Design Space Exploration
--with applications to single-stage amplifier design
Review from last lecture:

**Single-ended Op Amp Inverting Amplifier**

\[
V_O = (-A)(V_1 - V_{XQ}) + V_{YQ}
\]

\[
V_1 = \frac{R_1}{R_1 + R_2} V_O + \frac{R_2}{R_1 + R_2} V_{IN}
\]

**Summary:**

\[
V_O = -\frac{R_2}{R_1} V_{\text{inss}} + V_{XQ} + \frac{R_2}{R_1} (V_{XQ} - V_{\text{inQ}})
\]

What type of circuits have the transfer characteristic shown?
Review from last lecture:

Single-stage single-input **low-gain** op amp

Basic Structure

![Basic Structure Diagram]

Practical Implementation

![Practical Implementation Diagram]

Have added the load capacitance to include frequency dependence of the amplifier gain
Review from last lecture:
Single-stage single-input low-gain op amp

$$\text{GB and } A_{V0} \text{ are two of the most important parameters in an op amp}$$
Parameter Domains for Characterizing Amplifier Performance

Degrees of Freedom: 2

Small signal parameter domain:

\[ A_{v0} = \frac{-g_m}{g_0} \]
\[ GB = \frac{g_m}{C_L} \]
\[ \{g_m, g_0\} \]

Natural design parameter domain:

\[ A_{v0} = \left[ \frac{\sqrt{2 \mu C_{OX}}}{\lambda} \right] \frac{W}{L} \frac{1}{\sqrt{DQ}} \]
\[ GB = \left[ \frac{\sqrt{2 \mu C_{OX}}}{C_L} \right] \frac{W}{L} \sqrt{bQ} \]
\[ \left\{ \frac{W}{L}, I_{DQ} \right\} \]

Alternate parameter domain:

\[ A_{v0} = \left[ \frac{2}{\lambda} \right] \left[ \frac{1}{V_{EB}} \right] \]
\[ GB = \left[ \frac{2}{V_{DD} C_L} \right] \frac{P}{V_{EB}} \]
\[ \left\{ P, V_{EB} \right\} \]

Architecture Dependent
Parameter Domains for Characterizing Amplifier Performance

• Design often easier if approached in the alternate parameter domain

• How does one really get the design done, though? That is, how does one get back from the alternate parameter domain to the natural parameter domain?

Alternate parameter domain: \( \{P, V_{EB}\} \)

Natural design parameter domain: \( \left\{ \frac{W}{L}, I_{DQ} \right\} \)

\[ I_{DQ} = \frac{P}{V_{DD}} \]

\[ \frac{W}{L} = \frac{P}{V_{DD} \mu C_{OX} V_{EB}^2} \]
Consider basic op amp structure

Alternate Parameter Domain

\[ \{ P, V_{EB} \} \]

\[ A_{V0} = \left[ \frac{2}{\lambda} \right] \left[ \frac{1}{V_{EB}} \right] \]

\[ GB = \left[ \frac{2}{V_{DDCL}} \right] \left[ \frac{P}{V_{EB}} \right] \]

Design Degrees of Freedom: 1

Question: How can one meet two or more performance requirements with one design degree of freedom with this circuit?

Luck or Can’t
Design With the Basic Amplifier Structure

Consider basic op amp structure

Alternate Parameter Domain

\[ \{P, V_{EB}\} \]

\[
A_{V0} = \begin{bmatrix} 2/\lambda & 1/V_{EB} \\ \frac{2}{V_{DD}C_L} & \frac{P}{V_{EB}} \end{bmatrix}
\]

What do you do if you can’t meet the performance requirements?

Look for a different architecture that either has more favorable performance characteristics or more design degrees of freedom.
Design With the Basic Amplifier Structure

Consider basic op amp structure

\[ \begin{align*}
\{ P, V_{EB} \} \\
A_{V0} &= \frac{2}{\lambda} \frac{1}{V_{EB}} \\
GB &= \frac{2}{V_{DDCL}} \frac{P}{V_{EB}}
\end{align*} \]

So, what performance can the designer really get with this circuit?

\[ V_{EB} = -V_{SS} - V_T \]

Designer really has no control of \( V_{EB} \) with this circuit so

- Gain is fixed by the architecture
- \( P \) can be used to determine \( GB \)

If gain is adequate, designer got “lucky” but \( GB \) can be engineered
Design With the Basic Amplifier Structure

Consider basic op amp structure

\[
\begin{align*}
V_{DD} & \quad I_{DQ} & \quad V_{OUT} \\
M_1 & \quad C_L & \quad V_{SS} \\
V_{in} & \quad V_{SS} & \quad +
\end{align*}
\]

\[
\{P, V_{EB}\}
\]

\[
A_{V0} = \begin{bmatrix}
2 \\
\lambda
\end{bmatrix} \begin{bmatrix}
1 \\
V_{EB}
\end{bmatrix}
\]

\[
GB = \begin{bmatrix}
2 \\
V_{DD}C_L
\end{bmatrix} \begin{bmatrix}
P \\
V_{EB}
\end{bmatrix}
\]

\[
V_{EB} = -V_{SS} - V_T
\]

GB varies linearly with P!
GB is costly!
Over design will cost power!
Architectural Modification of the Basic Amplifier Structure

\( \{ P, V_{EB}, V_{XX} \} \)

Mathematical Degrees of Freedom : 3

\( V_{EB} = V_{XX} - V_{SS} - V_T \)

Circuit Constraints: 1

Effective Design Degrees of Freedom: 2

\[
A_{V0} = \begin{bmatrix} 2 \lambda \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} \\
\text{GB} = \begin{bmatrix} \frac{2}{V_{DDCL}} \end{bmatrix} \begin{bmatrix} P \end{bmatrix} \begin{bmatrix} V_{EB} \end{bmatrix}
\]

- \( V_{EB} \) used to determine the gain (\( P \) does not affect gain!)
- \( P \) used to determine GB (but \( V_{EB} \) does affect \( P \) needed for a given GB)
Consider the modified single-stage op amp

\[ A_V = \frac{-g_m}{sC_L + g_0} \]

\[ GB = \frac{g_m}{C_L} \]

\[ A_{V0} = \frac{-g_m}{g_0} \]
Design Space Exploration

Question: How does the GB of the modified single-stage amplifier change with bias current?

\[ GB = \frac{g_m}{C_L} \]

GB increases linearly with \( I_{DQ} \)

\[
GB = \left[ \frac{2}{C_L} \right] \left[ \frac{I_{DQ}}{V_{EB}} \right]
\]
Design Space Exploration

Question: How does the GB of the modified single-stage amplifier change with bias current?

\[
GB = \left[ \frac{\sqrt{2\mu C_{OX}}}{C_L} \right] \left[ \sqrt{\frac{W}{L}} \sqrt{I_{DQ}} \right]
\]

GB increases with the square root of \(I_{DQ}\)
Design Space Exploration

Question: How does the GB of the modified single-stage amplifier change with bias current?

\[ GB = \frac{2}{V_{DD} C_L} \left[ \frac{P}{V_{EB}} \right] \]

GB independent of \( I_{DQ} \)
Design Space Exploration

Question: How does the GB of the modified single-stage amplifier change with bias current?

\[ GB = \frac{1}{\sqrt{I_{DQ}}} \frac{P}{C_L} \sqrt{\frac{2\mu C_{Ox} W}{L}} \]

GB decreases with the reciprocal of the square root of \( I_{DQ} \)
Design Space Exploration

Question: How does the GB of the modified single-stage amplifier change with bias current?

\[ GB = \frac{\sqrt{2\mu C_{ox} W P^3}}{L V_{DD}} \frac{1}{I_{DQ} C_L} \]

GB decreases with the reciprocal of \( I_{DQ} \)
Design Space Exploration

Question: How does the GB of the modified single-stage amplifier change with bias current?

\[ GB = \sqrt{\frac{2 \mu C_{OX}}{C_L}} \left[ \frac{1}{V_{DD}} \right] \left[ \frac{P}{V_{EB}} \right] \]

- Increases Linearly
- Increases Quadratically
- Independent of \( I_{DQ} \)
- Decreases Quadratically
- Decreases Linearly

It depends upon how the design space is explored !!!
Design Space Exploration

Different trajectories through a design space.
Design Space Exploration

Issue becomes more involved for amplifiers or circuits with more than one transistor.

Choice of design parameters can have major impact on insight into design.

Size of parameter domain should agree with the number of degrees of freedom.

Affects of any parameter on performance whether it be in the identified parameter domain or not is strongly dependent on how design space is explored.

Small signal and natural parameter domains give little insight into design or performance.
Single-Stage Low-Gain Op Amps

- Single-ended input

Basic single-stage op amp
Single-Stage Low-Gain Op Amps

• Single-ended input

Observations:

• This circuit often known as a common source amplifier
• Gain in the 30dB to 45dB range
• Inherently a transconductance amplifier since output impedance is high
• Voltage gain is ratio of transconductance gain to output conductance
• Critical to know degrees of freedom in design and know how to systematically explore design space
• Alternative parameter domain much more useful for design than small-signal domain or natural domain
• Performance of differential circuits will be obtained by inspection from those of the single-ended structures
Review

• Multiple parameter domains can be used to characterize and explore a design space
• Performance characteristics of interest take on many different forms depending upon how design space is characterized
• Critical to identify the real number of degrees of freedom in design space (mathematical degrees of freedom minus the number of constraints)
• Performance characteristics often can be expressed as product of a process dependent term and an architecture dependent term
  – Facilitates comparison of different architectures
• Choice of characterization parameters can make a major difference on how hard it is to explore a design space
Review

• Design space is often a high-dimensional system with many local extrema (minimums or maximums)

• Be careful about drawing conclusions about how any parameter individually affects system performance because its affect will depend upon how the design space is explored
Design Space for Single-Stage Op Amp

\[
GB = \left[ \frac{2}{V_{DD}C_L} \right] \left[ \frac{P}{V_{EB}} \right]
\]

Plot of \( GB_N = \frac{P}{V_{EB}} \)
Basic Op Amp Design

- Fundamental Amplifier Design Issues
- Single-Stage Low Gain Op Amps
- Single-Stage High Gain Op Amps
- Two-Stage Op Amp
- Other Basic Gain Enhancement Approaches
Where we are at:

Single-Stage Low-Gain Op Amps

- Single-ended input

- Differential Input

(Symbol does not distinguish between different amplifier types)
Differential Input Low Gain Op Amps

Will Next Show That:

- Differential input opamps can be readily obtained from single-ended opamps

- Performance characteristics of differential opamps can be directly determined from those of the single-ended counterparts
Systematic strategies for designing and analyzing op amps

- Analytical expressions for even simple op amps can become very complicated if brute force analysis techniques are used.
- Considerable insight into both performance and design can be obtained from a systematic strategy for design and analysis of op amps.
- Most authors present operational amplifiers from an “appear and analyze” approach.

A systematic strategy for designing and analyzing op amps will now be developed.
Symmetric Networks

Theorem: If a linear network is symmetric, then for all differential symmetric excitations, the small signal voltage is zero at all points on the axis of symmetry.
Symmetric Networks

Theorem: Symmetric outputs of a symmetric network excited differentially have no common-mode components if biased at the axis of symmetry with an ideal current source.
End of Lecture 3