EE 435

Lecture 28

Data Converters

Spectral Performance
Spectral Characterization
INL Often Not a Good Measure of Linearity

Four identical INL with dramatically different linearity
Linearity Issues

- INL is often not adequate for predicting the linearity performance of a data converter
- Distortion (or lack thereof) is of major concern in many applications
- Distortion is generally characterized in terms of the harmonics that may appear in a waveform
Spectral Analysis

If \( f(t) \) is periodic

\[
f(t) = A_0 + \sum_{k=1}^{\infty} A_k \sin(k\omega t + \theta_k)
\]

alternately

\[
f(t) = A_0 + \sum_{k=1}^{\infty} a_k \sin(k\omega t) + \sum_{k=1}^{\infty} b_k \cos(k\omega t)
\]

\[
\omega = \frac{2\pi}{T}
\]

\[
A_k = \sqrt{a_k^2 + b_k^2}
\]

Termed the Fourier Series Representation of \( f(t) \)
Often the system of interest is ideally linear but practically it is weakly nonlinear.

Often the input is nearly periodic and often sinusoidal and in latter case desired output is also sinusoidal.

Weak nonlinearity will cause distortion of signal as is propagated through the system.

Spectral analysis often used to characterize effects of the weak nonlinearity.
Spectral Analysis

\[ X_{IN}(t) \xrightarrow{\text{Nonlinear System}} X_{OUT}(t) \]

If

\[ X_{IN}(t) = X_m \sin(\omega t + \theta) \]

\[ X_{OUT}(t) = A_0 + \sum_{k=1}^{\infty} A_k \sin(k \omega t + \theta_k) \]

All spectral performance metrics depend upon the sequence \( \langle A_k \rangle_{k=0}^{\infty} \)

Spectral performance metrics of interest: SNDR, SDR, THD, SFDR, IMOD
Distortion Analysis

\[ \langle A_k \rangle_{k=0}^{\infty} \]

Often termed the DFT coefficients (will show later)
Spectral lines, not a continuous function

\( A_1 \) is termed the fundamental
\( A_k \) is termed the kth harmonic
Distortion Analysis

Often ideal response will have only fundamental present and all remaining spectral terms will vanish
For a low distortion signal, the 2\textsuperscript{nd} and higher harmonics are generally much smaller than the fundamental.

The magnitude of the harmonics generally decrease rapidly with $k$ for low distortion signals.
Distortion Analysis

\[ f(t) \text{ is band-limited to frequency } 2\pi f_k \text{ if } A_k=0 \text{ for all } k>k_x \]
Distortion Analysis

Total Harmonic Distortion, THD

\[ THD = \frac{\text{RMS voltage in harmonics}}{\text{RMS voltage of fundamental}} \]

\[ THD = \frac{\sqrt{\left(\frac{A_2}{\sqrt{2}}\right)^2 + \left(\frac{A_3}{\sqrt{2}}\right)^2 + \left(\frac{A_4}{\sqrt{2}}\right)^2 + \ldots}}{\frac{A_1}{\sqrt{2}}} \]

\[ THD = \frac{\sqrt{\sum_{k=2}^{\infty} A_k^2}}{A_1} \]
Distortion Analysis

Spurious Free Dynamic Range, SFDR

The SFDR is the difference between the fundamental and the largest harmonic

\[ |A_k| \]

SFDR

SFDR is usually determined by either the second or third harmonic
Distortion Analysis

In a fully differential symmetric circuit, all even harmonics are absent in the differential output!
**Distortion Analysis**

**Theorem:** In a fully differential symmetric circuit, all even harmonics are absent in the differential output for symmetric differential excitations!

**Proof:** Expanding in a Taylor’s series around $V_{ID}=0$, we obtain

$$V_{OD} = f(V_{ID}) = \sum_{k=0}^{\infty} h_k V_{ID}^k$$

Assume $V_{ID}=K\sin(\omega t)$ W.L.O.G. assume $K=1$

$$V_{O1} = \sum_{k=0}^{\infty} h_k [\sin(\omega t)]^k$$

$$V_{O2} = \sum_{k=0}^{\infty} h_k [-\sin(\omega t)]^k$$

$$V_{OD} = V_{O1} - V_{O2} = \sum_{k=0}^{\infty} h_k \left([\sin(\omega t)]^k - [-\sin(\omega t)]^k \right) = \sum_{k=0}^{\infty} h_k \left([\sin(\omega t)]^k - (-1)^k [\sin(\omega t)]^k \right)$$

Observe the even-ordered harmonics are absent in this last sum.
Distortion Analysis

How are spectral components determined?

**By integral**

\[
A_k = \frac{1}{\omega T} \left( \int_{t_1}^{t_1+T} f(t) e^{-j k \omega t} \, dt + \int_{t_1}^{t_1+T} f(t) e^{j k \omega t} \, dt \right)
\]

or

\[
a_k = \frac{2}{\omega T} \int_{t_1}^{t_1+T} f(t) \sin(kt\omega) \, dt \quad b_k = \frac{2}{\omega T} \int_{t_1}^{t_1+T} f(t) \cos(kt\omega) \, dt
\]

Integral is very time consuming, particularly if large number of components are required

**By DFT** (with some restrictions that will be discussed)

**By FFT** (special computational method for obtaining DFT)
Distortion Analysis

How are spectral components determined?

Consider sampling f(t) at uniformly spaced points in time $T_s$ seconds apart.

This gives a sequence of samples $\left\{ f\left(kT_s\right) \right\}_{k=1}^{N}$.
Distortion Analysis

NOTATION:

- \( T \): Period of Excitation
- \( T_S \): Sampling Period
- \( N_P \): Number of periods over which samples are taken
- \( N \): Total number of samples

\[
N_P = \frac{N T_S}{T}
\]

Note: \( N_P \) is not an integer unless a specific relationship exists between \( N \), \( T_S \) and \( T \)
THEOREM: If $N_p$ is an integer and $x(t)$ is band limited to $f_{\text{MAX}}$, then
\[
|A_m| = \frac{2}{N} |X(mN_p + 1)| \quad 0 \leq m \leq h - 1
\]
and
\[X(k) = 0\]
for all $k$ not defined above.

where
\[
\left\langle X(k) \right\rangle_{k=0}^{N-1}
\]
is the DFT of the sequence
\[
\left\langle x(kT_s) \right\rangle_{k=0}^{N-1}
\]
f = $1/T$, and
\[
f_{\text{MAX}} = \frac{f}{2} \cdot \left[ \frac{N}{N_p} \right]
\]
If the hypothesis of the theorem are satisfied, we thus have
Distortion Analysis

If the hypothesis of the theorem are satisfied, we thus have

|X(k)|

FFT is a computationally efficient way of calculating the DFT, particularly when N is a power of 2
End of Lecture 28
FFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1. The sampling window be an integral number of periods

2. \[ N > \frac{2 f_{\text{max}}}{f_{\text{SIGNAL}}} N_p \]
Considerations for Spectral Characterization

• Tool Validation
• FFT Length
• Importance of Satisfying Hypothesis
• Windowing
Considerations for Spectral Characterization

• Tool Validation
• FFT Length
• Importance of Satisfying Hypothesis
• Windowing
FFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1. The sampling window be an integral number of periods
2. \[ N > \frac{2 f_{\text{max}}}{f_{\text{SIGNAL}}} N_p \]
Example

WLOG assume $f_{SIG} = 50\text{Hz}$

$$V_{IN} = \sin(\omega t) + 0.5 \sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

Consider $N_P = 20$  $N = 512$

Recall $20\log_{10}(0.5) = -6.0205999$
Input Waveform
Input Waveform
Input Waveform

Location of First Point if Extended Into Periodic Function
Spectral Response

Rect. Window N=512  Np =20

Frequency

Mag(dB)
Spectral Response

DFT Horizontal Axis Converter to Frequency:  
\[ f_{\text{AXIS}} = f_{\text{SIGNAL}} \frac{n - 1}{N_p} \]
Spectral Response
Fundamental will appear at position 1+Np = 21

Columns 1 through 5

-316.1458 -312.9517 -329.5203 -311.1473 -314.2615

Columns 6 through 10

-315.2584 -330.6258 -317.2896 -312.2316 -311.6335

Columns 11 through 15


Columns 16 through 20

-314.0088 -302.6391 -306.6650 -311.3733 -308.3689

Columns 21 through 25

\[
\begin{array}{c}
-0.0000 \\
307.7012 \\
312.9902 \\
312.8737 \\
305.4320
\end{array}
\]

Observe system noise floor due to both spectral limitations of signal generator and numerical limitations in FFT are below -300db
**Second Harmonic at 1+2Np = 41**

Columns 26 through 30

-307.8301  -309.0737  -305.8503  -312.2772  -315.7544

Columns 31 through 35

-311.9316  -316.0581  -318.3454  -306.4977  -308.6679

Columns 36 through 40

-309.9702  -305.9809  -322.1270  -310.6723  -310.3506

Columns 41 through 45

-6.0206    -309.6071  -314.1026  -307.6405  -302.9277

Columns 46 through 50

-313.0745  -304.2330  -310.8487  -317.7966  -316.3385
Third Harmonic at 1+3Np = 61

Columns 51 through 55

-307.0529 -312.7787 -312.9340 -323.2969 -314.9297

Columns 56 through 60

-318.7605 -303.5929 -305.2994 -310.6430 -306.7613

Columns 61 through 65

-304.8298 -301.4463 -301.1410 -303.1784 -317.8343

Columns 66 through 70

-308.6310 -307.0135 -321.6015 -316.6548 -309.8946

Columns 71 through 75

-306.3472 -323.0110 -319.3267 -314.7873 -310.4085
Fourth Harmonic at 1+4Np = 81

Columns 76 through 80


Columns 81 through 85


Columns 86 through 90

-313.4988 -303.4513 -310.4969 -317.9652 -312.5846

Columns 91 through 95

-309.8121 -311.6403 -312.8374 -310.5414 -308.7807

Columns 96 through 100

-316.7549 -316.3395 -308.4113 -307.3766 -311.0358
Question: How much noise is in the computational environment?

Is this due to quantization in the computational environment or to numerical rounding in the FFT?
Question: How much noise is in the computational environment?

Observation: This noise is nearly uniformly distributed. The level of this noise at each component is around -310dB.
Question: How much noise is in the computational environment?

Assume $A_k = -310$ dB for $0 \leq k \leq N$

$$A_{kDB} = 20 \log_{10} A_k$$

$$A_k \approx 10 \frac{-310}{20} = 10^{-15.5}$$

$$V_{Noise,RMS} \approx \sqrt{\sum_{k=1}^{N-1} A_k^2} = \sqrt{\overline{A}^2} = \sqrt{N \overline{A}}$$

$$V_{Noise,RMS} \approx \sqrt{N} \overline{A} = \sqrt{512} 10^{-15.5} = 1.8 \cdot 10^{-14} = 18fV$$

Note: This computational environment has a very low total computational noise and does not become significant until the 45-bit resolution level is reached !!
Considerations for Spectral Characterization

• Tool Validation

• FFT Length

• Importance of Satisfying Hypothesis

• Windowing
Example

WLOG assume $f_{\text{SIG}}=50\text{Hz}$

\[ V_{\text{IN}} = \sin(\omega t) + 0.5 \sin(2\omega t) \]

\[ \omega = 2\pi f_{\text{SIG}} \]

Consider $N_P=20$  $N=4096$
Spectral Response

Rect. Window N=4096  Np =20

Frequency 0 50 100 150 200

Mag(dB) -350 -300 -250 -200 -150 -100 -50 0

Plot showing the spectral response with a rectangular window and parameters N=4096 and Np=20.
Fundamental will appear at position 1+Np = 21

Columns 1 through 7


Columns 8 through 14

-319.7032 -317.4419 -327.4933 -321.1968 -318.2241 -312.7300 -316.8359

Columns 15 through 21

-315.5166 -316.1801 -307.8072 -304.3414 -301.3326 -301.7993

Columns 22 through 28


Columns 29 through 35

The $k$th harmonic will appear at position $1+k\cdot N_p$.

Columns 36 through 42


Columns 43 through 49

-300.8222 -301.6722 -304.8150 -313.0288 -313.5963 -312.1136 -310.7740

Columns 50 through 56


Columns 57 through 63

-320.2843 -320.9910 -316.8320 -318.3531 -318.4341 -322.1619 -321.6183

Columns 64 through 70

Example

WLOG assume $f_{\text{SIG}}=50\text{Hz}$

$$V_{\text{IN}} = \sin(\omega t) + 0.5 \sin(2\omega t)$$

$$\omega = 2\pi f_{\text{SIG}}$$

Consider $N_P=50$, $N=4096$
Spectral Response
Fundamental will appear at position $1+N_p = 51$

<table>
<thead>
<tr>
<th>Columns 1 through 7</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-322.4309</td>
<td>-325.5445</td>
<td>-322.2645</td>
<td>-321.6226</td>
<td>-319.5894</td>
<td>-323.4895</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 8 through 14</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-321.2981</td>
<td>-316.1855</td>
<td>-312.3071</td>
<td>-310.4889</td>
<td>-309.6790</td>
<td>-309.9436</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 15 through 21</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Columns 22 through 28</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-310.1735</td>
<td>-311.1633</td>
<td>-308.9079</td>
<td>-312.0709</td>
<td>-310.6683</td>
<td>-310.6908</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 29 through 35</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-312.9440</td>
<td>-310.5706</td>
<td>-316.2098</td>
<td>-318.9565</td>
<td>-327.6885</td>
<td>-326.4021</td>
</tr>
</tbody>
</table>
Fundamental will appear at position 1+Np = 51

Columns 36 through 42

Columns 43 through 49

Columns 50 through 56
-309.5231 0 -308.8842 -316.1343 -314.5406 -333.4024 -313.7342

Columns 57 through 63
-319.6023 -314.9029 -316.6932 -314.7123 -311.9567 -312.0200 -309.8825

Columns 64 through 70
-308.7103 -309.8064 -314.9393 -312.4610 -322.7229 -328.0350 -326.6767
$k$th harmonic will appear at position $1 + k \cdot N_p$

Columns 71 through 77


Columns 78 through 84


Columns 85 through 91


Columns 92 through 98

-313.6855 -313.3882 -330.4962 -324.4762 -333.2237 -325.8694 -313.9127

Columns 99 through 105

-315.4869 -308.6364 -6.0206 -309.2723 -314.4098 -316.3311 -328.2626
$k^{th}$ harmonic will appear at position $1+k \cdot Np$

Columns 106 through 112


Columns 113 through 119

-319.9292 -325.4840 -318.0998 -328.0000 -321.7632 -326.5097 -328.5867

Columns 120 through 126


Columns 127 through 133

-315.0684 -308.6315 -312.9640 -309.5056 -311.6251 -316.1369 -316.1064

Columns 134 through 140

-320.4989 -331.2686 -314.3479 -310.0891 -308.0023 -308.1556 -309.0616
The $k$th harmonic will appear at position $1+k\cdot Np$

Columns 141 through 147

-311.2372 -312.6180 -319.0565 -325.6750 -323.7759 -320.7444 -318.0752

Columns 148 through 154


Columns 155 through 161


Columns 162 through 168


Columns 169 through 175

Considerations for Spectral Characterization

**FFT Length**

- FFT Length does not affect the computational noise floor.
- Although not shown here yet, FFT length does reduce the quantization noise floor coefficients.

If we assume $E_{QUANT}$ is fixed:

$$E_{QUANT} \approx \sqrt{\sum_{k=2}^{2^{n_{DFT}}} A_k^2}$$

If the $A_k$'s are constant and equal:

$$E_{QUANT} \approx A_k \frac{2^{n_{DFT}}}{2}$$

Solving for $A_k$, obtain:

$$A_k \approx \frac{E_{QUANT}}{2^{n_{DFT}} / 2}$$

If input is full-scale sinusoid with only amplitude quantization with $n$-bit res,

$$E_{QUANT} \approx \frac{X_{LSB}}{\sqrt{12}} = \frac{X_{REF}}{\sqrt{3} \cdot 2^{n+1}}$$
Considerations for Spectral Characterization

**FFT Length**

\[
E_{QUANT} \approx \frac{X_{LSB}}{\sqrt{12}} = \frac{X_{REF}}{\sqrt{3} \cdot 2^{n+1}}
\]

Substituting for \(E_{QUANT}\), obtain

\[
A_k \approx \frac{X_{REF}}{\sqrt{3} \cdot 2^{n+1} 2^{n_{DFT}/2}}
\]

This value for \(A_k\) thus decreases with the length of the DFT window.
Considerations for Spectral Characterization

- Tool Validation
- FFT Length
- Importance of Satisfying Hypothesis
  - NP is an integer
  - Band-limited excitation
- Windowing
Example

WLOG assume $f_{SIG} = 50\text{Hz}$

$V_{IN} = \sin(\omega t) + 0.5 \sin(2\omega t)$

$\omega = 2\pi f_{SIG}$

Consider $N_P = 20.2$  $N = 4096$

Recall $20\log_{10}(0.5) = -6.0205999$
Input Waveform
Input Waveform
Input Waveform
Input Waveform
Spectral Response

Rect. Window  N=4096  Np =20.2

![Graph showing spectral response with peaks centered around 50 and 100, labeled with Mag(dB) on the y-axis and Frequency on the x-axis.](image-url)
<table>
<thead>
<tr>
<th>Columns 1 through 7</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-35.0366</td>
<td>-35.0125</td>
</tr>
<tr>
<td>-34.9400</td>
<td>-34.8182</td>
</tr>
<tr>
<td>-34.6458</td>
<td>-34.4208</td>
</tr>
<tr>
<td>-34.1403</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 8 through 14</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-33.8005</td>
<td>-33.3963</td>
</tr>
<tr>
<td>-32.9206</td>
<td>-32.3642</td>
</tr>
<tr>
<td>-31.7144</td>
<td>-30.9535</td>
</tr>
<tr>
<td>-30.0563</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 15 through 21</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-28.9855</td>
<td>-27.6830</td>
</tr>
<tr>
<td>-26.0523</td>
<td>-23.9155</td>
</tr>
<tr>
<td>-20.8888</td>
<td>-15.8561</td>
</tr>
<tr>
<td>-0.5309</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 22 through 28</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-12.8167</td>
<td>-20.1124</td>
</tr>
<tr>
<td>-24.2085</td>
<td>-27.1229</td>
</tr>
<tr>
<td>-29.4104</td>
<td>-31.2957</td>
</tr>
<tr>
<td>-32.8782</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 29 through 35</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-34.1902</td>
<td>-35.2163</td>
</tr>
<tr>
<td>-35.9043</td>
<td>-36.1838</td>
</tr>
<tr>
<td>-35.9965</td>
<td>-35.3255</td>
</tr>
<tr>
<td>-34.1946</td>
<td></td>
</tr>
</tbody>
</table>

**Note there is a dramatic increase in the noise floor and a significant change in and spreading of the fundamental!!**
\textit{k}^{\text{th}} \text{ harmonic will appear at position } 1+k\cdot N_p

<table>
<thead>
<tr>
<th>Columns 36 through 42</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Columns 43 through 49</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Columns 50 through 56</th>
</tr>
</thead>
<tbody>
<tr>
<td>-33.0833  -33.8720  -34.5759  -35.2113  -35.7902  -36.3218  -36.8133</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 57 through 63</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Columns 64 through 70</th>
</tr>
</thead>
</table>
$k^{\text{th}}$ harmonic will appear at position $1+k\cdot N_p$

Columns 36 through 42


Columns 43 through 49


Columns 50 through 56

-33.0833  -33.8720  -34.5759  -35.2113  -35.7902  -36.3218  -36.8133

Columns 57 through 63


Columns 64 through 70

Observations

- Modest change in sampling window of 0.2 out of 20 periods (1%) results in a big error in both fundamental and harmonic.
- More importantly, dramatic raise in the noise floor !!! (from over -300dB to only -12dB)
Example

WLOG assume $f_{SIG} = 50$Hz

$$V_{IN} = \sin(\omega t) + 0.5 \sin(2\omega t)$$

$$\omega = 2\pi f_{SIG}$$

Consider $N_p = 20.01$ $N = 4096$

Deviation from hypothesis is .05% of the sampling window
Input Waveform
Input Waveform
Input Waveform
Input Waveform
Spectral Response

![Graph of spectral response with Rectangular Window, N=4096, and Np=20.01. The x-axis represents frequency, and the y-axis represents magnitude in dB. The graph shows two peaks and a gradual decrease towards the right.]
Fundamental will appear at position $1+N_p = 21$

<table>
<thead>
<tr>
<th>Columns 1 through 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>-89.8679  -83.0583  -77.7239  -74.2607  -71.6830  -69.5948  -67.8044</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 8 through 14</th>
</tr>
</thead>
<tbody>
<tr>
<td>-66.2037  -64.7240  -63.3167  -61.9435  -60.5707  -59.1642  -57.6859</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 15 through 21</th>
</tr>
</thead>
<tbody>
<tr>
<td>-56.0866  -54.2966  -52.2035  -49.6015  -46.0326  -40.0441  -0.0007</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 22 through 28</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Columns 29 through 35</th>
</tr>
</thead>
<tbody>
<tr>
<td>-62.2078  -65.1175  -69.1845  -76.9560  -81.1539  -69.6230  -64.0636</td>
</tr>
</tbody>
</table>
The $k^{th}$ harmonic will appear at position $1+k\cdot N_p$.

Columns 36 through 42:

|----------|----------|----------|----------|----------|----------|----------|
Observations

• Modest change in sampling window of 0.01 out of 20 periods (.05%) still results in a modest error in both fundamental and harmonic

• More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -40dB)

• Errors at about the 6-bit level !
Example

WLOG assume $f_{\text{SIG}}=50\text{Hz}$

$$V_{\text{IN}} = \sin(\omega t) + 0.5 \sin(2\omega t)$$

$$\omega = 2\pi f_{\text{SIG}}$$

Consider $N_{\text{P}}=20.001$ $N=4096$

Deviation from hypothesis is .005% of the sampling window
Spectral Response

Rect. Window N=4096 Np =20.001

Frequency

Mag(dB)
Fundamental will appear at position $1+N_p = 21$

<table>
<thead>
<tr>
<th>Columns 1 through 7</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-112.2531</td>
<td>-103.4507</td>
<td>-97.8283</td>
<td>-94.3021</td>
<td>-91.7015</td>
<td>-89.6024</td>
<td>-87.8059</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 8 through 14</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-86.2014</td>
<td>-84.7190</td>
<td>-83.3097</td>
<td>-81.9349</td>
<td>-80.5605</td>
<td>-79.1526</td>
<td>-77.6726</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 15 through 21</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-76.0714</td>
<td>-74.2787</td>
<td>-72.1818</td>
<td>-69.5735</td>
<td>-65.9919</td>
<td>-59.9650</td>
<td>0.0001</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 22 through 28</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-60.0947</td>
<td>-66.2917</td>
<td>-70.0681</td>
<td>-72.9207</td>
<td>-75.3402</td>
<td>-77.5767</td>
<td>-79.8121</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 29 through 35</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-82.2405</td>
<td>-85.1651</td>
<td>-89.2710</td>
<td>-97.2462</td>
<td>-101.0487</td>
<td>-89.5195</td>
<td>-83.9851</td>
<td></td>
</tr>
</tbody>
</table>
\( k^{th} \) harmonic will appear at position \( 1 + k \cdot Np \)

Columns 36 through 42

-79.8472  -76.1160  -72.2601  -67.6621  -60.7642  -6.0220  -59.3448

Columns 43 through 49

-64.8177  -67.8520  -69.9156  -71.4625  -72.6918  -73.7078  -74.5718

Columns 50 through 56

-75.3225  -75.9857  -76.5796  -77.1173  -77.6087  -78.0613  -78.4809

Columns 57 through 63

-78.8721  -79.2387  -79.5837  -79.9096  -80.2186  -80.5125  -80.7927
Observations

- Modest change in sampling window of 0.01 out of 20 periods (.005%) results in a small error in both fundamental and harmonic
- More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -60dB)
- Errors at about the 10-bit level!
Spectral Response

Rect. Window  N=4096  Np =20.0001

![Spectral Response Graph](image-url)
**Fundamental will appear at position 1+Np = 21**

<table>
<thead>
<tr>
<th>Columns 1 through 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>-130.4427 -123.1634 -117.7467 -114.2649 -111.6804 -109.5888 -107.7965</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 8 through 14</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Columns 15 through 21</th>
</tr>
</thead>
<tbody>
<tr>
<td>-96.0691  -94.2764  -92.1793  -89.5706  -85.9878  -79.9571  0.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 22 through 28</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Columns 29 through 35</th>
</tr>
</thead>
</table>
The \( k \)th harmonic will appear at position \( 1+k \cdot N_p \)

Columns 36 through 42


Columns 43 through 49

-84.8247  -87.8566  -89.9190  -91.4652  -92.6940  -93.7098  -94.5736

Columns 50 through 56

-95.3241  -95.9872  -96.5810  -97.1187  -97.6100  -98.0625  -98.4821

Columns 57 through 63

-98.8732  -99.2398  -99.5847  -99.9107  -100.2197  -100.5135  -100.7937

Columns 64 through 70
Observations

• Modest change in sampling window of 0.001 out of 20 periods (0.0005%) results in a small error in both fundamental and harmonic

• More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -80dB)

• Errors at about the 13-bit level !
Lecture 33

Spectral Characterization

Distortion Analysis

• Time Quantization Effects
• Spectral Characteristic of DAC
  – Time and Amplitude Quantization
THEOREM: If $N_p$ is an integer and $x(t)$ is band limited to $hf$, then

$$|A_m| = \frac{2}{N}|X(mN_p + 1)| \quad 0 \leq m \leq h - 1$$

and $X(k) = 0$ for all $k$ not defined above.

where $\langle X(k) \rangle_{k=0}^{N-1}$ is the DFT of the sequence $\langle x(kT_s) \rangle_{k=0}^{N-1}$

and $f = 1/T$
Spectral Response
Review

**FFT Examples**

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1. The sampling window be an integral number of periods

2. 

\[ N > \frac{2f_{\text{max}}}{f_{\text{SIGNAL}}} N_p \]
Spectral Response

Rect. Window  N=4096  Np =20.01
Observations

- Modest change in sampling window of 0.001 out of 20 periods (.0005%) results in a small error in both fundamental and harmonic.
- More importantly, substantial raise in the computational noise floor!!! (from over -300dB to only -80dB).
- Errors at about the 13-bit level!
Considerations for Spectral Characterization

• Tool Validation

• FFT Length

• Importance of Satisfying Hypothesis
  - NP is an integer
  - Band-limited excitation

• Windowing
FFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1. The sampling window be an integral number of periods

2. $N > \frac{2f_{\text{max}}}{f_{\text{SIGNAL}}} N_P$
Example

If $f_{SIG}=50\text{Hz}$

and $N_P=20$  $N=512$

$$N > \frac{2f_{max}}{f_{SIGNAL}} N_P$$

$f_{max} < 640\text{Hz}$
Example

Consider \( N_P=20 \) \( N=512 \)

If \( f_{SIG}=50\text{Hz} \)

\[
V_{IN} = \sin(\omega t) + 0.5 \sin(2\omega t) + 0.5 \sin(14\omega t)
\]

\[
\omega = 2\pi f_{SIG}
\]

(i.e. a component at 700 Hz which violates the band limit requirement)

Recall \( 20\log_{10}(0.5)=-6.0205999 \)
Effects of High-Frequency Spectral Components
Effects of High-Frequency Spectral Components
Effects of High-Frequency Spectral Components
Effects of High-Frequency Spectral Components

\[ f_{\text{high}} = 14f_0 \]

<table>
<thead>
<tr>
<th>Columns 1 through 7</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Columns 8 through 14</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-299.0778</td>
<td>-292.3045</td>
<td>-297.0529</td>
<td>-301.4639</td>
<td>-297.3332</td>
<td>-309.6947</td>
<td>-308.2308</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 15 through 21</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-297.3710</td>
<td>-316.5113</td>
<td>-293.5661</td>
<td>-294.4045</td>
<td>-293.6881</td>
<td>-292.6872</td>
<td>0.0000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 22 through 28</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Columns 29 through 35</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
## Effects of High-Frequency Spectral Components

\[ f_{\text{high}} = 14 f_0 \]

<table>
<thead>
<tr>
<th>Columns 36 through 42</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Columns 43 through 49</th>
</tr>
</thead>
<tbody>
<tr>
<td>-298.9215  -309.4829  -306.7363  -293.0808  -300.0882  -306.5530  -302.9962</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 50 through 56</th>
</tr>
</thead>
<tbody>
<tr>
<td>-318.4706  -294.8956  -304.4663  -300.8919  -298.7732  -301.2474  -293.3188</td>
</tr>
</tbody>
</table>
Effects of High-Frequency Spectral Components

Aliased components at

\[ f_{\text{alias}} = 2f_{\text{sample}} - f \]

\[ f_{\text{alias}} = 2 \cdot 12.8f_{\text{sig}} - 14f_{\text{sig}} = 11.6f_{\text{sig}} \]

thus position in sequence = \(1 + N_p \frac{f_{\text{alias}}}{f_{\text{sig}}} = 1 + 20 \cdot 11.6 = 233\)

Columns 225 through 231

-296.8883 -292.8175 -295.8882 -286.7494 -300.3477 -284.4253 -282.7639

Columns 232 through 238


Columns 239 through 245

-299.1299 -305.8361 -295.1772 -295.1670 -300.2698 -293.6406 -304.2886

Columns 246 through 252

-302.0233 -306.6100 -297.7242 -305.4513 -300.4242 -298.1795 -299.0956
Effects of High-Frequency Spectral Components

Rect. Window N=512  Np =20

f_{high}=24 f_0
Effects of High-Frequency Spectral Components

Rect. Window N=512 Np =20

f_{high}=25 f_0
Effects of High-Frequency Spectral Components
Effects of High-Frequency Spectral Components

Rect. Window  N=512  Np =20

f_{high}=24.4f_0

Mag(dB)

Frequency
Effects of High-Frequency Spectral Components

Rect. Window $N=512$ $N_p=20$

$f_{\text{high}}=24.5f_0$

![Graph showing high-frequency spectral components](image)
Observations

• Aliasing will occur if the band-limited part of the hypothesis for using the DFT is not satisfied
• Modest aliasing will cause high frequency components that may or may not appear at a harmonic frequency
• More egregious aliasing can introduce components near or on top of fundamental and lower-order harmonics
• Important to avoid aliasing if the DFT is used for spectral characterization
End of Lecture 31
Considerations for Spectral Characterization

- Tool Validation

- FFT Length

- Importance of Satisfying Hypothesis
  - NP is an integer
  - Band-limited excitation

- Windowing
Are there any strategies to address the problem of requiring precisely an integral number of periods to use the FFT?

Windowing is sometimes used

Windowing is sometimes misused
Windowing

Windowing is the weighting of the time domain function to maintain continuity at the end points of the sample window

Well-studied window functions:

• Rectangular
• Triangular
• Hamming
• Hanning
• Blackman
Rectangular Window

- Sometimes termed a boxcar window
- Uniform weight
- Can append zeros
- Without appending zeros equivalent to no window
Rectangular Window

Assume $f_{\text{SIG}} = 50\text{Hz}$

$$V_{IN} = \sin(\omega t) + 0.5 \sin(2\omega t)$$

$$\omega = 2\pi f_{\text{SIG}}$$

Consider $N_P = 20.1$  $N = 512$
### Rectangular Window

Columns 1 through 7


Columns 8 through 14

-44.4065 -43.4052 -42.3602 -41.2670 -40.1146 -38.8851 -37.5520

Columns 15 through 21

-36.0756 -34.3940 -32.4043 -29.9158 -26.5087 -20.9064 -0.1352

Columns 22 through 28


Columns 29 through 35

Rectangular Window

Columns 1 through 7


Columns 8 through 14

-44.4065  -43.4052  -42.3602  -41.2670  -40.1146  -38.8851  -37.5520

Columns 15 through 21

-36.0756  -34.3940  -32.4043  29.9158  -26.5087  -20.9064  -0.1352

Columns 22 through 28


Columns 29 through 35


Energy spread over several frequency components
Triangular Window
Triangular Window

![Graph of Triangular Window N=512, Np=20.1]
Triangular Window
## Triangular Window

Columns 1 through 7

-100.8530  -72.0528  -99.1401  -68.0110  -95.8741  -63.9944  -92.5170

Columns 8 through 14

-60.3216  -88.7000  -56.7717  -85.8679  -52.8256  -82.1689  -48.3134

Columns 15 through 21

-77.0594  -42.4247  -70.3128  -33.7318  -58.8762  -15.7333  -6.0918

Columns 22 through 28

-12.2463  -57.0917  -32.5077  -68.9492  -41.3993  -74.6234  -46.8037

Columns 29 through 35

-77.0686  -50.1054  -77.0980  -51.5317  -75.1218  -50.8522  -71.2410
Hamming Window
Hamming Window
Comparison with Rectangular Window
# Hamming Window

Columns 1 through 7

-70.8278  -70.6955  -70.3703  -69.8555  -69.1502  -68.3632  -67.5133

Columns 8 through 14

-66.5945  -65.6321  -64.6276  -63.6635  -62.6204  -61.5590  -60.4199

Columns 15 through 21

-59.3204  -58.3582  -57.8735  -60.2994  -52.6273  -14.4702  -5.4343

Columns 22 through 28

-11.2659  -45.2190  -67.9926  -60.1662  -60.1710  -61.2796  -62.7277

Columns 29 through 35

-64.3642  -66.2048  -68.2460  -70.1835  -71.1529  -70.2800  -68.1145
Hanning Window
Hanning Window
Comparison with Rectangular Window
## Hanning Window

Columns 1 through 7


Columns 8 through 14

-92.4519  -90.4372  -87.7977  -84.9554  -81.8956  -79.3520  -75.8944

Columns 15 through 21

-72.0479  -67.4602  -61.7543  -54.2042  -42.9597  -13.4511  -6.0601

Columns 22 through 28

-10.8267  -40.4480  -53.3906  -61.8561  -68.3601  -73.9966  -79.0757

Columns 29 through 35

-84.4318  -92.7280  -99.4046  -89.0799  -83.4211  -78.5955  -73.9788
Comparison of 4 windows

- Rect. Window N=512  Np =20.1
- Hamming Window N=512  Np =20.1
- Hanning Window N=512  Np =20.1
- Triangular Window N=512  Np =20.1
Comparison of 4 windows
Preliminary Observations about Windows

• Provide separation of spectral components
• Energy can be accumulated around spectral components
• Simple to apply
• Some windows work much better than others

But – windows do not provide dramatic improvement and …
Comparison of 4 windows when sampling hypothesis are satisfied
Comparison of 4 windows
Preliminary Observations about Windows

- Provide separation of spectral components
- Energy can be accumulated around spectral components
- Simple to apply
- Some windows work much better than others

But – windows do not provide dramatic improvement and can significantly degrade performance if sampling hypothesis are met
Issues of Concern for Spectral Analysis

An integral number of periods is critical for spectral analysis

Not easy to satisfy this requirement in the laboratory

Windowing can help but can hurt as well

Out of band energy can be reflected back into bands of interest

Characterization of CAD tool environment is essential

Spectral Characterization of high-resolution data converters requires particularly critical consideration to avoid simulations or measurements from masking real performance
Spectral Characterization

- Distortion Analysis
- Time Quantization Effects
- Spectral Characteristic of DAC
  - Time and Amplitude Quantization
Quantization Effects on Spectral Performance and Noise Floor in DFT

- Assume the effective clock rate (for either an ADC or a DAC) is arbitrarily fast
- Without Loss of Generality it will be assumed that $f_{\text{SIG}}=50\text{Hz}$
- Index on DFT will be listed in terms of frequency (rather than index number)

Matlab File: afft_Quantization.m
Quantization Effects

16,384 pts  res = 4bits  N_p=25

20 msec
Quantization Effects

16,384 pts  res = 4bits  N_P=25

20 msec
Quantization Effects

16,384 pts   res = 4bits
Quantization Effects

Simulation environment:

\[ N_P = 23 \]
\[ f_{SIG} = 50 \text{Hz} \]
\[ V_{REF}: -1V, 1V \]
Res: will be varied
\[ N = 2^n \text{ will be varied} \]
Quantization Effects

Res = 4 bits
Quantization Effects

Res = 4 bits

Rect. Window N=4096 Np =23

Axis of Symmetry
Quantization Effects

Res = 4 bits

Some components very small
Quantization Effects

Res = 4 bits

Set lower display limit at -120dB
Quantization Effects
Res = 4 bits
Quantization Effects
Res = 4 bits
Quantization Effects

Res = 4 bits

Rect. Window N=65536 Np =23

Magnitude (dB) vs Frequency

[Graph showing the magnitude in decibels against frequency, with a peak near the lower end of the frequency range and a decrease as frequency increases.]
Quantization Effects

Res = 4 bits

Rect. Window N=65536 Np =23
Quantization Effects

Res = 4 bits
Quantization Effects

Res = 4 bits
Quantization Effects

Res = 4 bits

Fundamental
Quantization Effects
Res = 10 bits
Quantization Effects

Res = 10 bits

Rect. Window N=256  Np =23

Frequency

Mag(dB)
Quantization Effects

Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits

Rect. Window N=4096  Np =23

Magnitude (dB) vs Frequency

Frequency range: 0 to 9000 Hz
Magnitude range: -180 to 0 dB
Quantization Effects

Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects

Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits

Rect. Window  N=65536  Np =23
Quantization Effects

Res = 10 bits

Res 10 No. points 256 fsig= 50.00 No. Periods 23.00
Rectangular Window

Columns 1 through 5

-55.7419 -120.0000 -85.1461 -106.1614 -89.2395

Columns 6 through 10

-102.3822 -99.5653 -85.7335 -89.1227 -83.0851
<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-87.5203</td>
<td>-78.5459</td>
<td>-93.9801</td>
<td>-89.8324</td>
<td>-94.5461</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-77.6478</td>
<td>-80.8867</td>
<td>-100.8153</td>
<td>-78.7936</td>
<td>-86.2954</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-85.8697</td>
<td>-79.5073</td>
<td>-101.6929</td>
<td>-0.0004</td>
<td>-83.6600</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-83.3148</td>
<td>-74.8410</td>
<td>-89.7384</td>
<td>-91.5556</td>
<td>-86.9109</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-93.0155</td>
<td>-82.1062</td>
<td>-78.4561</td>
<td>-98.7568</td>
<td>-109.4766</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Columns 36 through 40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-98.2999 -84.9383 -115.7328</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-100.0758 -77.1246</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Columns 41 through 45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-86.6455 -82.5379 -98.8707</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-111.1638 -85.9572</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Columns 46 through 50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-85.7575 -92.6227 -83.7312</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-83.4865 -82.4473</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Columns 51 through 55</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-77.4085 -88.0611 -84.5256</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-98.4813 -82.7990</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Columns 56 through 60</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-86.0396 -83.8284 -87.2621</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-97.6189 -94.7694</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Columns 61 through 65</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------------------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-86.9239  -89.5881  -82.8701  -95.5137  -82.3502</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 66 through 70</th>
</tr>
</thead>
<tbody>
<tr>
<td>-74.9482  -83.4468  -94.0629  -95.3199  -95.4482</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 71 through 75</th>
</tr>
</thead>
<tbody>
<tr>
<td>-107.0215  -98.3102  -87.4623  -82.4935  -98.6972</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 76 through 80</th>
</tr>
</thead>
<tbody>
<tr>
<td>-83.1902  -82.2598  -103.0396  -87.2043  -79.1829</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 81 through 85</th>
</tr>
</thead>
<tbody>
<tr>
<td>-76.6723  -87.0770  -91.5964  -82.1222  -78.7656</td>
</tr>
</tbody>
</table>
Columns 86 through 90

-82.9621  -93.0224  -116.8549  -93.7327  -75.6231

Columns 91 through 92

-94.4914  -81.0819
Rectangular Window

Columns 1 through 5

-55.6060  -97.9951  -107.4593  -103.4508  -120.0000

Columns 6 through 10

-96.7808  -105.2905  -96.7395  -104.5281  -90.7582
<table>
<thead>
<tr>
<th>Columns 11 through 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>-85.6641 -101.5338 -120.0000 -87.9656 -99.8947</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 16 through 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>-108.1949 -90.9072 -111.7312 -120.0000 -117.6276</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 21 through 25</th>
</tr>
</thead>
<tbody>
<tr>
<td>-97.1804 -102.6126 -111.4008 -0.0003 -97.1838</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 26 through 30</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Columns 31 through 35</th>
</tr>
</thead>
<tbody>
<tr>
<td>-104.3215 -100.3451 -97.1556 -86.0534 -94.7263</td>
</tr>
<tr>
<td>Columns 36 through 40</td>
</tr>
<tr>
<td>----------------------</td>
</tr>
<tr>
<td>-96.6002</td>
</tr>
<tr>
<td>-105.9608</td>
</tr>
<tr>
<td>-91.7843</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 41 through 45</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-96.9903</td>
<td>-91.2626</td>
</tr>
<tr>
<td>-102.3499</td>
<td>-97.1841</td>
</tr>
<tr>
<td>-99.2579</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 46 through 50</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-91.7837</td>
<td>-102.1146</td>
</tr>
<tr>
<td>-98.7668</td>
<td>-98.8830</td>
</tr>
<tr>
<td>-120.0000</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 51 through 55</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-108.2877</td>
<td>-110.9318</td>
</tr>
<tr>
<td>-97.5933</td>
<td>-94.4604</td>
</tr>
<tr>
<td>-99.6057</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 56 through 60</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-91.1056</td>
<td>-101.5798</td>
</tr>
<tr>
<td>-94.1031</td>
<td>-95.9163</td>
</tr>
<tr>
<td>-83.8407</td>
<td></td>
</tr>
<tr>
<td>Columns 61 through 65</td>
<td></td>
</tr>
<tr>
<td>----------------------</td>
<td></td>
</tr>
<tr>
<td>-93.2650 -103.4274 -103.9702 -98.4092 -91.1825</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 66 through 70</th>
</tr>
</thead>
<tbody>
<tr>
<td>-98.0638 -93.7989 -107.7453 -93.4277 -88.0409</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 71 through 75</th>
</tr>
</thead>
<tbody>
<tr>
<td>-107.3584 -102.5984 -95.3312 -102.9342 -108.5206</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 76 through 80</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Columns 81 through 85</th>
</tr>
</thead>
<tbody>
<tr>
<td>-96.5194 -85.8129 -95.1970 -94.8699 -104.9224</td>
</tr>
</tbody>
</table>
Quantization Effects

With $\text{Vin}=2v \text{ pp}$

$\text{Res} = 10 \text{ bits}$
With $\text{Vin}=1^*0.99$ and $\text{Vos}=0.25\text{LSB}$
With $Vin = 1.999999$ pp
With $V_{in} = 1^{*}.99$ and $V_{os} = .35\text{LSB}$
Res  10   No. points 4096   fsig=  50.00   No. Periods  25.00   Tstep  1.220703e-004
Magnitude of   Fundamental 1.000      2nd Harmonic 0.000

<table>
<thead>
<tr>
<th>Columns 1 through 7</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-56.6785</td>
<td>-65.4098</td>
</tr>
<tr>
<td>-66.2097</td>
<td>-65.5916</td>
</tr>
<tr>
<td>-66.2436</td>
<td>-66.0461</td>
</tr>
<tr>
<td>-66.2097</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 8 through 14</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-66.9055</td>
<td>-66.2436</td>
</tr>
<tr>
<td>-66.2436</td>
<td>-66.1762</td>
</tr>
<tr>
<td>-66.2097</td>
<td>-65.6639</td>
</tr>
<tr>
<td>-66.2436</td>
<td>-65.7315</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 15 through 21</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-66.2097</td>
<td>-66.2800</td>
</tr>
<tr>
<td>-66.2436</td>
<td>-66.6393</td>
</tr>
<tr>
<td>-66.2097</td>
<td>-65.4202</td>
</tr>
<tr>
<td>-66.2436</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 22 through 28</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-66.1363</td>
<td>-66.2097</td>
</tr>
<tr>
<td>-65.6765</td>
<td>-66.2436</td>
</tr>
<tr>
<td>-0.0044</td>
<td>-66.2097</td>
</tr>
<tr>
<td>-65.7635</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 29 through 35</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-66.2436</td>
<td>-66.2196</td>
</tr>
<tr>
<td>-66.2097</td>
<td>-66.0852</td>
</tr>
<tr>
<td>-66.2436</td>
<td>-66.4771</td>
</tr>
<tr>
<td>-66.2097</td>
<td></td>
</tr>
</tbody>
</table>
Quantization Effects

Res = 10 bits
Quantization Effects

Res = 10 bits

[Graph showing frequency response with peak at specific frequency]
Quantization Effects

Res = 10 bits
Quantization Effects

Res = 10 bits

![Graph showing quantization effects with a rectangular window, N=65536, Np=25. The graph plots frequency on the x-axis and magnitude in dB on the y-axis, with peaks indicating quantization effects.](image-url)
Quantization Effects

Res = 5 bits
Quantization Effects

Res = 4 bits
Quantization Effects

Res = 4 bits
Quantization Effects

16,384 pts  res = 4bits
Quantization Effects

16,384 pts  res = 4bits
Quantization Effects

Res = 10 bits
Spectral Characterization

- Distortion Analysis
- Time Quantization Effects
  - Spectral Characteristic of DAC
    - Time and Amplitude Quantization
Spectral Characteristics of DACs and ADCs
Spectral Characteristics of DAC

Periodic Input Signal

Sampling Clock

Sampled Input Signal (showing time points where samples taken)
Spectral Characteristics of DAC

Quantized Sampled Input Signal (with zero-order sample and hold)
Spectral Characteristics of DAC

$T_{DFT\ WINDOW}$

$T_{PERIOD}$

$T_{SIG}$

$T_{CLOCK}$

Sampling Clock

$T_{DFT\ CLOCK}$

DFT Clock
Spectral Characteristics of DAC
Spectral Characteristics of DAC

- Sampling Clock
- DFT Clock
Spectral Characteristics of DAC

Sampled Quantized Signal (zoomed)

DFT Clock

Sampling Clock
Spectral Characteristics of DAC

Consider the following example
- $f_{\text{SIG}} = 50\text{Hz}$
- $k_1 = 230$
- $k_2 = 23$
- $N_P = 1$
- $n_{\text{res}} = 8\text{bits}$
- $X_{\text{in}}(t) = 0.95\sin(2\pi f_{\text{SIG}} t)$ (-0.4455dB)

Thus
- $N_{P1} = 23$
- $\theta_{\text{SR}} = 5$
- $f_{\text{CL}}/f_{\text{SIG}} = 10$

Matlab File: afft_Quantization_DAC.m
DFT Simulation from Matlab

$n_{\text{sam}} = 142.4696$
DFT Simulation from Matlab

Expanded View

Rect. Window N=32768 Np =1

n_{\text{sam}} = 142.4696

Width of this region is f_{\text{CL}}

Analogous to the overall DFT window when directly sampled but modestly asymmetric
nsam = 142.4696

DFT Simulation from Matlab

Expanded View

Rect. Window N=32768 Np=1

n_res=8 bits

n_{sam} = 142.4696
DFT Simulation from Matlab

Expanded View

Rect. Window N=32768 Np =1

n_{sam} = 142.4696

n_{res} = 8 bits
DFT Simulation from Matlab
Expanded View

Rect. Window  N=32768  Np =1

\[ n_{\text{sam}} = 142.4696 \]
\[ f_{\text{SIG}} = 50\text{Hz}, \ k_1 = 23, \ k_2 = 23, \ N_P = 1, \ n_{\text{res}} = 8\text{bits} \quad \text{Xin}(t) = \sin(2\pi f_{\text{SIG}}t) \quad N = 32768 \]

<table>
<thead>
<tr>
<th>Columns 1 through 7</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-44.0825</td>
<td>-84.2069</td>
<td>-118.6751</td>
<td>-89.2265</td>
<td>-120.0000</td>
<td>-76.0893</td>
<td>-120.0000</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 8 through 14</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-90.3321</td>
<td>-120.0000</td>
<td>-69.9163</td>
<td>-120.0000</td>
<td>-88.9097</td>
<td>-120.0000</td>
<td>-85.1896</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 15 through 21</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.0000</td>
<td>-83.0183</td>
<td>-109.4722</td>
<td>-89.4980</td>
<td>-120.0000</td>
<td>-79.6110</td>
<td>-120.0000</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 22 through 28</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-90.2992</td>
<td>-120.0000</td>
<td>-0.5960</td>
<td>-120.0000</td>
<td>-88.5446</td>
<td>-120.0000</td>
<td>-86.0169</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 29 through 35</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.0000</td>
<td>-81.5409</td>
<td>-109.6386</td>
<td>-89.7275</td>
<td>-120.0000</td>
<td>-81.8340</td>
<td>-120.0000</td>
<td></td>
</tr>
</tbody>
</table>
\( f_{\text{SIG}} = 50 \text{Hz} \), \( k_1 = 23 \), \( k_2 = 23 \), \( N_p = 1 \), \( n_{\text{res}} = 8 \text{bits} \)
\[ X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t) \]

\( N = 32768 \)

<table>
<thead>
<tr>
<th>Columns 36 through 42</th>
<th>-90.2331</th>
<th>-120.0000</th>
<th>-69.4356</th>
<th>-120.0000</th>
<th>-88.1400</th>
<th>-120.0000</th>
<th>-86.7214</th>
</tr>
</thead>
<tbody>
<tr>
<td>Columns 43 through 49</td>
<td>-120.0000</td>
<td>-79.6273</td>
<td>-119.1428</td>
<td>-89.9175</td>
<td>-56.7024</td>
<td>-83.0511</td>
<td>-120.0000</td>
</tr>
<tr>
<td>Columns 50 through 56</td>
<td>-90.1331</td>
<td>-120.0000</td>
<td>-75.1821</td>
<td>-120.0000</td>
<td>-87.5706</td>
<td>-120.0000</td>
<td>-87.3205</td>
</tr>
<tr>
<td>Columns 57 through 63</td>
<td>-120.0000</td>
<td>-76.9769</td>
<td>-120.0000</td>
<td>-90.0703</td>
<td>-119.0588</td>
<td>-83.2950</td>
<td>-113.3964</td>
</tr>
<tr>
<td>Columns 64 through 70</td>
<td>-89.9982</td>
<td>-120.0000</td>
<td>-78.4288</td>
<td>-120.0000</td>
<td>-87.0328</td>
<td>-120.0000</td>
<td>-64.5409</td>
</tr>
</tbody>
</table>
\[ f_{\text{SIG}} = 50\text{Hz} , \ k_1 = 23, k_2 = 23, N_p = 1, n_{\text{res}} = 8\text{bits} \quad \text{Xin}(t) = \sin(2\pi f_{\text{SIG}}t) \]

\[ N = 32768 \]

<table>
<thead>
<tr>
<th>Columns 71 through 77</th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.0000  -72.8111  -120.0000  -90.1876  -120.0000  -82.5616  -114.0867</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 78 through 84</th>
</tr>
</thead>
<tbody>
<tr>
<td>-89.8269  -115.6476  -80.6553  -120.0000  -86.3818  -120.0000  -88.3454</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 85 through 91</th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.0000  -63.5207  -120.0000  -90.2704  -120.0000  -80.8524  -120.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 92 through 98</th>
</tr>
</thead>
<tbody>
<tr>
<td>-89.6174  -58.5435  -82.3253  -120.0000  -85.6188  -120.0000  -88.7339</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 99 through 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.0000  -63.8165</td>
</tr>
</tbody>
</table>
DFT Simulation from Matlab

Rect. Window N=131072  Np =1

nres=8 bits

\[ n_{sam} = 569.8783 \]
DFT Simulation from Matlab

Expanded View

Rect. Window \( N=131072 \quad N_p = 1 \)

\[ n_{\text{res}}=8 \text{ bits} \]

\[ n_{\text{sam}} = 569.8783 \]
DFT Simulation from Matlab

Expanded View

Rect. Window  N=131072  Np =1

nres=8 bits

n_{sam} = 569.8783
DFT Simulation from Matlab

Expanded View

Rect. Window N=131072  Np =1

nres=8 bits

nsam = 569.8783
\[ f_{\text{SIG}} = 50\text{Hz} \, , \, k_1 = 23, \, k_2 = 23, \, N_P = 1, \, n_{\text{res}} = 8\text{bits} \quad X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t) \]

\[ N = 131072 \]

Columns 1 through 7

-44.0824  -97.0071  -120.0000  -110.6841  -120.0000  -76.0276  -120.0000

Columns 8 through 14

-103.5227  -120.0000  -109.7590  -120.0000  -89.7127  -120.0000  -107.6334

Columns 15 through 21

-120.0000  -107.8772  -120.0000  -90.3300  -120.0000  -109.5748  -120.0000

Columns 22 through 28

-104.0809  -120.0000  \boxed{-0.5960}  -120.0000  -110.6201  -120.0000  -98.0920

Columns 29 through 35

-120.0000  -95.8006  -120.0000  -110.7338  -120.0000  -82.3448  -120.0000
\[ f_{\text{SIG}}=50\text{Hz} \, , \, k_1=23, \, k_2=23, \, N_{P}=1, \, n_{\text{res}}=8\text{bits} \quad X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t) \]

\[ N=131072 \]

Columns 36 through 42

-102.9185 -120.0000 -109.9276 -120.0000 -88.8778 -120.0000 -107.5734

Columns 43 through 49

-120.0000 -108.1493 -120.0000 -90.7672 -56.7029 -109.3748 -120.0000

Columns 50 through 56

-104.5924 -120.0000 -75.3784 -120.0000 -110.5416 -120.0000 -99.0764

Columns 57 through 63

-120.0000 -94.4432 -120.0000 -110.7692 -120.0000 -86.1442 -120.0000

Columns 64 through 70

-102.2661 -120.0000 -110.0806 -120.0000 -87.7635 -120.0000 -64.4072
\[ f_{\text{SIG}} = 50\text{Hz}, \ k_1 = 23, \ k_2 = 23, \ N_P = 1, \ n_{\text{res}} = 8\text{bits} \quad X_{in}(t) = \sin(2\pi f_{\text{SIG}} t) \]

\[ N = 131072 \]

Columns 71 through 77
-120.0000 -108.4202 -120.0000 -91.0476 -120.0000 -109.1589 -120.0000

Columns 78 through 84
-105.0508 -120.0000 -81.0390 -120.0000 -110.4486 -120.0000 -99.9756

Columns 85 through 91
-120.0000 -92.8919 -120.0000 -110.7904 -120.0000 -88.9028 -120.0000

Columns 92 through 98
-101.5617 \fbox{-58.5437} -110.2183 -120.0000 -86.2629 -120.0000 -105.5980

Columns 99 through 100
-120.0000 -108.6808
Consider the following example

- $f_{\text{SIG}} = 50\text{Hz}$
- $k_1 = 50$
- $k_2 = 5$
- $N_P = 2$
- $n_{\text{res}} = 8\text{bits}$
- $X_{\text{in}}(t) = .95 \sin(2\pi f_{\text{SIG}} t) \quad (-.4455\text{dB})$

Thus

- $N_{P1} = 5$
- $\theta_{\text{SR}} = 5$
- $N_{P2} = 10$
DFT Simulation from Matlab

Rect. Window  N=32768  Np =2

nsam = 327.6800

n_res = 8
DFT Simulation from Matlab

Expanded View

Rect. Window N=32768  Np =2

nsam = 327.6800

n_{res} = 8
DFT Simulation from Matlab

Image Description:
- Rectangular Window, N=256, Np=2
- nsam = 2.5600
- n_res = 8
- Graph showing frequency vs. magnitude in decibels.
DFT Simulation from Matlab

Expanded View

Rect. Window  N=256  Np =2
nsam =  2.5600

\( n_{\text{res}} = 8 \)
\[ f_{\text{SIG}} = 50 \text{Hz}, \quad k_1 = 50, \quad k_2 = 5, \quad N_P = 2, \quad n_{\text{res}} = 8 \text{bits}, \quad X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t) \]
\[ N = 131072 \]

<table>
<thead>
<tr>
<th>Columns 1 through 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>-44.1164 -120.0000 -36.9868 -120.0000 -74.6451 -120.0000 -50.4484</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 8 through 14</th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.0000 -80.1218 -120.0000 -0.6543 -120.0000 -90.0332 -120.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 15 through 21</th>
</tr>
</thead>
<tbody>
<tr>
<td>-43.9537 -120.0000 -73.3311 -120.0000 -49.2755 -120.0000 -56.5832</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 22 through 28</th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.0000 -30.4886 -120.0000 -80.8472 -120.0000 -47.9795 -120.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 29 through 35</th>
</tr>
</thead>
<tbody>
<tr>
<td>-78.0140 -120.0000 -47.7412 -120.0000 -85.9233 -120.0000 -27.8207</td>
</tr>
</tbody>
</table>
\[ f_{\text{SIG}}=50\text{Hz}, \ k_1=50, \ k_2=5, \ N_P=2, \ n_{\text{res}}=8\text{bits}, \ Xin(t) = \sin(2\pi f_{\text{SIG}}t) \]

\[ N=131072 \]

<table>
<thead>
<tr>
<th>Columns 36 through 42</th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.0000  -75.9471  -120.0000  -49.8914  -120.0000  \boxed{-58.4761}  -120.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 43 through 49</th>
</tr>
</thead>
<tbody>
<tr>
<td>-41.7535  -120.0000  -91.4791  -120.0000  -28.1314  -120.0000  -79.7024</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 50 through 56</th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.0000  -50.5858  -120.0000  -78.7241  -120.0000  -31.9459  -120.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 57 through 63</th>
</tr>
</thead>
<tbody>
<tr>
<td>-91.9095  -120.0000  -40.4010  -120.0000  \boxed{-62.1214}  -120.0000  -50.1249</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 64 through 70</th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.0000  -78.2678  -120.0000  -24.9258  -120.0000  -87.6235  -120.0000</td>
</tr>
</tbody>
</table>
\[ f_{\text{SIG}} = 50\text{Hz}, \quad k_1 = 50, \quad k_2 = 5, \quad N_P = 2, \quad n_{\text{res}} = 8\text{bits}, \quad X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t) \]
\[ N = 131072 \]

<table>
<thead>
<tr>
<th>Columns 71 through 77</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-45.3926</td>
<td>-120.0000</td>
</tr>
<tr>
<td>-77.2183</td>
<td>-120.0000</td>
</tr>
<tr>
<td>-48.4567</td>
<td>-120.0000</td>
</tr>
<tr>
<td>-76.6666</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 78 through 84</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.0000</td>
<td>-30.9406</td>
</tr>
<tr>
<td>-120.0000</td>
<td>-69.1777</td>
</tr>
<tr>
<td>-48.8912</td>
<td>-120.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 85 through 91</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-75.7581</td>
<td>-120.0000</td>
</tr>
<tr>
<td>-44.8212</td>
<td>-120.0000</td>
</tr>
<tr>
<td>-88.9694</td>
<td>-120.0000</td>
</tr>
<tr>
<td>-19.1255</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 92 through 98</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.0000</td>
<td>-79.5390</td>
</tr>
<tr>
<td>-120.0000</td>
<td>-50.3103</td>
</tr>
<tr>
<td>-70.6123</td>
<td>-120.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 99 through 105</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-38.8332</td>
<td>-120.0000</td>
</tr>
<tr>
<td>-92.1633</td>
<td>-120.0000</td>
</tr>
<tr>
<td>-34.7560</td>
<td>-120.0000</td>
</tr>
<tr>
<td>-77.1229</td>
<td></td>
</tr>
</tbody>
</table>
DFT Simulation from Matlab

Rect. Window N=1024  Np =2

nsam = 10.2400

n_{res} = 8
DFT Simulation from Matlab

Expanded View

Rect. Window  N=1024  Np =2

\[ n_{\text{sam}} = 10.2400 \]
\[ n_{\text{res}} = 8 \]
\[ f_{\text{SIG}} = 50\text{Hz}, \ k_1 = 50, \ k_2 = 5, \ N_P = 2, \ n_{\text{res}} = 8\text{bits}, \ Xin(t) = \sin(2\pi f_{\text{SIG}}t) \]
\[ N = 1024 \]

Columns 1 through 7

\[-44.0739 \ -120.0000 \ -53.8586 \ -120.0000 \ -91.9997 \ -120.0000 \ -50.3884 \]

Columns 8 through 14

\[-120.0000 \ -91.3235 \ -120.0000 \ -0.6017 \ -120.0000 \ -89.9100 \ -120.0000 \]

Columns 15 through 21

\[-41.0786 \ -120.0000 \ -86.6863 \ -120.0000 \ -48.5379 \ -120.0000 \ -56.7320 \]

Columns 22 through 28

\[-120.0000 \ -53.4112 \ -120.0000 \ -103.7582 \ -120.0000 \ -54.1209 \ -120.0000 \]

Columns 29 through 35

\[-98.4283 \ -120.0000 \ -51.2204 \ -120.0000 \ -92.1630 \ -120.0000 \ -39.9145 \]
\[ f_{\text{SIG}}=50\text{Hz}, \ k_1=50, \ k_2=5, \ N_P=2, \ n_{res}=8\text{bits}, \ Xin(t) =\sin(2\pi f_{\text{SIG}}t) \]

\[ N=1024 \]

<table>
<thead>
<tr>
<th>Columns 36 through 42</th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.0000  -86.0994  -120.0000  -46.4571  -120.0000  -58.5568  -120.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 43 through 49</th>
</tr>
</thead>
<tbody>
<tr>
<td>-45.7332  -120.0000  -88.7034  -120.0000  -52.7530  -120.0000  -102.0744</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 50 through 56</th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.0000  -54.2124  -120.0000  -101.8321  -120.0000  -52.6742  -120.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 57 through 63</th>
</tr>
</thead>
<tbody>
<tr>
<td>-89.3186  -120.0000  -45.3675  -120.0000  -62.0430  -120.0000  -46.7029</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 64 through 70</th>
</tr>
</thead>
<tbody>
<tr>
<td>-120.0000  -85.3723  -120.0000  -40.6886  -120.0000  -92.0718  -120.0000</td>
</tr>
</tbody>
</table>
\[ f_{SIG}=50\text{Hz}, \ k_1=50, \ k_2=5, \ N_P=2, \ n_{res}=8\text{bits}, \ Xin(t) = \sin(2\pi f_{SIG}t) \]

\[ N=1024 \]

Columns 71 through 77

\[-51.9029 \ -120.0000 \ -98.8650 \ -120.0000 \ -54.1376 \ -120.0000 \ -103.6450 \]

Columns 78 through 84

\[-120.0000 \ -53.3554 \ -120.0000 \ -68.6244 \ -120.0000 \ -48.3107 \ -120.0000 \]

Columns 85 through 91

\[-85.8692 \ -120.0000 \ -41.9049 \ -120.0000 \ -89.7301 \ -120.0000 \ -19.6301 \]

Columns 92 through 98

\[-120.0000 \ -91.5501 \ -120.0000 \ -50.5392 \ -120.0000 \ -92.8884 \ -120.0000 \]

Columns 99 through 105

\[-53.8928 \ -120.0000 \ -104.2832 \ -120.0000 \ -53.8225 \ -120.0000 \ -91.0209 \]
Spectral Characteristics of DAC

Consider the following example

- \( f_{\text{SIG}} = 50\text{Hz} \)
- \( k_1 = 11 \)
- \( k_2 = 1 \)
- \( N_P = 2 \)
- \( n_{\text{res}} = 12\text{bits} \)
- \( X_{\text{in}}(t) = 0.95\sin(2\pi f_{\text{SIG}}t) \) (-0.4455dB)

Thus

- \( N_{P_1} = 1 \)
- \( \theta_{SR} = 11 \)
- \( N_{P_2} = 2 \)
DFT Simulation from Matlab

Rec Win N=4096 Np =2 Nsam = 186.181818 nres = 12 fCL/fsig = 11 fDFT/fsig = 2048
DFT Simulation from Matlab
DFT Simulation from Matlab

![Graph showing the magnitude of frequency response with various parameters such as N=4096, Np=2, Nsam=186.181818, nres=12, fCL/fsig=11, fDFT/fsig=2048. The graph plots frequency on the x-axis and magnitude in dB on the y-axis, displaying a series of peaks and troughs.]
DFT Simulation from Matlab
DFT Simulation from Matlab

Rec Win  N=65536  Np =2  Nsam = 2978.90909  nres = 12  fCL/fsig = 11  fDFT/fsig = 32768

![DFT Simulation Graph](image)

- Frequency (Hz)
- Magnitude (dB)
DFT Simulation from Matlab

Rec Win N=65536 Np =2 Nsam = 2978.90909 nres = 12 fCL/fsig = 11 fDFT/fsig = 32768

Magnitude (dB) vs Frequency
Spectral Characteristics of DAC

Consider the following example

- $f_{\text{SIG}} = 50\text{Hz}$
- $k_1 = 230$
- $k_2 = 23$
- $N_P = 1$
- $n_{\text{res}} = 12\text{bits}$
- $X_{\text{in}}(t) = .95\sin(2\pi f_{\text{SIG}} t) (-.4455\text{dB})$

Thus

- $N_{P1} = 23$
- $\theta_{\text{SR}} = 10$
- $N_{P2} = 23$
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab

Rec Win  N=65536  Np =1  Nsam = 284.93913  nres = 12  fCL/fsig = 10  fDFT/fsig = 2849.3913

![Graph showing DFT Simulation from Matlab](image)
DFT Simulation from Matlab
DFT Simulation from Matlab

\[ f_{\text{SIG}} = 50\text{Hz} \quad k_1 = 230 \quad k_2 = 23 \quad N_P = 1 \quad n_{\text{res}} = 12\text{bits} \quad X_{\text{in}}(t) = 0.95\sin(2\pi f_{\text{SIG}}t) \quad (-0.4455\text{dB}) \quad N_{P_1} = 23 \quad \theta_{SR} = 10 \quad N_{P_2} = 23 \]

Columns 1 through 7

\[-68.1646 \quad -94.7298 \quad -120.0000 \quad -90.8893 \quad -120.0000 \quad -75.8402 \quad -120.0000\]

Columns 8 through 14

\[-97.7128 \quad -120.0000 \quad -69.7549 \quad -120.0000 \quad -90.5257 \quad -120.0000 \quad -95.1113\]

Columns 15 through 21

\[-120.0000 \quad -94.3119 \quad -120.0000 \quad -91.2004 \quad -120.0000 \quad -79.4167 \quad -120.0000\]

Columns 22 through 28

\[-97.6931 \quad -120.0000 \quad \boxed{-0.5886} \quad -120.0000 \quad -90.1044 \quad -120.0000 \quad -95.4585\]

Columns 29 through 35

\[-120.0000 \quad -93.8547 \quad -120.0000 \quad -91.4631 \quad -120.0000 \quad -81.9608 \quad -120.0000\]
DFT Simulation from Matlab

Columns 36 through 42

-97.6535 -120.0000 -69.6068 -120.0000 -89.6188 -120.0000 -95.7721

Columns 43 through 49

-120.0000 -93.3545 -120.0000 -91.6806 -80.7859 -83.9353 -120.0000

Columns 50 through 56

-97.5940 -120.0000 -75.5346 -120.0000 -89.0602 -120.0000 -96.0458

Columns 57 through 63

-120.0000 -92.8067 -120.0000 -91.8555 -85.5462 -120.0000

Columns 64 through 70

-97.5144 -120.0000 -78.9551 -120.0000 -88.4176 -120.0000 -88.0509
DFT Simulation from Matlab

Columns 71 through 77
-120.0000  -92.2056  -120.0000  -91.9896  -120.0000  -86.9037  -120.0000

Columns 78 through 84
-97.4143  -120.0000  -81.3430  -120.0000  -87.6762  -120.0000  -96.6112

Columns 85 through 91
-120.0000  -91.5441  -120.0000  -92.0844  -120.0000  -88.0732  -120.0000

Columns 92 through 98
-97.2936  **-82.6264**  -83.1604  -120.0000  -86.8155  -120.0000  -96.8068

Columns 99 through 100
-120.0000  -90.8133
Spectral Characteristics of DAC

Consider the following example

- \( f_{\text{SIG}} = 50\text{Hz} \)
- \( k_1 = 230 \)
- \( k_2 = 23.1 \)
- \( N_P = 1 \)
- \( n_{\text{res}} = 12\text{bits} \)
- \( X_{\text{in}}(t) = .95\sin(2\pi f_{\text{SIG}} t) \) \((-0.4455\text{dB})\)

Thus

- \( N_{P1} = 23.1 \)
- \( \theta_{SR} = 9.957 \)
- \( N_{P2} = 23.1 \)
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab
Spectral Characteristics of DAC

Consider the following example

- $f_{\text{SIG}}=50\text{Hz}$
- $k_1=230$
- $k_2=23$
- $N_P=1$
- $n_{\text{res}}=12\text{bits}$
- $X_{\text{in}}(t) = 0.88\sin(2\pi f_{\text{SIG}}t) + 0.1\sin(2\pi f_{\text{SIG}}t)$
- (-1.11\text{db fundamental, } -20\text{dB } 2^{\text{nd}} \text{ harmonic})

Thus

- $N_{P_1}=23$
- $\theta_{SR}=10$
- $N_{P_2}=23$
DFT Simulation from Matlab

Rec Win N=65536 Np =1 Nsam = 284.93913 nres = 12 fCL/fsig = 10 fDFT/fsig = 2849.3913
DFT Simulation from Matlab
DFT Simulation from Matlab

\[ f_{\text{SIG}} = 50 \text{Hz} \quad k_1 = 230 \quad k_2 = 23 \quad N_p = 1 \quad n_{\text{res}} = 12 \text{bits} \quad X(t) = 0.88 \sin(2\pi f_{\text{SIG}} t) + 0.1 \sin(2\pi f_{\text{SIG}} t) \quad (-1.11 \text{dB fundamental, } -20 \text{dB 2nd harmonic}) \quad N_{p_1} = 23 \quad \theta_{\text{SR}} = 10 \quad N_{p_2} = 23 \]

<table>
<thead>
<tr>
<th>Columns 1 through 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>-68.2448  -95.4048 -103.0624 -91.5534 -94.3099 -76.5052 -107.8586</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 8 through 14</th>
</tr>
</thead>
<tbody>
<tr>
<td>-98.3634 -107.7150 -70.4198 -97.2597 -91.1898 -103.5449 -95.7898</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 15 through 21</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Columns 22 through 28</th>
</tr>
</thead>
<tbody>
<tr>
<td>-98.3435 -107.5614 1.2534 -99.6919 -90.7685 -103.9860 -96.1429</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Columns 29 through 35</th>
</tr>
</thead>
</table>
DFT Simulation from Matlab

Columns 36 through 42

-98.3035 -107.3983 -70.2715 -101.8108 -90.2829 -104.3909 -96.4685

Columns 43 through 49


Columns 50 through 56

-98.2433 -107.2276 -76.1993 -103.7144 -89.7244 -104.7634 -96.7781

Columns 57 through 63

-108.8537 -93.4756 -100.5602 -92.5195 -83.3389 -86.2119 -108.3343

Columns 64 through 70

-98.1627 -107.0564 -79.6196 -105.4341 -89.0818 -105.1065 -82.5417
## DFT Simulation from Matlab

Columns 71 through 77

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>

Columns 78 through 84

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>

Columns 85 through 91

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>

Columns 92 through 98

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-97.9383</td>
<td><strong>-82.1713</strong></td>
<td>-83.8248</td>
<td>-108.1091</td>
<td>-87.4797</td>
<td>-105.7091</td>
<td>-97.4305</td>
</tr>
</tbody>
</table>

Columns 99 through 105

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>