EE 435

Lecture 29

Spectral Performance
  – Windowing

Spectral Performance of Data Converters
  - Time Quantization
  - Amplitude Quantization

Quantization Noise
**THEOREM?:** If \( N_P \) is an integer and \( x(t) \) is band limited to \( f_{\text{MAX}} \), then

\[
|A_m| = \frac{2}{N} |X(mN_P + 1)| \quad 0 \leq m \leq h
\]

and

\[
X(k) = 0 \quad \text{for all } k \text{ not defined above}
\]

where \( \langle X(k) \rangle_{k=0}^{N-1} \) is the DFT of the sequence \( \langle x(kT_S) \rangle_{k=0}^{N-1} \)

\[
f = \frac{1}{T}, \quad f_{\text{MAX}} = \frac{f}{2} \cdot \left[ \frac{N}{N_P} \right], \text{ and } \quad h = \text{Int}\left( \frac{f_{\text{MAX}}}{f} \right)
\]
Spectral Response (expressed in dB)

Note Magnitude is Symmetric wrt $f_{\text{SAMPLE}}$

$\text{Rect. Window } N=512 \quad N_p=20$

(Actually Stem plots but points connected in plotting program)

$\text{Frequency}$

$\text{Mag(dB)}$

$f_{\text{AXIS}} = f_{\text{SIGNAL}} \frac{n - 1}{N_p}$
Spectral Response with Non-coherent Sampling

Review from last lecture.

Rect. Window N=4096   Np =20.01

(zoomed in around fundamental)
Observations

• Modest change in sampling window of 0.001 out of 20 periods (.0005%) results in a small error in both fundamental and harmonic

• More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -80dB)

• Errors at about the 13-bit level !
Effects of High-Frequency Spectral Components

Rect. Window N=512 Np =20

fhigh=25 fo

Review from last lecture.
Observations

- Aliasing will occur if the band-limited part of the hypothesis for using the DFT is not satisfied.
- Modest aliasing will cause high frequency components that may or may not appear at a harmonic frequency.
- More egregious aliasing can introduce components near or on top of fundamental and lower-order harmonics.
- Important to avoid aliasing if the DFT is used for spectral characterization.
Considerations for Spectral Characterization

• Tool Validation

• FFT Length

• Importance of Satisfying Hypothesis
  - NP is an integer
  - Band-limited excitation

• Windowing
Are there any strategies to address the problem of requiring precisely an integral number of periods to use the FFT?

Windowing is sometimes used

Windowing is sometimes misused
Windowing

Windowing is the weighting of the time domain function to maintain continuity at the end points of the sample window.

Well-studied window functions:

- Rectangular (also with appended zeros)
- Triangular
- Hamming
- Hanning
- Blackman
Rectangular Window

Assume \( f_{\text{SIG}} = 50 \text{Hz} \)

\[
V_{\text{IN}} = \sin(\omega t) + 0.5 \sin(2\omega t)
\]

\[
\omega = 2\pi f_{\text{SIG}}
\]

Consider \( N_p = 20.1 \) \( N = 512 \)
Rectangular Window
Spectral Response with Non-coherent sampling

Rect. Window N=512  Np =20.1

(zoomed in around fundamental)
Rectangular Window

Columns 1 through 7


Columns 8 through 14

-44.4065  -43.4052  -42.3602  -41.2670  -40.1146  -38.8851  -37.5520

Columns 15 through 21

-36.0756  -34.3940  -32.4043  -29.9158  -26.5087  -20.9064  -0.1352

Columns 22 through 28


Columns 29 through 35

## Rectangular Window

### Columns 1 through 7


### Columns 8 through 14

-44.4065  -43.4052  -42.3602  -41.2670  -40.1146  -38.8851  -37.5520

### Columns 15 through 21

-36.0756  -34.3940  -32.4043  -29.9158  -26.5087  -20.9064  -0.1352

### Columns 22 through 28


### Columns 29 through 35


**Energy spread over several frequency components**
Rectangular Window (with appended zeros)
Triangular Window
Triangular Window
Spectral Response with Non-Coherent Sampling and Windowing

Triangular Window  \( N=512 \), \( N_p=20.1 \)

(zoomed in around fundamental)
Triangular Window

Graph showing the magnitude response of a triangular window.
### Triangular Window

**Columns 1 through 7**

-100.8530  -72.0528  -99.1401  -68.0110  -95.8741  -63.9944  -92.5170

**Columns 8 through 14**

-60.3216  -88.7000  -56.7717  -85.8679  -52.8256  -82.1689  -48.3134

**Columns 15 through 21**

-77.0594  -42.4247  -70.3128  -33.7318  -58.8762  -15.7333  -6.0918

**Columns 22 through 28**

-12.2463  -57.0917  -32.5077  -68.9492  -41.3993  -74.6234  -46.8037

**Columns 29 through 35**

-77.0686  -50.1054  -77.0980  -51.5317  -75.1218  -50.8522  -71.2410
Hamming Window
Hamming Window
Spectral Response with Non-Coherent Sampling and Windowing

Hamming Window  N=512  Np =20.1

(zoomed in around fundamental)
Comparison with Rectangular Window
# Hamming Window

Columns 1 through 7

-70.8278  -70.6955  -70.3703  -69.8555  -69.1502  -68.3632  -67.5133

Columns 8 through 14

-66.5945  -65.6321  -64.6276  -63.6635  -62.6204  -61.5590  -60.4199

Columns 15 through 21

-59.3204  -58.3582  -57.8735  -60.2994  -52.6273  -14.4702  -5.4343

Columns 22 through 28

-11.2659  -45.2190  -67.9926  -60.1662  -60.1710  -61.2796  -62.7277

Columns 29 through 35

-64.3642  -66.2048  -68.2460  -70.1835  -71.1529  -70.2800  -68.1145
Hanning Window
Hanning Window
Spectral Response with Non-Coherent Sampling and Windowing

Hanning Window  \( N=512 \)  \( N_p=20.1 \)

(zoomed in around fundamental)
Comparison with Rectangular Window

![Graph showing comparison between Henning Window and Rectangular Window.](image-url)
## Hanning Window

Columns 1 through 7


Columns 8 through 14

-92.4519  -90.4372  -87.7977  -84.9554  -81.8956  -79.3520  -75.8944

Columns 15 through 21

-72.0479  -67.4602  -61.7543  -54.2042  -42.9597  -13.4511  -6.0601

Columns 22 through 28

-10.8267  -40.4480  -53.3906  -61.8561  -68.3601  -73.9966  -79.0757

Columns 29 through 35

-84.4318  -92.7280  -99.4046  -89.0799  -83.4211  -78.5955  -73.9788
Comparison of 4 windows

Rect. Window  N=512  Np =20.1

Hamming Window  N=512  Np =20.1

Hanning Window  N=512  Np =20.1

Triangular Window  N=512  Np =20.1
Comparison of 4 windows

Rect. Window  N=512  Np =20.01

Hamming Window  N=512  Np =20.01

Hannning Window  N=512  Np =20.01

Triangular Window  N=512  Np =20.01
Preliminary Observations about Windows

- Provide separation of spectral components
- Energy can be accumulated around spectral components
- Simple to apply
- Some windows work much better than others

But – windows do not provide dramatic improvement and …
Comparison of 4 windows when sampling hypothesis are satisfied
Comparison of 4 windows

Rect. Window N=512 Np =20

Hamming Window N=512 Np =20

Hanning Window N=512 Np =20

Triangular Window N=512 Np =20

Frequency

Mag(dB)

Frequency

Mag(dB)

Frequency

Mag(dB)

Frequency
Preliminary Observations about Windows

- Provide separation of spectral components
- Energy can be accumulated around spectral components
- Simple to apply
- Some windows work much better than others

But – windows do not provide dramatic improvement and can significantly degrade performance if sampling hypothesis are met
Issues of Concern for Spectral Analysis

An integral number of periods is critical for spectral analysis. Not easy to satisfy this requirement in the laboratory. Windowing can help but can hurt as well. Out of band energy can be reflected back into bands of interest. Characterization of CAD tool environment is essential. Spectral Characterization of high-resolution data converters requires particularly critical consideration to avoid simulations or measurements from masking real performance.
Spectral Characterization of Data Converters

• Distortion Analysis
• Time Quantization Effects
  – of DACs
  – of ADCs
• Amplitude Quantization Effects
  – of DACs
  – of ADCs
Will leave the issues of time-quantization and DAC characterization to the student

These concepts are investigated in the following slides

Concepts are important but time limitations preclude spending more time on these topics in this course
Skip to Next Yellow Slide

Few comments from slides 103-107
Spectral Characterization of Data Converters

- Distortion Analysis
  - Time Quantization Effects
    - of DACs
    - of ADCs
- Amplitude Quantization Effects
  - of DACs
  - of ADCs
Quantization Effects on Spectral Performance and Noise Floor in DFT

- Assume the effective clock rate (for either an ADC or a DAC) is arbitrarily fast
- Without Loss of Generality it will be assumed that $f_{\text{SIG}}=50\text{Hz}$
- Index on DFT will be listed in terms of frequency (rather than index number)

Matlab File: afft_Quantization.m
Quantization Effects

16,384 pts  res = 4 bits  \( N_p = 25 \)

\( \text{20 msec} \)
Quantization Effects

16,384 pts  res = 4bits  $N_p=25$

20 msec
Quantization Effects

16,384 pts  res = 4bits
Quantization Effects

Simulation environment:

\[ N_p = 23 \]
\[ f_{SIG} = 50 \text{Hz} \]
\[ V_{REF}: -1V, 1V \]
Res: will be varied
\[ N = 2^n \text{ will be varied} \]
Quantization Effects

Res = 4 bits
Quantization Effects

Res = 4 bits

Rect. Window N=4096  Np =23

Mag(dB)

0  1000  2000  3000  4000  5000  6000  7000  8000  9000  Frequency

-150 -100 -50  0

Axis of Symmetry
Quantization Effects

Res = 4 bits

Some components very small
Quantization Effects

Res = 4 bits

Set lower display limit at -120dB
Quantization Effects

Res = 4 bits
Quantization Effects
Res = 4 bits
Quantization Effects

Res = 4 bits

Rect. Window N=65536  Np =23
Quantization Effects

Res = 4 bits
Quantization Effects

Res = 4 bits
Quantization Effects

Res = 4 bits
Quantization Effects

Res = 4 bits

Fundamental
Quantization Effects

Res = 10 bits

Rect. Window N=256  Np =23

Frequency

Magnitude (dB)
Quantization Effects

Res = 10 bits
Quantization Effects
Res = 10 bits

Rect. Window N=256  Np =23
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits

Rect. Window N=4096 Np =23
Quantization Effects

Res = 10 bits
Quantization Effects

Res = 10 bits

Rect. Window  N=4096  Np = 23
Quantization Effects

Res = 10 bits
Quantization Effects

Res = 10 bits

Rect. Window N=16384 Np =23
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits

Rect. Window N=16384  Np =23

Magnitude (dB) vs Frequency
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits

Res 10 No. points 256 fsig= 50.00 No. Periods 23.00
Rectangular Window

Columns 1 through 5

-55.7419 -120.0000 -85.1461 -106.1614 -89.2395

Columns 6 through 10

-102.3822 -99.5653 -85.7335 -89.1227 -83.0851
Columns 11 through 15
-87.5203  -78.5459  -93.9801  -89.8324  -94.5461

Columns 16 through 20
-77.6478  -80.8867  -100.8153  -78.7936  -86.2954

Columns 21 through 25
-85.8697  -79.5073  -101.6929  -0.0004  -83.6600

Columns 26 through 30
-83.3148  -74.8410  -89.7384  -91.5556  -86.9109

Columns 31 through 35
-93.0155  -82.1062  -78.4561  -98.7568  -109.4766
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<td>Columns 56 through 60</td>
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<td>-86.0396 -83.8284 -87.2621 -97.6189 -94.7694</td>
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<td>Columns 61 through 65</td>
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<td>-86.9239</td>
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<td>-83.1902</td>
<td>-82.2598</td>
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<tr>
<td>-103.0396</td>
<td>-87.2043</td>
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<td>-79.1829</td>
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<td>-91.5964</td>
<td>-82.1222</td>
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<td>-78.7656</td>
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</table>
Columns 86 through 90

-82.9621  -93.0224  -116.8549  -93.7327  -75.6231

Columns 91 through 92

-94.4914  -81.0819
Res  10  No. points 4096  fsig= 50.00  No. Periods 23.00

Rectangular Window

Columns 1 through 5

-55.6060  -97.9951  -107.4593  -103.4508  -120.0000

Columns 6 through 10

-96.7808  -105.2905  -96.7395  -104.5281  -90.7582
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Columns 61 through 65

-93.2650  -103.4274  -103.9702  -98.4092  -91.1825

Columns 66 through 70

-98.0638  -93.7989  -107.7453  -93.4277  -88.0409

Columns 71 through 75

-107.3584  -102.5984  -95.3312  -102.9342  -108.5206

Columns 76 through 80


Columns 81 through 85

-96.5194  -85.8129  -95.1970  -94.8699  -104.9224
Quantization Effects

Res = 10 bits

With Vin=2v pp
With $V_{in} = 1^{*}.99$ and $V_{os} = .25$ LSB

Rectangular Window $N = 4096$ $N_p = 25$
With $Vin = 1.9999999$ pp
With $\text{Vin}=1^{*}.99$ and $\text{Vos}=.35\text{LSB}$
<table>
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Quantization Effects

Res = 10 bits
Quantization Effects

Res = 10 bits
Quantization Effects

Res = 10 bits
Quantization Effects

Res = 10 bits
Quantization Effects

Res = 5 bits
Quantization Effects

Res = 4 bits
Quantization Effects

Res = 4 bits
Quantization Effects

16,384 pts  res = 4bits
Quantization Effects

16,384 pts  res = 4bits
Quantization Effects

Res = 10 bits
Spectral Characterization of Data Converters

• Distortion Analysis

→ Time Quantization Effects
  – of DACs
  – of ADCs

• Amplitude Quantization Effects
  – of DACs
  – of ADCs
Spectral Characteristics of DACs and ADCs
Spectral Characteristics of DAC

Periodic Input Signal

Sampling Clock

Sampled Input Signal (showing time points where samples taken)
Spectral Characteristics of DAC

Quantized Sampled Input Signal (with zero-order sample and hold)

Quantization Levels

$T_{SIG}$

$T_{PERIOD}$
Spectral Characteristics of DAC

- $T_{DFT\ WINDOW}$
- $T_{PERIOD}$
- $T_{SIG}$
- $T_{CLOCK}$
- $T_{DFT\ CLOCK}$

Sampling Clock

DFT Clock
Spectral Characteristics of DAC

\[ T_{DFT\ WINDOW} \]

\[ T_{PERIOD} \]

\[ T_{SIG} \]

\[ T_{CLOCK} \]

Sampling Clock

\[ T_{DFT\ CLOCK} \]

DFT Clock
Spectral Characteristics of DAC
Spectral Characteristics of DAC

Sampled Quantized Signal (zoomed)

DFT Clock

Sampling Clock
Spectral Characteristics of DAC

Consider the following example

- \( f_{\text{SIG}} = 50\text{Hz} \)
- \( k_1 = 230 \)
- \( k_2 = 23 \)
- \( N_P = 1 \)
- \( n_{\text{res}} = 8\text{bits} \)
- \( X_{in(t)} = .95 \sin(2\pi f_{\text{SIG}}t) \) (-.4455dB)

Thus

- \( N_{P1} = 23 \)
- \( \theta_{\text{SR}} = 5 \)
- \( f_{\text{CL}} / f_{\text{SIG}} = 10 \)

Matlab File: afft_Quantization_DAC.m
DFT Simulation from Matlab

Rect. Window N=32768  Np =1

\[ n_{\text{sam}} = 142.4696 \]
Width of this region is $f_{CL}$

Analogous to the overall DFT window when directly sampled but modestly asymmetric
DFT Simulation from Matlab

Expanded View

Rect. Window N=32768 Np =1

nres=8 bits

$n_{sam} = 142.4696$
DFT Simulation from Matlab

Expanded View

Rect. Window N=32768 Np =1

$n_{\text{sam}} = 142.4696$

$n_{\text{res}}=8 \text{ bits}$
DFT Simulation from Matlab
Expanded View

Rect. Window N=32768 Np =1

nres=8 bits

\( n_{\text{sam}} = 142.4696 \)
\[ f_{\text{SIG}} = 50\text{Hz}, \; k_1 = 23, \; k_2 = 23, \; N_P = 1, \; n_{\text{res}} = 8\text{bits} \]

\[
X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t) \]

N = 32768

Columns 1 through 7

\[-44.0825 \quad -84.2069 \quad -118.6751 \quad -89.2265 \quad -120.0000 \quad -76.0893 \quad -120.0000\]

Columns 8 through 14

\[-90.3321 \quad -120.0000 \quad -69.9163 \quad -120.0000 \quad -88.9097 \quad -120.0000 \quad -85.1896\]

Columns 15 through 21

\[-120.0000 \quad -83.0183 \quad -109.4722 \quad -89.4980 \quad -120.0000 \quad -79.6110 \quad -120.0000\]

Columns 22 through 28

\[-90.2992 \quad -120.0000 \quad \boxed{-0.5960} \quad -120.0000 \quad -88.5446 \quad -120.0000 \quad -86.0169\]

Columns 29 through 35

\[-120.0000 \quad -81.5409 \quad -109.6386 \quad -89.7275 \quad -120.0000 \quad -81.8340 \quad -120.0000\]
\[ f_{SIG} = 50 \text{Hz}, \ k_1 = 23, \ k_2 = 23, \ N_P = 1, \ n_{res} = 8 \text{bits} \quad \text{Xin}(t) = \sin(2\pi f_{SIG} t) \]

\[ N = 32768 \]

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<tr>
<th>Columns 36 through 42</th>
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</thead>
<tbody>
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<table>
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<thead>
<tr>
<th>Columns 50 through 56</th>
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<tbody>
<tr>
<td>-90.1331 -120.0000 -75.1821 -120.0000 -87.5706 -120.0000 -87.3205</td>
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<table>
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<tr>
<th>Columns 64 through 70</th>
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<tbody>
<tr>
<td>-89.9982 -120.0000 -78.4288 -120.0000 -87.0328 -120.0000 -64.5409</td>
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</table>
\( f_{\text{SIG}}=50\text{Hz} \), \( k_1=23, k_2=23, N_p=1, n_{\text{res}}=8\text{bits} \) \( X(t) = \sin(2\pi f_{\text{SIG}} t) \)

\[ N=32768 \]

Columns 71 through 77

-120.0000 -72.8111 -120.0000 -90.1876 -120.0000 -82.5616 -114.0867

Columns 78 through 84

-89.8269 -115.6476 -80.6553 -120.0000 -86.3818 -120.0000 -88.3454

Columns 85 through 91

-120.0000 -63.5207 -120.0000 -90.2704 -120.0000 -80.8524 -120.0000

Columns 92 through 98

-89.6174 -58.5435 -82.3253 -120.0000 -85.6188 -120.0000 -88.7339

Columns 99 through 100

-120.0000 -63.8165
DFT Simulation from Matlab

Rect. Window N=131072, Np=1

nres=8 bits

n_{sam} = 569.8783
DFT Simulation from Matlab

Expanded View

Rect. Window N=131072  Np =1

nres=8 bits

n_{\text{sam}} = 569.8783
DFT Simulation from Matlab

Expanded View

Rect. Window N=131072 Np =1

nres=8 bits

\( n_{\text{sam}} = 569.8783 \)
DFT Simulation from Matlab

Expanded View

Rect. Window  N=131072  Np =1

nres=8 bits

nsam = 569.8783
\[ f_{\text{SIG}} = 50\text{Hz}, \; k_1 = 23, \; k_2 = 23, \; N_P = 1, \; n_{\text{res}} = 8\text{bits} \quad X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t) \]

\[ N = 131072 \]

<table>
<thead>
<tr>
<th>Columns 1 through 7</th>
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<tbody>
<tr>
<td>-44.0824  -97.0071 -120.0000 -110.6841 -120.0000 -76.0276 -120.0000</td>
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<td>-120.0000 -107.8772 -120.0000 -90.3300 -120.0000 -109.5748 -120.0000</td>
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<tr>
<th>Columns 22 through 28</th>
</tr>
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<tbody>
<tr>
<td>-104.0809 -120.0000 <strong>-0.5960</strong> -120.0000 -110.6201 -120.0000 -98.0920</td>
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<td>-120.0000  -95.8006 -120.0000 -110.7338 -120.0000 -82.3448 -120.0000</td>
</tr>
</tbody>
</table>
\[ f_{\text{SIG}} = 50 \text{Hz} , \quad k_1 = 23 , \quad k_2 = 23 , \quad N_P = 1 , \quad n_{\text{res}} = 8 \text{bits} \]

\[ X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t) \]

\[ N = 131072 \]

<table>
<thead>
<tr>
<th>Columns 36 through 42</th>
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<tbody>
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<td>-102.9185 -120.0000 -109.9276 -120.0000 -88.8778 -120.0000 -107.5734</td>
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<tbody>
<tr>
<td>-120.0000 -108.1493 -120.0000 -90.7672 \textbf{-56.7029} -109.3748 -120.0000</td>
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<td>-120.0000 -94.4432 -120.0000 -110.7692 -120.0000 -86.1442 -120.0000</td>
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<th>Columns 64 through 70</th>
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<tbody>
<tr>
<td>-102.2661 -120.0000 -110.0806 -120.0000 -87.7635 -120.0000 \textbf{-64.4072}</td>
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</tbody>
</table>
\[ f_{\text{SIG}} = 50Hz, \ k_1 = 23, k_2 = 23, N_p = 1, n_{\text{res}} = 8\text{bits} \quad X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t) \]

\[ N = 131072 \]

Columns 71 through 77

\[-120.0000 -108.4202 -120.0000 \quad -91.0476 -120.0000 -109.1589 -120.0000\]

Columns 78 through 84

\[-105.0508 -120.0000 \quad -81.0390 -120.0000 -110.4486 -120.0000 -99.9756\]

Columns 85 through 91

\[-120.0000 -92.8919 -120.0000 -110.7904 -120.0000 -88.9028 -120.0000\]

Columns 92 through 98

\[-101.5617 \boxed{-58.5437} -110.2183 -120.0000 -86.2629 -120.0000 -105.5980\]

Columns 99 through 100

\[-120.0000 -108.6808\]
Consider the following example

- \( f_{\text{SIG}} = 50 \text{Hz} \)
- \( k_1 = 50 \)
- \( k_2 = 5 \)
- \( N_P = 2 \)
- \( n_{\text{res}} = 8 \text{bits} \)
- \( X_{\text{in}}(t) = 0.95 \sin(2\pi f_{\text{SIG}} t) \) \((-0.4455 \text{dB})\)

Thus

- \( N_{P1} = 5 \)
- \( \theta_{\text{SR}} = 5 \)
- \( N_{P2} = 10 \)
DFT Simulation from Matlab

Rect. Window $N=32768$, $N_p=2$

$$n_{\text{sam}} = 327.6800$$

$$n_{\text{res}} = 8$$
DFT Simulation from Matlab

Expanded View

Rect. Window N=32768  Np =2

$n_{\text{sam}} = 327.6800$

$n_{\text{res}} = 8$
DFT Simulation from Matlab

Rect. Window  N=256  Np =2

\( n_{\text{sam}} = 2.5600 \)

\( n_{\text{res}} = 8 \)
DFT Simulation from Matlab

Expanded View

Rect. Window  N=256  Np =2

$n_{\text{res}} = 8$

Mag(dB)

Frequency

nsam = 2.5600
\[ f_{\text{SIG}} = 50\,\text{Hz}, \quad k_1 = 50, \quad k_2 = 5, \quad N_p = 2, \quad n_{\text{res}} = 8\text{bits}, \quad X(t) = \sin(2\pi f_{\text{SIG}} t) \]
\[ N = 131072 \]

<table>
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<th>Columns 29 through 35</th>
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<tbody>
<tr>
<td>-78.0140 -120.0000</td>
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</table>
\[
\begin{align*}
f_{\text{SIG}} &= 50\text{Hz}, \quad k_1 = 50, \quad k_2 = 5, \quad N_P = 2, \quad n_{\text{res}} = 8\text{bits}, \quad X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t) \\
N &= 131072
\end{align*}
\]

<table>
<thead>
<tr>
<th>Columns 36 through 42</th>
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<tbody>
<tr>
<td>-120.0000 - 75.9471 -120.0000 -49.8914 -120.0000 <strong>-58.4761</strong> -120.0000</td>
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<thead>
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<tbody>
<tr>
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<thead>
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<th>Columns 50 through 56</th>
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<thead>
<tr>
<th>Columns 57 through 63</th>
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<tbody>
<tr>
<td>-91.9095 -120.0000 -40.4010 -120.0000 <strong>-62.1214</strong> -120.0000 -50.1249</td>
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<thead>
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<tbody>
<tr>
<td>-120.0000 -78.2678 -120.0000 -24.9258 -120.0000 -87.6235 -120.0000</td>
</tr>
</tbody>
</table>
\[ f_{\text{SIG}} = 50\text{Hz}, \ k_1 = 50, \ k_2 = 5, \ N_P = 2, \ n_{\text{res}} = 8\text{bits}, \ Xin(t) = \sin(2\pi f_{\text{SIG}}t) \]
\[ N = 131072 \]

<table>
<thead>
<tr>
<th>Columns 71 through 77</th>
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<tbody>
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<tr>
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<tbody>
<tr>
<td>-120.0000 -30.9406 -120.0000 -69.1777 -120.0000 -48.8912 -120.0000</td>
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<thead>
<tr>
<th>Columns 85 through 91</th>
</tr>
</thead>
<tbody>
<tr>
<td>-75.7581 -120.0000 -44.8212 -120.0000 -88.9694 -120.0000 -19.1255</td>
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<table>
<thead>
<tr>
<th>Columns 92 through 98</th>
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<tbody>
<tr>
<td>-120.0000 -79.5390 -120.0000 -50.3103 -120.0000 -70.6123 -120.0000</td>
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<thead>
<tr>
<th>Columns 99 through 105</th>
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<tbody>
<tr>
<td>-38.8332 -120.0000 -92.1633 -120.0000 -34.7560 -120.0000 -77.1229</td>
</tr>
</tbody>
</table>
DFT Simulation from Matlab

Expanded View

Rect. Window N=1024  Np =2

\[ nsam = 10.2400 \]
\[ n_{res} = 8 \]
\[ f_{\text{SIG}} = 50 \text{Hz}, \ k_1 = 50, \ k_2 = 5, \ N_P = 2, \ n_{\text{res}} = 8 \text{bits}, \ X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t) \]
\[ N = 1024 \]

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<thead>
<tr>
<th>Column 1</th>
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<th>Column 4</th>
<th>Column 5</th>
<th>Column 6</th>
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<th>Column 11</th>
<th>Column 12</th>
<th>Column 13</th>
<th>Column 14</th>
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<tr>
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<td>-91.3235</td>
<td>-120.0000</td>
<td>-0.6017</td>
<td>-120.0000</td>
<td>-89.9100</td>
<td>-120.0000</td>
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<th>Column 17</th>
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<th>Column 19</th>
<th>Column 20</th>
<th>Column 21</th>
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<td>-86.6863</td>
<td>-120.0000</td>
<td>-48.5379</td>
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<td>-56.7320</td>
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<tr>
<td>-120.0000</td>
<td>-53.4112</td>
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<td>-103.7582</td>
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<td>-54.1209</td>
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<th>Column 31</th>
<th>Column 32</th>
<th>Column 33</th>
<th>Column 34</th>
<th>Column 35</th>
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</thead>
<tbody>
<tr>
<td>-98.4283</td>
<td>-120.0000</td>
<td>-51.2204</td>
<td>-120.0000</td>
<td>-92.1630</td>
<td>-120.0000</td>
<td>-39.9145</td>
</tr>
</tbody>
</table>
\[ f_{\text{SIG}} = 50 \text{Hz}, \ k_1 = 50, \ k_2 = 5, \ N_P = 2, \ n_{\text{res}} = 8 \text{bits}, \ X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t) \]

\[ N = 1024 \]

Columns 36 through 42

\[-120.0000 \ -86.0994 \ -120.0000 \ -46.4571 \ -120.0000 \ -58.5568 \ -120.0000\]

Columns 43 through 49

\[-45.7332 \ -120.0000 \ -88.7034 \ -120.0000 \ -52.7530 \ -120.0000 \ -102.0744\]

Columns 50 through 56

\[-120.0000 \ -54.2124 \ -120.0000 \ -101.8321 \ -120.0000 \ -52.6742 \ -120.0000\]

Columns 57 through 63

\[-89.3186 \ -120.0000 \ -45.3675 \ -120.0000 \ -62.0430 \ -120.0000 \ -46.7029\]

Columns 64 through 70

\[-120.0000 \ -85.3723 \ -120.0000 \ -40.6886 \ -120.0000 \ -92.0718 \ -120.0000\]
\[ f_{\text{SIG}} = 50 \text{Hz}, \; k_1 = 50, \; k_2 = 5, \; N_P = 2, \; n_{\text{res}} = 8 \text{bits}, \; X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t) \]

\[
N = 1024
\]

Columns 71 through 77

-51.9029 -120.0000 -98.8650 -120.0000 -54.1376 -120.0000 -103.6450

Columns 78 through 84

-120.0000 -53.3554 -120.0000 \textbf{-68.6244} -120.0000 -48.3107 -120.0000

Columns 85 through 91

-85.8692 -120.0000 -41.9049 -120.0000 -89.7301 -120.0000 -19.6301

Columns 92 through 98

-120.0000 -91.5501 -120.0000 -50.5392 -120.0000 -92.8884 -120.0000

Columns 99 through 105

-53.8928 -120.0000 \textbf{-104.2832} -120.0000 -53.8225 -120.0000 -91.0209
Spectral Characteristics of DAC

Consider the following example

- $f_{\text{SIG}} = 50\text{Hz}$
- $k_1 = 11$
- $k_2 = 1$
- $N_P = 2$
- $n_{\text{res}} = 12\text{bits}$
- $X_{\text{in}}(t) = .95\sin(2\pi f_{\text{SIG}}t)$ ($-0.4455\text{dB}$)

Thus

- $N_{P1} = 1$
- $\theta_{\text{SR}} = 11$
- $N_{P2} = 2$
DFT Simulation from Matlab

Rec Win  N=4096 Np =2  Nsam = 186.181818 nres = 12  fCL/fsig = 11  fDFT/fsig = 2048
DFT Simulation from Matlab

Rec Win  N=4096 Np =2  Nsam = 186.181818 nres = 12  fCL/fsig = 11  fDFT/fsig = 2048

![Graph showing frequency and magnitude in dB](image-url)
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab

Rec Win  N=65536  Np =2  Nsam = 2978.90909  nres = 12  fCL/fsig = 11  fDFT/fsig = 32768

![Graph of DFT Simulation](image)
Spectral Characteristics of DAC

Consider the following example

- $f_{SIG}=50\text{Hz}$
- $k_1=230$
- $k_2=23$
- $N_P=1$
- $n_{res}=12\text{bits}$
- $X_{in}(t) = .95\sin(2\pi f_{SIG}t)$ ($-0.4455\text{dB}$)

Thus

- $N_{P1}=23$
- $\theta_{SR}=10$
- $N_{P2}=23$
DFT Simulation from Matlab

![DFT Simulation Graph](image-url)
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab

![Graph showing a spectrum analysis with frequency on the x-axis and magnitude in dB on the y-axis. The graph displays a prominent peak at a certain frequency and has a noisy spectrum with multiple peaks at different frequencies.]

Parameters:
- Rec Win: N=65536 Np =1
- Nsam = 284.93913
- nres = 12
- fCL/ fsig = 10
- fDFT/ fsig = 2849.3913
DFT Simulation from Matlab

\[ f_{\text{SIG}} = 50\text{Hz} \quad k_1 = 230 \quad k_2 = 23 \quad N_p = 1 \quad n_{\text{res}} = 12\text{bits} \quad X_{\text{in}}(t) = 0.95\sin(2\pi f_{\text{SIG}}t) \quad (-0.4455\text{dB}) \quad N_{p_1} = 23 \quad \theta_{\text{SR}} = 10 \quad N_{p_2} = 23 \]

Columns 1 through 7

-68.1646 -94.7298 -120.0000 -90.8893 -120.0000 -75.8402 -120.0000

Columns 8 through 14

-97.7128 -120.0000 -69.7549 -120.0000 -90.5257 -120.0000 -95.1113

Columns 15 through 21

-120.0000 -94.3119 -120.0000 -91.2004 -120.0000 -79.4167 -120.0000

Columns 22 through 28

-97.6931 -120.0000 -0.5886 -120.0000 -90.1044 -120.0000 -95.4585

Columns 29 through 35

-120.0000 -93.8547 -120.0000 -91.4631 -120.0000 -81.9608 -120.0000
### DFT Simulation from Matlab

**Columns 36 through 42**

| -97.6535 | -120.0000 | -69.6068 | -120.0000 | -89.6188 | -120.0000 | -95.7721 |

**Columns 43 through 49**

| -120.0000 | -93.3545 | -120.0000 | -91.6806 | -80.7859 | -83.9353 | -120.0000 |

**Columns 50 through 56**

| -97.5940 | -120.0000 | -75.5346 | -120.0000 | -89.0602 | -120.0000 | -96.0458 |

**Columns 57 through 63**

| -120.0000 | -92.8067 | -120.0000 | -91.8555 | -85.5462 | -120.0000 |

**Columns 64 through 70**

| -97.5144 | -120.0000 | -78.9551 | -120.0000 | -88.4176 | -120.0000 | -88.0509 |
## DFT Simulation from Matlab

Columns 71 through 77

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Columns 78 through 84

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Columns 85 through 91

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Columns 92 through 98

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Columns 99 through 100

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<tbody>
<tr>
<td>-120.000</td>
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</table>
Spectral Characteristics of DAC

Consider the following example

- $f_{SIG} = 50$Hz
- $k_1 = 230$
- $k_2 = 23.1$
- $N_P = 1$
- $n_{res} = 12$bits
- $X_{in}(t) = 0.95\sin(2\pi f_{SIG} t)\quad (-0.4455\text{dB})$

Thus

- $N_{P1} = 23.1$
- $\theta_{SR} = 9.957$
- $N_{P2} = 23.1$
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab
Spectral Characteristics of DAC

Consider the following example

- $f_{\text{SIG}} = 50\text{Hz}$
- $k_1 = 230$
- $k_2 = 23$
- $N_P = 1$
- $n_{\text{res}} = 12\text{bits}$
- $X_{\text{in}}(t) = 0.88\sin(2\pi f_{\text{SIG}} t) + 0.1\sin(2\pi f_{\text{SIG}} t)$
- (-1.11\text{db fundamental}, -20\text{dB 2rd harmonic})

Thus

- $N_{P_1} = 23$
- $\theta_{\text{SR}} = 10$
- $N_{P_2} = 23$
DFT Simulation from Matlab

Rec Win N=65536 Np =1 Nsam = 284.93913 nres = 12 fCL/fsig = 10 fDFT/fsig = 2849.3913

Magnitude (dB) vs Frequency
DFT Simulation from Matlab

Rec Win  N=65536 Np =1  Nsam = 284.93913 nres = 12  fCL/fsig = 10  fDFT/fsig = 2849.3913

-110 -100 -90 -80 -70 -60 -50 -40 -30 -20 -10

Frequency

Mag(dB)
DFT Simulation from Matlab

\[ f_{\text{SIG}}=50\text{Hz} \quad k_1=230 \quad k_2=23 \quad N_p=1 \quad n_{\text{res}}=12\text{bits} \quad X(t)=0.88\sin(2\pi f_{\text{SIG}}t)+0.1\sin(2\pi f_{\text{SIG}}t) \quad (-1.11\text{dB fundamental, -20dB 2nd harmonic}) \quad N_{P_1}=23 \quad \theta_{\text{SR}}=10 \quad N_{P_2}=23 \]

Columns 1 through 7

-68.2448   -95.4048   -103.0624   -91.5534   -94.3099   -76.5052   -107.8586

Columns 8 through 14

-98.3634   -107.7150   -70.4198   -97.2597   -91.1898   -103.5449   -95.7898

Columns 15 through 21


Columns 22 through 28


Columns 29 through 35

DFT Simulation from Matlab

Columns 36 through 42

-98.3035 -107.3983 -70.2715 -101.8108 -90.2829 -104.3909 -96.4685

Columns 43 through 49


Columns 50 through 56

-98.2433 -107.2276 -76.1993 -103.7144 -89.7244 -104.7634 -96.7781

Columns 57 through 63

-108.8537 -93.4756 -100.5602 -92.5195 -83.3389 -86.2119 -108.3343

Columns 64 through 70

-98.1627 -107.0564 -79.6196 -105.4341 -89.0818 -105.1065 -82.5417
DFT Simulation from Matlab

Columns 71 through 77


Columns 78 through 84


Columns 85 through 91


Columns 92 through 98

-97.9383  **-82.1713**  -83.8248  -108.1091  -87.4797  -105.7091  -97.4305

Columns 99 through 105

Skip from Previous Yellow Slide
Summary of time and amplitude quantization assessment

Time and amplitude quantization do not introduce harmonic distortion

Time and amplitude quantization do increase the noise floor
Quantization Noise

- DACs and ADCs generally quantize both amplitude and time
- If converting a continuous-time signal (ADC) or generating a desired continuous-time signal (DAC) these quantizations cause a difference in time and amplitude from the desired signal
- First a few comments about Noise
We will define “Noise” to be the difference between the actual output and the desired output of a system.

Types of noise:

• Random noise due to movement of electrons in electronic circuits
• Interfering signals generated by other systems
• Interfering signals generated by a circuit or system itself
• Error signals associated with imperfect signal processing algorithms or circuits
We will define “Noise” to be the difference between the actual output and the desired output of a system.

All of these types of noise are present in data converters and are of concern when designing most data converters.

Can not eliminate any of these noise types but with careful design can manage their effects to certain levels.

Noise (in particular the random noise) is often the major factor limiting the ultimate performance potential of many if not most data converters.
Noise

We will define “Noise” to be the difference between the actual output and the desired output of a system.

Types of noise:

- Random noise due to movement of electrons in electronic circuits
- Interfering signals generated by other systems
- Interfering signals generated by a circuit or system itself
- Error signals associated with imperfect signal processing algorithms or circuits

Quantization noise is a significant component of this noise in ADCs and DACs and is present even if the ADC or DAC is ideal.
Quantization Noise in ADC
(same concepts apply to DACs)

Consider an ideal ADC with first transition point at $0.5X_{\text{LSB}}$

If the input is a low frequency sawtooth waveform of period $T$ that goes from 0 to $X_{\text{REF}}$, the error signal in the time domain will be:

$$\varepsilon_Q^-$$

\[ .5 X_{\text{LSB}} \quad T_1 \quad 2T_1 \quad 3T_1 \quad 4T_1 \ldots \quad T \]

where $T_1 = T/2^n$

This time-domain waveform is termed the Quantization Noise for the ADC with a sawtooth (or triangular) input.
For large \( n \), this periodic waveform behaves much like a random noise source that is uncorrelated with the input and can be characterized by its RMS value which can be obtained by integrating over any interval of length \( T_1 \). For notational convenience, shift the waveform by \( T_1/2 \) units

\[
E_{\text{RMS}} = \sqrt{\frac{1}{T_1} \int_{-T_1/2}^{T_1/2} \varepsilon_Q^2(t) \, dt}
\]
Quantization Noise in ADC

In this interval, \( \varepsilon_Q \) can be expressed as

\[
\varepsilon_Q(t) = -\left( \frac{X_{\text{LSB}}}{T_1} \right) t
\]
Quantization Noise in ADC

\[ E_{\text{RMS}} = \sqrt{\frac{1}{T_1} \int_{-T_1/2}^{T_1/2} \varepsilon_Q^2(t) dt} \]

\[ E_{\text{RMS}} = \sqrt{\frac{1}{T_1} \int_{-T_1/2}^{T_1/2} \left(- \frac{X_{\text{LSB}}}{T_1} \right)^2 t^2 dt} \]

\[ E_{\text{RMS}} = X_{\text{LSB}} \sqrt{\frac{1}{T_1^3} \frac{t^3}{3} \bigg|_{-T_1/2}^{T_1/2}} \]

\[ E_{\text{RMS}} = \frac{X_{\text{LSB}}}{\sqrt{12}} \]
Quantization Noise in ADC

The signal to quantization noise ratio (SNR) can now be determined. Since the input signal is a sawtooth waveform of period $T$ and amplitude $X_{REF}$, it follows by the same analysis that it has an RMS value of

$$X_{RMS} = \frac{X_{REF}}{\sqrt{12}}$$

Thus the SNR is given by

$$SNR = \frac{X_{RMS}}{E_{RMS}} = \frac{X_{RMS}}{X_{LSB}} = 2^n$$

or, in dB,

$$SNR_{dB} = 20(n \cdot \log_2) = 6.02n$$

Note: dB subscript often neglected when not concerned about confusion
Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $X_{REF}$ centered at $X_{REF}/2$?

SNR $= 20(n \cdot \log_2) = 6.02n$
Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $X_{\text{REF}}$ centered at $X_{\text{REF}}/2$?

Time and amplitude quantization points
How does the SNR change if the input is a sinusoid that goes from 0 to $X_{\text{REF}}$ centered at $X_{\text{REF}}/2$?
Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $X_{\text{REF}}$ centered at $X_{\text{REF}}/2$?
Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $X_{\text{REF}}$ centered at $X_{\text{REF}}/2$?

- Appears to be highly uncorrelated with input even though deterministic
- Mathematical expression for $\varepsilon_Q$ very messy
- Excursions exceed $X_{\text{LSB}}$ (but will be smaller and bounded by $\pm X_{\text{LSB}}/2$ for lower frequency signal/frequency clock ratios)
- For lower frequency inputs and higher resolution, at any time, errors are approximately uniformly distributed between $-X_{\text{LSB}}/2$ and $X_{\text{LSB}}/2$
- Analytical form for $\varepsilon_{\text{QRMS}}$ essentially impossible to obtain from $\varepsilon_Q(t)$
Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $X_{\text{REF}}$ centered at $X_{\text{REF}}/2$?

For low $f_{\text{SIG}}/f_{\text{CL}}$ ratios, bounded by $\pm X_{\text{LB}}$ and at any point in time, behaves almost as if a uniformly distributed random variable

$$
\varepsilon_Q \sim U[-0.5X_{\text{LSB}}, 0.5X_{\text{LSB}}]
$$
Quantization Noise in ADC

Recall:

If the random variable \( f \) is uniformly distributed in the interval \([A,B]\)
\( f : U[A,B] \) then the mean and standard deviation of \( f \) are given by

\[
\mu_f = \frac{A + B}{2} \quad \sigma_f = \frac{B - A}{\sqrt{12}}
\]

Theorem: If \( n(t) \) is a random process and \( \langle n(kT_S) \rangle \) is a sequence
of samples of \( n(t) \) then for large \( T/T_S \),

\[
V_{RMS} = \sqrt{\frac{1}{T} \int_{t_1}^{t_1 + T} n^2(t) \, dt} = \sqrt{\sigma^2_{n(kT_S)} + \mu^2_{n(kT_S)}}
\]
Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to \( X_{\text{REF}} \) centered at \( X_{\text{REF}}/2 \)?

\[ \varepsilon_Q \sim U[-0.5X_{\text{LSB}}, 0.5X_{\text{LSB}}] \]

\[ \mu_{\varepsilon_Q} = \frac{A+B}{2} = 0 \quad \sigma_f = \frac{B-A}{\sqrt{12}} = \frac{X_{\text{LSB}}}{\sqrt{12}} \]

\[ V_{\text{RMS}} = \sqrt{\frac{1}{T} \int_{t_1}^{t_1+T} n^2(t) dt} = \sqrt{\sigma_n^2 + \mu_n^2} \]

\[ V_{\text{RMS}} = \sigma_{\varepsilon_Q} = \frac{X_{\text{LSB}}}{\sqrt{12}} \]

Note this is the same RMS noise that was present with a triangular input.
Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $X_{\text{REF}}$ centered at $X_{\text{REF}}/2$?

$$V_{\text{RMS}} = \frac{X_{\text{LSB}}}{\sqrt{12}}$$

But

$$V_{\text{INRMS}} = \left(\frac{X_{\text{REF}}}{2}\right) \frac{1}{\sqrt{2}}$$

Thus obtain

$$\text{SNR} = \frac{X_{\text{REF}}}{2 \sqrt{2} \frac{X_{\text{LSB}}}{\sqrt{12}}} = 2^n \sqrt{\frac{3}{2}}$$

Finally, in db,

$$\text{SNR}_{\text{dB}} = 20\log\left(2^n \sqrt{\frac{3}{2}}\right) = 6.02 \, n + 1.76$$
ENOB based upon Quantization Noise

\[ SNR = 6.02 \, n + 1.76 \]

Solving for \( n \), obtain

\[ ENOB = \frac{SNR_{dB} - 1.76}{6.02} \]

Note: could have used the \( SNR_{dB} \) for a triangle input and would have obtained the expression

\[ ENOB = \frac{SNR_{dB}}{6.02} \]

But the earlier expression is more widely used when specifying the ENOB based upon the noise level present in a data converter.
ENOB based upon Quantization Noise

For very low resolution levels, the assumption that the quantization noise is uncorrelated with the signal is not valid and the ENOB expression will cause a modest error

\[ \text{SNR}_{\text{corr}} \approx \left( 2^n - 2 + \frac{4}{\pi} \right) \sqrt{\frac{3}{2}} \]

from van de Plassche (p13)

<table>
<thead>
<tr>
<th>Res (n)</th>
<th>SNR\textsubscript{corr}</th>
<th>SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.86</td>
<td>7.78</td>
</tr>
<tr>
<td>2</td>
<td>12.06</td>
<td>13.8</td>
</tr>
<tr>
<td>3</td>
<td>19.0</td>
<td>19.82</td>
</tr>
<tr>
<td>4</td>
<td>25.44</td>
<td>25.84</td>
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<td>8</td>
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<td>49.92</td>
</tr>
<tr>
<td>10</td>
<td>61.95</td>
<td>61.96</td>
</tr>
</tbody>
</table>

Table values in dB

Almost no difference for n ≥ 3

\[ \text{SNR} = 6.02 \, n + 1.76 \]
End of Lecture 29