Design Space Exploration
--with applications to single-stage amplifier design
Single-ended Op Amp Inverting Amplifier

\[ V_O = (-A)(V_1-V_{XQ})+V_{YQ} \]
\[ V_1 = \frac{R_1}{R_1+R_2} V_O + \frac{R_2}{R_1+R_2} V_{IN} \]

Summary:
\[ V_O = -\frac{R_2}{R_1} V_{inss} + V_{XQ} + \frac{R_2}{R_1} (V_{XQ}-V_{inQ}) \]

What type of circuits have the transfer characteristic shown?
Review from last lecture:
Single-stage single-input low-gain op amp

Basic Structure

Practical Implementation

Have added the load capacitance to include frequency dependence of the amplifier gain
Review from last lecture:
Single-stage single-input low-gain op amp

A_v = \frac{-G_M}{sC_L + G}

A_{v0} = \frac{-G_M}{G}

BW = \frac{G}{C_L}

GB = \left(\frac{G_M}{G}\right)\left(\frac{G}{C_L}\right) = \frac{G_M}{C_L}

GB and A_{v0} are two of the most important parameters in an op amp
Review from last lecture:
How do we design an amplifier with a given architecture in general or this architecture in particular?

What is the design space?

Generally $V_{SS}$, $V_{DD}$, $C_L$ (and possibly $V_{OUTQ}$) will be fixed.

Must determine $\{W_1, L_1, I_{DQ} \text{ and } V_{INQ}\}$.

Thus there are 4 design variables.

But $W_1$ and $L_1$ appear as a ratio in almost all performance characteristics of interest.

And $I_{DQ}$ is related to $V_{INQ}$, $W_1$ and $L_1$ (this is a constraint).

Thus the design space generally has only two independent variables or two degrees of freedom $\left\{\frac{W_1}{L_1} , I_{DQ}\right\}$.

Thus design or “synthesis” with this architecture involves exploring the two-dimensional design space $\left\{\frac{W_1}{L_1} , I_{DQ}\right\}$.
Parameter Domains for Characterizing Amplifier Performance

Degrees of Freedom: 2

Small signal parameter domain:

\[ A_{v0} = \frac{-g_m}{g_0} \]

\[ \text{GB} = \frac{g_m}{C_L} \]

Natural design parameter domain:

\[ A_{v0} = \left[ \sqrt{\frac{2\mu C_{OX}}{\lambda}} \right] \frac{W}{\sqrt{L}} \right] \sqrt{\frac{W}{L}} \sqrt{I_{DQ}} \]

\[ \text{GB} = \left[ \sqrt{\frac{2\mu C_{OX}}{C_L}} \right] \sqrt{\frac{W}{L}} \sqrt{I_{DQ}} \]

Alternate parameter domain:

\[ A_{v0} = \left[ \frac{2}{\lambda} \right] \frac{1}{V_{EB}} \]

\[ \text{GB} = \left[ \frac{2}{V_{DD} C_L} \right] \frac{P}{V_{EB}} \]

Architecture Dependent
Parameter Domains for Characterizing Amplifier Performance

- Design often easier if approached in the alternate parameter domain

- How does one really get the design done, though? That is, how does one get back from the alternate parameter domain to the natural parameter domain?

**Alternate parameter domain:** \( \{P, V_{EB}\} \)

\[
\begin{align*}
W &= \? \\
L &= \? \\
I_{DQ} &= \? \\
V_{INQ} &= \?
\end{align*}
\]
Parameter Domains for Characterizing Amplifier Performance

- Design often easier if approached in the alternate parameter domain

- How does one really get the design done, though? That is, how does one get back from the alternate parameter domain to the natural parameter domain?

Alternate parameter domain: \( \{ P, V_{EB} \} \)

Natural design parameter domain: \( \left\{ \frac{W}{L}, I_{DQ} \right\} \)

\[
I_{DQ} = \frac{P}{V_{DD}} \\
\frac{W}{L} = \frac{P}{V_{DD} \mu C_{OX} V_{EB}^2} \\
V_{INQ} = V_{SS} + V_{T} + \sqrt{I_{DQ} \frac{2}{\mu C_{OX}}} \frac{L}{W}
\]
Design With the Basic Amplifier Structure

Consider basic op amp structure

\[ \begin{align*}
V_{in} & \quad I_{DQ} \quad V_{OUT} \\
V_{DD} & \quad C_L & \quad V_{SS} \\
M_1 & & \\
\end{align*} \]

Alternate parameter domain: \( \{P, V_{EB}\} \)

Degrees of Freedom: 2

\[ \begin{align*}
A_{V0} &= \begin{bmatrix} 2 \lambda \\ \frac{1}{V_{EB}} \end{bmatrix} \\
G_B &= \begin{bmatrix} 2 \lambda \frac{P}{V_{DD}C_L} \end{bmatrix} \begin{bmatrix} P \\ V_{EB} \end{bmatrix} \\
I_{DQ} &= \frac{P}{V_{DD}} \\
W/L &= \frac{P}{V_{DD}\mu C_OX V_{EB}^2} \\
V_{INQ} &= V_{SS} + V_T + \sqrt{\frac{2}{\mu C_OX \frac{L}{W}}} \\
\end{align*} \]

But what if the design requirement dictates that \( V_{INQ}=0? \)

- Increase the number of constraints from 2 to 3
- Decrease the Degrees of Freedom from 2 to 1

Question: How can one meet two or more performance requirements with one design degree of freedom with this circuit?
Design With the Basic Amplifier Structure

Consider basic op amp structure

\[ I_{DQ} = \frac{P}{V_{DD}} \]

Degree of Freedom: 1

Alternate parameter domain: \( \{P, V_{EB}\} \)

Degrees of Freedom: 2

\[ A_{V0} = \left[ \frac{2}{\lambda} \right] \left[ \frac{1}{V_{EB}} \right] \]

\[ GB = \left[ \frac{2}{V_{DD}C_L} \right] \left[ \frac{P}{V_{EB}} \right] \]

But what if the design requirement dictates that \( V_{\text{INQ}} = 0 \)?

Question: How can one meet two or more performance requirements with one design degree of freedom with this circuit?

Degrees of Freedom: 1

Luck or Can’t

\[ W = \frac{P}{V_{DD} \mu C_{OX} V_{EB}^2} \]

\[ V_{\text{INQ}} = V_{SS} + V_T + \sqrt{\frac{2}{\mu C_{OX}} \frac{L}{W}} \]
How do we design an amplifier with a given architecture?

1. Determine the design space
2. Identify the constraints
3. Determine the entire set of unknown variables and the Degrees of Freedom
4. Determine an appropriate parameter domain
5. Explore the resultant design space with the identified number of Degrees of Freedom
Design Space Exploration

Question: How does the GB of the modified single-stage amplifier change with bias current?

$$GB = \frac{g_m}{C_L}$$

GB increases linearly with $I_{DQ}$

$$GB = \left[ \frac{2}{C_L} \right] \left[ \frac{I_{DQ}}{V_{EB}} \right]$$
Design Space Exploration

Question: How does the GB of the modified single-stage amplifier change with bias current?

$$GB = \left[ \frac{\sqrt{2\mu C_{OX}}}{C_L} \right] \left[ \sqrt{\frac{W}{L}} \sqrt{I_{DQ}} \right]$$

GB increases with the square root of $I_{DQ}$

![Graph showing GB vs I_{DQ}](image)
Design Space Exploration

Question: How does the GB of the modified single-stage amplifier change with bias current?

\[ GB = \frac{2}{V_{DDC_L}} \left[ \frac{P}{V_{EB}} \right] \]

GB independent of \( I_{DQ} \)
Question: How does the GB of the modified single-stage amplifier change with bias current?

\[ GB = \frac{1}{P} \frac{1}{\sqrt{I_{DQ} C_L}} \sqrt{\frac{2 \mu C_{OX} W}{L}} \]

GB decreases with the reciprocal of the square root of \( I_{DQ} \).
Design Space Exploration

Question: How does the GB of the modified single-stage amplifier change with bias current?

\[ GB = \sqrt[3]{\frac{2\mu C_{ox} W P^3}{L V_{DD} I_{DQ} C_L}} \]

GB decreases with the reciprocal of \( I_{DQ} \)
Design Space Exploration

Question: How does the GB of the modified single-stage amplifier change with bias current?

\[ GB = \left[ \frac{2}{C_L} \right] \left[ \frac{I_{DQ}}{V_{EB}} \right] \]

- Increases Linearly

\[ GB = \left[ \frac{2}{V_{DD}C_L} \right] \left[ \frac{P}{V_{EB}} \right] \]

- Increases Quadratically

- Independent of \( I_{DQ} \)

\[ GB = \frac{1}{\sqrt{I_{DQ}C_L}} \sqrt{\frac{2\mu COX W}{L}} \]

- Decreases Quadratically

\[ GB = \frac{LV_{DD}}{I_{QCL}} \]

- Decreases Linearly

It depends upon how the design space is explored !!!
Design Space Exploration

Different trajectories through a design space
Design Space Exploration

Issue becomes more involved for amplifiers or circuits with more than one transistor

Choice of design parameters can have major impact on insight into design

Size of parameter domain should agree with the number of degrees of freedom

Affects of any parameter on performance whether it be in the identified parameter domain or not is strongly dependent on how design space is explored

Small signal and natural parameter domains give little insight into design or performance
Single-Stage Low-Gain Op Amps

- Single-ended input

![Basic single-stage op amp diagram]

Basic single-stage op amp
Single-Stage Low-Gain Op Amps

• Single-ended input

\[ \text{Av} \]

Observations:

• This circuit often known as a common source amplifier
• Gain in the 30dB to 45dB range
• Inherently a transconductance amplifier since output impedance is high
• Voltage gain is ratio of transconductance gain to output conductance
• Critical to know degrees of freedom in design and know how to systematically explore design space
• Alternative parameter domain much more useful for design than small-signal domain or natural domain
• Performance of differential circuits will be obtained by inspection from those of the single-ended structures
Review

- Multiple parameter domains can be used to characterize and explore a design space
- Performance characteristics of interest take on many different forms depending upon how design space is characterized
- Critical to identify the real number of degrees of freedom in design space (mathematical degrees of freedom minus the number of constraints)
- Performance characteristics often can be expressed as product of a process dependent term and an architecture dependent term
  - Facilitates comparison of different architectures
- Choice of characterization parameters can make a major difference on how hard it is to explore a design space
Review

• Design space is often a high-dimensional system with many local extrema (minimums or maximums)

• Be careful about drawing conclusions about how any parameter individually affects system performance because its affect will depend upon how the design space is explored
Design Space for Single-Stage Op Amp

\[
GB = \left[ \frac{2}{V_{DD}C_L} \right] \left[ \frac{P}{V_{EB}} \right] 
\]

Plot of \( GB_N = \frac{P}{V_{EB}} \)
Where we are at:

Basic Op Amp Design

- Fundamental Amplifier Design Issues
- Single-Stage Low Gain Op Amps
- Single-Stage High Gain Op Amps
- Two-Stage Op Amp
- Other Basic Gain Enhancement Approaches
Where we are at:

Single-Stage Low-Gain Op Amps

- Single-ended input

- Differential Input

(Symbol does not distinguish between different amplifier types)
Differential Input Low Gain Op Amps

Will Next Show That:

• Differential input op amps can be readily obtained from single-ended op amps

• Performance characteristics of differential op amps can be directly determined from those of the single-ended counterparts
Systematic strategies for designing and analyzing op amps

- Analytical expressions for even simple op amps can become very complicated if brute force analysis techniques are used.
- Considerable insight into both performance and design can be obtained from a systematic strategy for design and analysis of op amps.
- Most authors present operational amplifiers from an “appear and analyze” approach.

A systematic strategy for designing and analyzing op amps will now be developed.
Theorem: If a linear network is symmetric, then for all differential symmetric excitations, the small signal voltage is zero at all points on the axis of symmetry.

\[ V_X = 0 \]
Symmetric Networks

Theorem: Symmetric outputs of a symmetric network excited differentially have no common-mode components if biased at the axis of symmetry with an ideal current source.
Counterpart Networks

Definition: The counterpart network of a network is obtained by replacing all n-channel devices with p-channel devices, replacing all p-channel devices with n-channel devices, replacing $V_{SS}$ biases with $V_{DD}$ biases, and replacing all $V_{DD}$ biases with $V_{SS}$ biases.
Counterpart Networks

Example:
Counterpart Networks

the counterpart network is unique

the counterpart of the counterpart is the original network
Counterpart Networks

Theorem: The parametric expressions for all small-signal characteristics, such as voltage gain, output impedance, and transconductance of a network and its counterpart network are the same.
Synthesis of fully-differential op amps from symmetric networks and counterpart networks

Theorem: If F is any network with a single input and P is its counterpart network, then the following circuits are fully differential circuits --- “op amps”.

\[ V_d = V_1 - V_2 \]
Synthesis of fully-differential op amps from symmetric networks and counterpart networks

What do we do with the extra output?
What do we do with the extra output?

Use it or ignore it!!
Synthesis of fully-differential op amps from symmetric networks and counterpart networks

**Terminology**

\[ v_d = V_1 - V_2 \]
Synthesis of fully-differential op amps from symmetric networks and counterpart networks

A fully differential op amp is derived from any quarter circuit by combining it with its counterpart to obtain a half-circuit, combining two half-circuits to form a differential symmetric circuit and then biasing the symmetric differential circuit on the axis of symmetry.

Further, most of the properties of the operational amplifier can be obtained by inspection, from those of the quarter circuit.
A fully differential op amp is derived from any quarter circuit by combining it with its counterpart to obtain a half-circuit, combining two half-circuits to form a differential symmetric circuit and then biasing the symmetric differential circuit on the axis of symmetry.

Further, most of the properties of the operational amplifier can be obtained by inspection, from those of the quarter circuit.

Implications: Much Op Amp design can be reduced to designing much simpler quarter-circuits where it is much easier to get insight into circuit performance.
End of Lecture 3