N-2} \]

\[ \text{DNL}_i = \frac{\hat{C}_i - \bar{C}}{C} \]

\[ \text{INL}_i = \begin{cases} 0 & i = 0, N-2 \\ \left[ \sum_{k=1}^{i} \hat{C}_k \right] \frac{-i\bar{C}}{C} & 1 \leq i \leq N-3 \end{cases} \]

\[ \text{DNL} = \max_{1 \leq i \leq N-2} \{ |DNL_i| \} \]

\[ \text{INL} = \max_{1 \leq i \leq N-3} \{ |INL_i| \} \]

- This measurement is widely used
- Does not keep track of order bins are filled
- Some weird things can occasionally happen with this approach
End of Lecture 27