Lecture 28

Data Converters

- INL of ADC
- Differential Nonlinearity
- Spectral Performance
Integral Nonlinearity (DAC)

Nonideal DAC

INL often expressed in LSB

\[ \text{INL}_k = \frac{x_{\text{OUT}}(k) - x_{\text{OF}}(k)}{x_{\text{LSB}}} \]

\[ \text{INL} = \max_{0 \leq k \leq N-1} \left\{ |\text{INL}_k| \right\} \]

- INL is often the most important parameter of a DAC
- INL_0 and INL_{N-1} are 0 (by definition)
- There are N-2 elements in the set of INL_k that are of concern
- INL is almost always nominally 0 (i.e. designers try to make it 0)
- INL is a random variable at the design stage
- INL_k is a random variable for 0<k<N-1
- INL_k and INL_{k+j} are almost always correlated for all k,j (not incl 0, N-1)
- Fit Line is a random variable
- INL is the N-2 order statistic of a set of N-2 correlated random variables
**ENOBO of DAC**

Nonideal DAC

- Concept of Equivalent Number of Bits (ENOBO) is to assess performance of an actual DAC to that of an ideal DAC at an “equivalent” resolution level.

- Several different definitions of ENOB exist for a DAC.

- Here will define ENOB as determined by the actual INL performance.

- Will use subscript to define this ENOB, e.g. $\text{ENOBO}_{\text{INL}}$. 

![Diagram showing INL and equivalent number of bits](image)
Premise: A good DAC is often designed so that the INL is equal to $\frac{1}{2}$ LSB. Thus will assume that if an n-bit DAC has an INL of $\frac{1}{2}$ LSB that the $\text{ENOB}_{\text{INL}}=n$.

Thus define the effective number of bits, $n_{\text{EFF}}$ by the expression

$$\frac{\text{INL}}{V_{\text{REF}}} = \frac{1}{2} \cdot \frac{1}{2^{n_{\text{EFF}}}} = \frac{1}{2^{n_{\text{EFF}}+1}} \quad \quad \Rightarrow \quad \quad n_{\text{EFF}} = \text{ENOB}_{\text{INL}} = \log_2 \left( \frac{V_{\text{REF}}}{\text{INL}} \right) - 1$$

where INL is in volts.

Thus, if an n-bit DAC has an INL of $\frac{1}{2}$ LSB

$$\text{ENOB}_{\text{INL}} = \log_2 \left( \frac{V_{\text{REF}}}{\text{INL}} \right) - 1 = \log_2 \left( \frac{2^n V_{\text{LSB}}}{V_{\text{LSB}}} \right) - 1 = \log_2 \left( 2^{n+1} \right) - 1 = n$$
Integral Nonlinearity (ADC)

Integral Non-Linearity (INL)

In more fundamental terms, INL represents the curvature in the Actual Transfer Function relative to a baseline transfer function, or the difference between the current and the ideal transition voltages. There are three primary definitions of INL in common use. They all have the same fundamental definition except they are measured against different transfer functions. This fundamental definition is:

\[
\text{Code INL} = V(\text{Current Transition}) - V(\text{Baseline Transition})
\]

\[
\text{INL} = \text{Max}(\text{Code INL})
\]

Actually probably more than 3
Integral Nonlinearity (ADC)

Nonideal ADC

Transition points are not uniformly spaced!
More than one definition for INL exists!
Will give two definitions here
Integral Nonlinearity (ADC)

Nonideal ADC

Consider end-point fit line with interpreted output axis

\[ x_{\text{INF}}(x_{\text{IN}}) = m x_{\text{IN}} + \left( \frac{x_{\text{LSB}}}{2} - m x_{T1} \right) \]

\[ m = \frac{(N-2)x_{\text{LSB}}}{x_{T7} - x_{T1}} \]
Integral Nonlinearity (ADC)

Nonideal ADC

Continuous-input based INL definition

\[ \text{INL}(x_{IN}) = x_{IN}(x_{IN}) - x_{\text{INF}}(x_{IN}) \]

\[ \text{INL} = \max_{0 \leq x_{IN} \leq x_{REF}} \left\{ |\text{INL}(x_{IN})| \right\} \]
Integral Nonlinearity (ADC)

Nonideal ADC

Continuous-input based INL definition

Often expressed in LSB

\[
\text{INL}(x_{IN}) = \frac{x_{IN}^e(x_{IN}) - x_{INF}(x_{IN})}{x_{LSB}}
\]

\[
\text{INL} = \max_{0 \leq x_{IN} \leq x_{REF}} \left\{ \left| \text{INL}(x_{IN}) \right| \right\}
\]
With this definition of INL, the INL of an ideal ADC is $X_{\text{LSB}}/2$ (for $X_{T1}=X_{\text{LSB}}$). This is effective at characterizing the overall nonlinearity of the ADC but does not vanish when the ADC is ideal and the effects of the breakpoints are not explicit.
Integral Nonlinearity (ADC)

Nonideal ADC

Break-point INL definition (most popular)

\[
\text{INL} = \max_{2 \leq k \leq N-2} \left\{ |\text{INL}_k| \right\}
\]

Place \( N-3 \) uniformly spaced points between \( X_{T1} \) and \( X_{T(N-1)} \) designated \( X_{FTk} \)

\[
\text{INL}_k = X_{Tk} - X_{FTk} \quad 1 \leq k \leq N-2
\]
Integral Nonlinearity (ADC)

Nonideal ADC

Break-point INL definition

Often expressed in LSB

$$\text{INL}_k = \frac{x_{T_k} - x_{FT_l}}{x_{\text{LSB}}} \quad 1 \leq k \leq N-2$$

$$\text{INL} = \max_{2 \leq k \leq N-2} \{ ||\text{INL}_k|| \}$$

For an ideal ADC, INL is ideally 0
Integral Nonlinearity (ADC)

Nonideal ADC

Break-point INL definition

\[ \text{INL}_k = \frac{X_{T_k} - X_{F_{T_l}}}{X_{\text{LSB}}} \quad 1 \leq k \leq N-2 \]

\[ \text{INL} = \max \left\{ \left| \text{INL}_k \right| \right\} \quad 2 \leq k \leq N-2 \]

- INL is often the most important parameter of an ADC
- INL\(_1\) and INL\(_{N-1}\) are 0 (by definition)
- There are N-3 elements in the set of INL\(_k\) that are of concern
- INL is a random variable at the design stage
- INL\(_k\) is a random variable for 0<k<N-1
- INL\(_k\) and INL\(_{k+j}\) are correlated for all k,j (not incl 0, N-1) for most architectures
- Fit Line (for cont INL) and uniformly spaced break pts (breakpoint INL) are random variables
- INL is the N-3 order statistic of a set of N-3 correlated random variables (breakpoint INL)
Integral Nonlinearity (ADC)

Nonideal ADC

Break-point INL definition

\[
\text{INL}_k = \frac{X_{T_k} - X_{FT_k}}{X_{\text{LSB}}} \quad 1 \leq k \leq N-2
\]

\[
\text{INL} = \max \left\{ |\text{INL}_k| \right\} \quad 2 \leq k \leq N-2
\]

- At design stage, INL characterized by standard deviation of the random variable
- Closed-form expressions for INL almost never exist because PDF of order statistics of correlated random variables is extremely complicated
- Simulation of INL very time consuming if n is very large (large sample size required to establish reasonable level of confidence)
  - Model parameters become random variables
  - Process parameters affect multiple model parameters causing model parameter correlation
  - Simulation times can become very large
Integral Nonlinearity (ADC)

Nonideal ADC

Break-point INL definition

\[ \text{INL}_k = \frac{X_{T_k} - X_{F_{T_l}}}{X_{\text{LSB}}} \quad 1 \leq k \leq N-2 \]

\[ \text{INL} = \max_{2 \leq k \leq N-2} \{|\text{INL}_k|\} \]

- INL can be readily measured in laboratory but often dominates test costs because of number of measurements needed when \( n \) is large
- INL is a random variable and is a major contributor to yield loss in many designs
- Expected value of \( \text{INL}_k \) at \( k=(N-1)/2 \) is largest for many architectures
- This definition does not account for missing transitions
- Major effort in ADC design is in obtaining an acceptable yield
INL-based ENOB

Consider initially the continuous INL definition for an ADC where the INL of an ideal ADC is $X_{\text{LSB}}/2$

Assume $\text{INL} = \theta X_{\text{REF}} = \nu X_{\text{LSBR}}$

where $X_{\text{LSBR}}$ is the LSB based upon the defined resolution

Define the effective LSB by

$$x_{\text{LSBEFF}} = \frac{x_{\text{REF}}}{2^n_{\text{EQ}}}$$

Thus

$$\text{INL} = \theta 2^{n_{\text{EQ}}} x_{\text{LSBEFF}}$$

Since an ideal ADC has an INL of $X_{\text{LSB}}/2$, express INL in terms of ideal ADC

$$\text{INL} = \left[ \theta 2^{(n_{\text{EQ}}+1)} \right] \left( \frac{x_{\text{LSBEFF}}}{2} \right)$$

Setting term in $[]$ to 1, can solve for $n_{\text{EQ}}$ to obtain

$$\text{ENOB} = n_{\text{EQ}} = \log_2 \left( \frac{1}{2\theta} \right) = n_R - \log_2 (\nu)$$

where $n_R$ is the defined resolution
INL-based ENOB

$$ENOB = n_R - 1 - \log_2(\nu)$$

Consider an ADC with specified resolution of $n_R$ and INL of $\nu$ LSB

<table>
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<tbody>
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<td>$\frac{1}{2}$</td>
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End of Lecture 28