EE 435

Lecture 31

Quantization Noise
Absolute and Relative Accuracy
THEOREM: If $N_p$ is an integer and $x(t)$ is band limited to $f_{\text{MAX}}$, then

$$|A_m| = \frac{2}{N} |X(mN_p + 1)| \quad 0 \leq m \leq h - 1$$

and

$$X(k) = 0 \quad \text{for all } k \text{ not defined above}$$

where $\langle X(k) \rangle_{k=0}^{N-1}$ is the DFT of the sequence $\langle x(kT_s) \rangle_{k=0}^{N-1}$

$$f = 1/T, \text{ and } f_{\text{MAX}} = \frac{f}{2} \cdot \left[ \frac{N}{N_p} \right]$$
Observations

• Modest change in sampling window of 0.01 out of 20 periods (.05%) still results in a modest error in both fundamental and harmonic.

• More importantly, substantial raise in the computational noise floor !!! (from over -300dB to only -40dB).

• Errors at about the 6-bit level !
FFT Examples

Recall the theorem that provided for the relationship between the DFT terms and the Fourier Series Coefficients required

1. The sampling window be an integral number of periods

2. 

$$N > \frac{2f_{\text{max}}}{f_{\text{SIGNAL}}} N_p$$
Example

If \( f_{\text{SIG}} = 50 \text{Hz} \)

and \( N_p = 20 \) \( N = 512 \)

\[ N > \frac{2f_{\text{max}}}{f_{\text{SIGNAL}}} N_p \quad \rightarrow \quad f_{\text{max}} < 640 \text{Hz} \]
Example

Consider \( N_p = 20 \) \( N = 512 \)

If \( f_{\text{SIG}} = 50 \text{Hz} \)

\[
V_{\text{IN}} = \sin(\omega t) + 0.5 \sin(2\omega t) + 0.5 \sin(14\omega t)
\]

\[
\omega = 2\pi f_{\text{SIG}}
\]

(i.e. a component at 700 Hz which violates the band limit requirement)

Recall \( 20\log_{10}(0.5) = -6.0205999 \)
Effects of High-Frequency Spectral Components

Review from last lecture.
Effects of High-Frequency Spectral Components

Review from last lecture.
Effects of High-Frequency Spectral Components

Rect. Window N=512  Np =20

$f_{\text{high}} = 14f_0$
### Effects of High-Frequency Spectral Components

\[ f_{\text{high}} = 14f_0 \]

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<th>Columns 1 through 7</th>
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<tbody>
<tr>
<td>-299.0778  -292.3045  -297.0529  -301.4639  -297.3332  -309.6947  -308.2308</td>
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<tbody>
<tr>
<td>-297.3710  -316.5113  -293.5661  -294.4045  -293.6881  -292.6872  ( \text{-0.0000} )</td>
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<th>Columns 22 through 28</th>
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Effects of High-Frequency Spectral Components

\[ f_{\text{high}} = 14f_0 \]

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<th>Columns 36 through 42</th>
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<th>Columns 43 through 49</th>
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<tbody>
<tr>
<td>-298.9215 -309.4829</td>
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<tr>
<th>Columns 50 through 56</th>
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<tbody>
<tr>
<td>-318.4706 -294.8956</td>
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</table>

Review from last lecture.
Effects of High-Frequency Spectral Components

Aliased components at

\[ f_{\text{alias}} = 2f_{\text{sample}} - f \]

\[ f_{\text{alias}} = 2 \cdot 12.8f_{\text{sig}} - 14f_{\text{sig}} = 11.6f_{\text{sig}} \]

thus position in sequence = \( 1 + N_p \frac{f_{\text{alias}}}{f_{\text{sig}}} = 1 + 20 \cdot 11.6 = 233 \)

Columns 225 through 231

-296.8883 -292.8175 -295.8882 -286.7494 -300.3477 -284.4253 -282.7639

Columns 232 through 238


Columns 239 through 245

-299.1299 -305.8361 -295.1772 -295.1670 -300.2698 -293.6406 -304.2886

Columns 246 through 252

-302.0233 -306.6100 -297.7242 -305.4513 -300.4242 -298.1795 -299.0956
Effects of High-Frequency Spectral Components

Rect. Window N=512  Np =20

f_{\text{high}}=24 \, f_0
Effects of High-Frequency Spectral Components

Rect. Window N=512 Np =20

fhigh=25 fo
Effects of High-Frequency Spectral Components

Rect. Window  N=512  Np =20

f_{high}=25 f_0
Effects of High-Frequency Spectral Components

Rect. Window N=512  Np =20

f_{high}=24.4f_o
Effects of High-Frequency Spectral Components

Rect. Window N=512 Np =20

$\text{f}_{\text{high}} = 24.5f_0$

Mag(dB)

Frequency
Observations

- Aliasing will occur if the band-limited part of the hypothesis for using the DFT is not satisfied.
- Modest aliasing will cause high frequency components that may or may not appear at a harmonic frequency.
- More egregious aliasing can introduce components near or on top of fundamental and lower-order harmonics.
- Important to avoid aliasing if the DFT is used for spectral characterization.
Windowing

Windowing is the weighting of the time domain function to maintain continuity at the end points of the sample window.

Well-studied window functions:

- Rectangular
- Triangular
- Hamming
- Hanning
- Blackman
Comparison of 4 windows

Rect. Window N=512  Np =20.01

Hamming Window N=512  Np =20.01

Hanning Window N=512  Np =20.01

Triangular Window N=512  Np =20.01
Comparison of 4 windows when sampling hypothesis are satisfied

- Rect. Window, N=512, Np=20
- Hamming Window, N=512, Np=20
- Hanning Window, N=512, Np=20
- Triangular Window, N=512, Np=20
Issues of Concern for Spectral Analysis

An integral number of periods is critical for spectral analysis.

Not easy to satisfy this requirement in the laboratory.

Windowing can help but can hurt as well.

Out of band energy can be reflected back into bands of interest.

Characterization of CAD tool environment is essential.

Spectral Characterization of high-resolution data converters requires particularly critical consideration to avoid simulations or measurements from masking real performance.
Spectral Characterization

- Distortion Analysis
- Time Quantization Effects
- Spectral Characteristic of DAC
  - Time and Amplitude Quantization
Will leave the issues of time-quantization and DAC characterization to the student

These concepts are investigated in the following slides

Concepts are important but time limitations preclude spending more time on these topics in this course
Skip to Slide 145

Few comments from slides 103-107
Spectral Characterization

- Distortion Analysis

Time Quantization Effects

- Spectral Characteristic of DAC
  - Time and Amplitude Quantization
Quantization Effects on Spectral Performance and Noise Floor in DFT

- Assume the effective clock rate (for either an ADC or a DAC) is arbitrarily fast
- Without Loss of Generality it will be assumed that $f_{\text{SIG}}=50$Hz
- Index on DFT will be listed in terms of frequency (rather than index number)

Matlab File: afft_Quantization.m
Quantization Effects

16,384 pts  res = 4bits  N_p=25

20 msec
Quantization Effects

16,384 pts  res = 4bits  N_p=25

20 msec
Quantization Effects

16,384 pts  res = 4bits
Quantization Effects

Simulation environment:

$N_P = 23$

$f_{SIG} = 50\text{Hz}$

$V_{REF}: -1\text{V, 1V}$

Res: will be varied

$N = 2^n$ will be varied
Quantization Effects

Res = 4 bits
Quantization Effects

Res = 4 bits

Axis of Symmetry
Quantization Effects

Res = 4 bits

Some components very small
Quantization Effects

Res = 4 bits

Set lower display limit at -120dB
Quantization Effects

Res = 4 bits
Quantization Effects
Res = 4 bits
Quantization Effects

Res = 4 bits

Rect. Window N=65536 Np =23

Mag(dB)

Frequency x 10^4
Quantization Effects

Res = 4 bits

Rect. Window N=65536 Np = 23
Quantization Effects

Res = 4 bits
Quantization Effects

Res = 4 bits
Quantization Effects
Res = 4 bits
Quantization Effects
Res = 10 bits

Rect. Window N=256 Np=23

Mag(dB)

0 100 200 300 400 500 600
Frequency

-150 -100 -50 0
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits

Rect. Window N=1024   Np =23
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits

Rect. Window N=4096 Np =23

Frequency

Mag(dB)

0 1000 2000 3000 4000 5000 6000 7000 8000 9000
-120 -110 -100 -90 -80 -70 -60 -50 -40 -30 -20 0
Quantization Effects
Res = 10 bits

Rect. Window N=4096 Np =23

Mag(dB)

Frequency
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects
Res = 10 bits
Quantization Effects

Res = 10 bits

Rectangular Window

Columns 1 through 5

-55.7419 -120.0000 -85.1461 -106.1614 -89.2395

Columns 6 through 10

-102.3822 -99.5653 -85.7335 -89.1227 -83.0851
<table>
<thead>
<tr>
<th>Columns 11 through 15</th>
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<tbody>
<tr>
<td>-87.5203  -78.5459  -93.9801  -89.8324  -94.5461</td>
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<td>-83.3148  -74.8410  -89.7384  -91.5556  -86.9109</td>
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<td>Columns 36 through 40</td>
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<td>-98.2999  -84.9383  -115.7328  -100.0758  -77.1246</td>
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<td>-86.6455  -82.5379  -98.8707  -111.1638  -85.9572</td>
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<td>Columns 61 through 65</td>
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<tr>
<td>-86.9239</td>
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<tr>
<td>Columns 66 through 70</td>
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<td>-74.9482</td>
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<td>Columns 71 through 75</td>
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<td>-107.0215</td>
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<td>Columns 76 through 80</td>
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<td>-83.1902</td>
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<tr>
<td>Columns 81 through 85</td>
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<td>-76.6723</td>
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Columns 86 through 90

-82.9621   -93.0224   -116.8549   -93.7327   -75.6231

Columns 91 through 92

-94.4914   -81.0819
Res  10  No. points 4096  fsig=  50.00  No. Periods  23.00

Rectangular Window

Columns 1 through 5

-55.6060  -97.9951  -107.4593  -103.4508  -120.0000

Columns 6 through 10

-96.7808  -105.2905  -96.7395  -104.5281  -90.7582
<table>
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<td>-85.6641 -101.5338 -120.0000 -87.9656 -99.8947</td>
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<td>Columns 16 through 20</td>
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<tr>
<td>-108.1949 -90.9072 -111.7312 -120.0000 -117.6276</td>
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<td>Columns 21 through 25</td>
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<tr>
<td>-97.1804 -102.6126 -111.4008 -0.0003 -97.1838</td>
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<tr>
<td>Columns 26 through 30</td>
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<tr>
<td>Columns 31 through 35</td>
</tr>
<tr>
<td>-104.3215 -100.3451 -97.1556 -86.0534 -94.7263</td>
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<tr>
<td>Columns 36 through 40</td>
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<tr>
<td>-96.6002  -91.5631  -105.9608 -116.1846  -91.7843</td>
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<tr>
<td>Columns 41 through 45</td>
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<tr>
<td>Columns 46 through 50</td>
</tr>
<tr>
<td>-91.7837  -102.1146  -98.7668  -98.8830  -120.0000</td>
</tr>
<tr>
<td>Columns 51 through 55</td>
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<tr>
<td>Columns 56 through 60</td>
</tr>
<tr>
<td>-91.1056  -101.5798  -94.1031  -95.9163  -83.8407</td>
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</tbody>
</table>
Columns 61 through 65

-93.2650  -103.4274  -103.9702  -98.4092  -91.1825

Columns 66 through 70

-98.0638  -93.7989  -107.7453  -93.4277  -88.0409

Columns 71 through 75

-107.3584  -102.5984  -95.3312  -102.9342  -108.5206

Columns 76 through 80


Columns 81 through 85

-96.5194  -85.8129  -95.1970  -94.8699  -104.9224
Quantization Effects

$\text{Res} \,= \, 10 \, \text{bits}$

With $V_{\text{in}} = 2v \, \text{pp}$
With $V_{in}=1*.99$ and $V_{os}=.25$LSB
With $V_{\text{in}} = 1.99999999$ pp
With $V_{\text{in}} = 1 \times 0.99$ and $V_{\text{os}} = 0.35\,\text{LSB}$
Res  10  No. points 4096  fsig= 50.00  No. Periods 25.00  Tstep 1.220703e-004
Magnitude of  Fundamental 1.000  2nd Harmonic 0.000

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<th>Columns 8 through 14</th>
<th>Columns 15 through 21</th>
<th>Columns 22 through 28</th>
<th>Columns 29 through 35</th>
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</table>
Quantization Effects

Res = 10 bits
Quantization Effects

Res = 10 bits
Quantization Effects

Res = 10 bits
Quantization Effects

Res = 10 bits

Rect. Window N=65536 Np =25

![Graph showing quantization effects](image-url)
Quantization Effects

Res = 5 bits
Quantization Effects

Res = 4 bits
Quantization Effects

Res = 4 bits
Quantization Effects

16,384 pts  res = 4bits
Quantization Effects

16,384 pts  res = 4bits
Quantization Effects

Res = 10 bits
Spectral Characterization

• Distortion Analysis
• Time Quantization Effects
  – Spectral Characteristic of DAC
    – Time and Amplitude Quantization
Spectral Characteristics of DACs and ADCs
Spectral Characteristics of DAC

Periodic Input Signal

Sampling Clock

Sampled Input Signal (showing time points where samples taken)
Spectral Characteristics of DAC

Quantized Sampled Input Signal (with zero-order sample and hold)
Spectral Characteristics of DAC

$T_{\text{DFT WINDOW}}$

$T_{\text{PERIOD}}$

$T_{\text{SIG}}$

Sampling Clock

$T_{\text{CLOCK}}$

$T_{\text{DFT CLOCK}}$

DFT Clock
Spectral Characteristics of DAC
Spectral Characteristics of DAC
Spectral Characteristics of DAC

Sampled Quantized Signal (zoomed)

DFT Clock

Sampling Clock
Consider the following example
- \( f_{\text{SIG}} = 50 \text{Hz} \)
- \( k_1 = 230 \)
- \( k_2 = 23 \)
- \( N_P = 1 \)
- \( n_{\text{res}} = 8 \text{bits} \)
- \( Xin(t) = .95 \sin(2\pi f_{\text{SIG}}t) \quad (-.4455 \text{dB}) \)

Thus
- \( N_{P1} = 23 \)
- \( \theta_{\text{SR}} = 5 \)
- \( f_{\text{CL}} / f_{\text{SIG}} = 10 \)

Matlab File: afft_Quantization_DAC.m
DFT Simulation from Matlab

\[ n_{\text{sam}} = 142.4696 \]
Width of this region is $f_{CL}$

Analogous to the overall DFT window when directly sampled but modestly asymmetric

$n_{sam} = 142.4696$
DFT Simulation from Matlab

Expanded View

Rect. Window  N=32768  Np =1

n_{\text{sam}} = 142.4696
DFT Simulation from Matlab

n_{\text{sam}} = 142.4696
DFT Simulation from Matlab

Expanded View

Rect. Window  N=32768  Np =1

nres=8 bits

n_{sam} = 142.4696
$$f_{SIG}=50Hz, \ k_1=23, \ k_2=23, \ N_P=1, \ n_{res}=8\text{bits} \quad Xin(t) = \sin(2\pi f_{SIG} t)$$

\[ N=32768 \]

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<thead>
<tr>
<th>Columns 1 through 7</th>
<th>-44.0825</th>
<th>-84.2069</th>
<th>-118.6751</th>
<th>-89.2265</th>
<th>-120.0000</th>
<th>-76.0893</th>
<th>-120.0000</th>
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<tbody>
<tr>
<td>Columns 8 through 14</td>
<td>-90.3321</td>
<td>-120.0000</td>
<td>-69.9163</td>
<td>-120.0000</td>
<td>-88.9097</td>
<td>-120.0000</td>
<td>-85.1896</td>
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<td>-109.4722</td>
<td>-89.4980</td>
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<td>-0.5960</td>
<td>-120.0000</td>
<td>-88.5446</td>
<td>-120.0000</td>
<td>-86.0169</td>
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<tr>
<td>Columns 29 through 35</td>
<td>-120.0000</td>
<td>-81.5409</td>
<td>-109.6386</td>
<td>-89.7275</td>
<td>-120.0000</td>
<td>-81.8340</td>
<td>-120.0000</td>
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</tbody>
</table>
\[ f_{SIG} = 50\text{Hz}, \ k_1 = 23, \ k_2 = 23, \ N_P = 1, \ n_{res} = 8\text{bits} \]
\[ X_{in}(t) = \sin(2\pi f_{SIG} t) \]

Columns 36 through 42

-90.2331 -120.0000 -69.4356 -120.0000 -88.1400 -120.0000 -86.7214

Columns 43 through 49

-120.0000 -79.6273 -119.1428 -89.9175 -56.7024 -83.0511 -120.0000

Columns 50 through 56

-90.1331 -120.0000 -75.1821 -120.0000 -87.5706 -120.0000 -87.3205

Columns 57 through 63

-120.0000 -76.9769 -120.0000 -90.0703 -119.0588 -83.2950 -113.3964

Columns 64 through 70

-89.9982 -120.0000 -78.4288 -120.0000 -87.0328 -120.0000 -64.5409
\[ f_{\text{SIG}} = 50\text{Hz}, \; k_1 = 23, \; k_2 = 23, \; N_p = 1, \; n_{\text{res}} = 8\text{bits} \quad \text{Xin}(t) = \sin(2\pi f_{\text{SIG}} t) \]

N = 32768

Columns 71 through 77

-120.0000 -72.8111 -120.0000 -90.1876 -120.0000 -82.5616 -114.0867

Columns 78 through 84

-89.8269 -115.6476 -80.6553 -120.0000 -86.3818 -120.0000 -88.3454

Columns 85 through 91

-120.0000 -63.5207 -120.0000 -90.2704 -120.0000 -80.8524 -120.0000

Columns 92 through 98

-89.6174 -58.5435 -82.3253 -120.0000 -85.6188 -120.0000 -88.7339

Columns 99 through 100

-120.0000 -63.8165
DFT Simulation from Matlab

Rect. Window N=131072  Np =1

nres=8 bits

\[ n_{\text{sam}} = 569.8783 \]
DFT Simulation from Matlab

Expanded View

Rect. Window N=131072  Np =1

nres=8 bits

\( n_{\text{sam}} = 569.8783 \)
DFT Simulation from Matlab

Expanded View

Rect. Window  N=131072  Np =1

nres=8 bits

\( n_{\text{sam}} = 569.8783 \)
DFT Simulation from Matlab

Expanded View

Rect. Window  N=131072  Np =1

nres=8 bits

nsam = 569.8783
\[ f_{\text{SIG}} = 50\text{Hz}, \quad k_1 = 23, \quad k_2 = 23, \quad N_p = 1, \quad n_{\text{res}} = 8\text{bits} \quad X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t) \]

\[ N = 131072 \]

Columns 1 through 7

-44.0824  -97.0071 -120.0000 -110.6841 -120.0000 -76.0276 -120.0000

Columns 8 through 14

-103.5227 -120.0000 -109.7590 -120.0000 -89.7127 -120.0000 -107.6334

Columns 15 through 21

-120.0000 -107.8772 -120.0000 -90.3300 -120.0000 -109.5748 -120.0000

Columns 22 through 28

-104.0809 -120.0000 \[ \boxed{-0.5960} \] -120.0000 -110.6201 -120.0000 -98.0920

Columns 29 through 35

-120.0000 -95.8006 -120.0000 -110.7338 -120.0000 -82.3448 -120.0000
\[ f_{\text{SIG}} = 50 \text{Hz} , \quad k_1 = 23 , \quad k_2 = 23 , \quad N_P = 1 , \quad n_{\text{res}} = 8 \text{bits} \quad X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}}t) \]

\[ N = 131072 \]

Columns 36 through 42

\[-102.9185 \quad -120.0000 \quad -109.9276 \quad -120.0000 \quad -88.8778 \quad -120.0000 \quad -107.5734 \]

Columns 43 through 49

\[-120.0000 \quad -108.1493 \quad -120.0000 \quad -90.7672 \quad -56.7029 \quad -109.3748 \quad -120.0000 \]

Columns 50 through 56

\[-104.5924 \quad -120.0000 \quad -75.3784 \quad -120.0000 \quad -110.5416 \quad -120.0000 \quad -99.0764 \]

Columns 57 through 63

\[-120.0000 \quad -94.4432 \quad -120.0000 \quad -110.7692 \quad -120.0000 \quad -86.1442 \quad -120.0000 \]

Columns 64 through 70

\[-102.2661 \quad -120.0000 \quad -110.0806 \quad -120.0000 \quad -87.7635 \quad -120.0000 \quad -64.4072 \]
$f_{\text{SIG}}=50\text{Hz}, \ k_1=23, \ k_2=23, \ N_P=1, \ n_{\text{res}}=8\text{bits} \quad X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t)$

$N=131072$

Columns 71 through 77

-120.0000 -108.4202 -120.0000 -91.0476 -120.0000 -109.1589 -120.0000

Columns 78 through 84

-105.0508 -120.0000 -81.0390 -120.0000 -110.4486 -120.0000 -99.9756

Columns 85 through 91

-120.0000 -92.8919 -120.0000 -110.7904 -120.0000 -88.9028 -120.0000

Columns 92 through 98

-101.5617 $\boxed{-58.5437}$ -110.2183 -120.0000 -86.2629 -120.0000 -105.5980

Columns 99 through 100

-120.0000 -108.6808
Spectral Characteristics of DAC

Consider the following example

- $f_{\text{SIG}}=50\text{Hz}$
- $k_1=50$
- $k_2=5$
- $N_P=2$
- $n_{\text{res}}=8\text{bits}$
- $X_{\text{in}}(t) = .95\sin(2\pi f_{\text{SIG}}t)$ (-.4455dB)

Thus

- $N_{P1}=5$
- $\theta_{SR}=5$
- $N_{P2}=10$
DFT Simulation from Matlab

Rect. Window N=32768  Np =2

nsam = 327.6800

n_res = 8
DFT Simulation from Matlab
Expanded View

Rect. Window N=32768  Np =2

\[ n_{\text{sam}} = 327.6800 \]

\[ n_{\text{res}} = 8 \]
DFT Simulation from Matlab

Rect. Window \( N=256 \) \( N_p =2 \)

\[ \text{nres} = 8 \]

\[ \text{nsam} = 2.5600 \]
DFT Simulation from Matlab

Expanded View

Rect. Window  N=256  Np =2

\[ n_{res} = 8 \]

Mag(dB)

Frequency
\begin{align*}
\text{f}_\text{SIG} &= 50\text{Hz}, \enspace k_1 = 50, \enspace k_2 = 5, \enspace N_P = 2, \enspace n_{\text{res}} = 8\text{bits}, \enspace \text{Xin}(t) = \sin(2\pi f_{\text{SIG}}t) \\
N &= 131072
\end{align*}

Columns 1 through 7

\begin{align*}
-44.1164 & \quad -120.0000 & -36.9868 & \quad -120.0000 & - 74.6451 & \quad -120.0000 & -50.4484
\end{align*}

Columns 8 through 14

\begin{align*}
-120.0000 & \quad -80.1218 & \quad -120.0000 & \quad -0.6543 & \quad -120.0000 & \quad -90.0332 & \quad -120.0000
\end{align*}

Columns 15 through 21

\begin{align*}
-43.9537 & \quad -120.0000 & \quad -73.3311 & \quad -120.0000 & \quad -49.2755 & \quad -120.0000 & \quad \boxed{-56.5832}
\end{align*}

Columns 22 through 28

\begin{align*}
-120.0000 & \quad -30.4886 & \quad -120.0000 & \quad -80.8472 & \quad -120.0000 & \quad -47.9795 & \quad -120.0000
\end{align*}

Columns 29 through 35

\begin{align*}
-78.0140 & \quad -120.0000 & \quad -47.7412 & \quad -120.0000 & \quad -85.9233 & \quad -120.0000 & \quad -27.8207
\end{align*}
The document contains the following information:

- $f_{\text{SIG}}=50\text{Hz}$, $k_1=50$, $k_2=5$, $N_P=2$, $n_{\text{res}}=8\text{bits}$, $X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}} t)$
- $N=131072$

The data is presented in columns, with values ranging from -120.0000 to 120.0000.

<table>
<thead>
<tr>
<th>Columns 36 through 42</th>
</tr>
</thead>
<tbody>
<tr>
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<tbody>
<tr>
<td>-120.0000 -78.2678 -120.0000 -24.9258 -120.0000 -87.6235 -120.0000</td>
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</table>
$f_{\text{SIG}}=50\text{Hz, } k_1=50, k_2=5, N_P=2, n_{\text{res}}=8\text{bits, } X\text{in}(t) = \sin(2\pi f_{\text{SIG}} t)$

$N=131072$

Columns 71 through 77

\[-45.3926 \ -120.0000 \ -77.2183 \ -120.0000 \ -48.4567 \ -120.0000 \ -76.6666\]

Columns 78 through 84

\[-120.0000 \ -30.9406 \ -120.0000 \ -69.1777 \ -120.0000 \ -48.8912 \ -120.0000\]

Columns 85 through 91

\[-75.7581 \ -120.0000 \ -44.8212 \ -120.0000 \ -88.9694 \ -120.0000 \ -19.1255\]

Columns 92 through 98

\[-120.0000 \ -79.5390 \ -120.0000 \ -50.3103 \ -120.0000 \ -70.6123 \ -120.0000\]

Columns 99 through 105

\[-38.8332 \ -120.0000 \ -92.1633 \ -120.0000 \ -34.7560 \ -120.0000 \ -77.1229\]
DFT Simulation from Matlab

Rect. Window  N=1024  Np =2

\[ n_{\text{sam}} = 10.2400 \]

\[ n_{\text{res}} = 8 \]
DFT Simulation from Matlab

Expanded View

Rect. Window N=1024 Np =2

\[ \text{nsam} = 10.2400 \]
\[ n_{res} = 8 \]
\[ f_{\text{SIG}}=50\text{Hz}, \; k_1=50, \; k_2=5, \; N_P=2, \; n_{\text{res}}=8\text{bits}, \; X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}}t) \]

\[ N=1024 \]

Columns 1 through 7

\[-44.0739 \; -120.0000 \; -53.8586 \; -120.0000 \; -91.9997 \; -120.0000 \; -50.3884\]

Columns 8 through 14

\[-120.0000 \; -91.3235 \; -120.0000 \; -0.6017 \; -120.0000 \; -89.9100 \; -120.0000\]

Columns 15 through 21

\[-41.0786 \; -120.0000 \; -86.6863 \; -120.0000 \; -48.5379 \; -120.0000 \; -56.7320\]

Columns 22 through 28

\[-120.0000 \; -53.4112 \; -120.0000 \; -103.7582 \; -120.0000 \; -54.1209 \; -120.0000\]

Columns 29 through 35

\[-98.4283 \; -120.0000 \; -51.2204 \; -120.0000 \; -92.1630 \; -120.0000 \; -39.9145\]
\( f_{\text{SIG}}=50 \text{Hz}, \ k_1=50, \ k_2=5, \ N_P=2, \ n_{\text{res}}=8\text{bits}, \ Xin(t) = \sin(2\pi f_{\text{SIG}}t) \)

\[
N=1024
\]

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</table>
\[
f_{\text{SIG}} = 50 \text{Hz}, \quad k_1 = 50, \quad k_2 = 5, \quad N_P = 2, \quad n_{\text{res}} = 8\text{bits}, \quad X_{\text{in}}(t) = \sin(2\pi f_{\text{SIG}}t)
\]

\[N = 1024\]

Columns 71 through 77

\[-51.9029 \quad -120.0000 \quad -98.8650 \quad -120.0000 \quad -54.1376 \quad -120.0000 \quad -103.6450\]

Columns 78 through 84

\[-120.0000 \quad -53.3554 \quad -120.0000 \quad -68.6244 \quad -120.0000 \quad -48.3107 \quad -120.0000\]

Columns 85 through 91

\[-85.8692 \quad -120.0000 \quad -41.9049 \quad -120.0000 \quad -89.7301 \quad -120.0000 \quad -19.6301\]

Columns 92 through 98

\[-120.0000 \quad -91.5501 \quad -120.0000 \quad -50.5392 \quad -120.0000 \quad -92.8884 \quad -120.0000\]

Columns 99 through 105

\[-53.8928 \quad -120.0000 \quad -104.2832 \quad -120.0000 \quad -53.8225 \quad -120.0000 \quad -91.0209\]
Spectral Characteristics of DAC

Consider the following example

- $f_{\text{SIG}} = 50\text{Hz}$
- $k_1 = 11$
- $k_2 = 1$
- $N_P = 2$
- $n_{\text{res}} = 12\text{bits}$
- $X_{\text{in}}(t) = 0.95\sin(2\pi f_{\text{SIG}}t) (-0.4455\text{dB})$

Thus

- $N_{P1} = 1$
- $\theta_{\text{SR}} = 11$
- $N_{P2} = 2$
DFT Simulation from Matlab

[Graph showing frequency response with various parameters specified]
DFT Simulation from Matlab

Rec Win N=4096 Np =2 Nsam = 186.181818 nres = 12 fCL/ftsig = 11 fDFT/ftsig = 2048

![DFT Simulation Diagram](image-url)
DFT Simulation from Matlab
DFT Simulation from Matlab

Rec Win  N=65536 Np =2  Nsam = 2978.90909 nres = 12  fCL/fsig = 11  fDFT/fsig = 32768

![DFT Simulation Graph](image-url)
DFT Simulation from Matlab

Rec Win  N=65536 Np =2  Nsam = 2978.90909 nres = 12  fCL/fsig = 11  fDFT/fsig = 32768
DFT Simulation from Matlab

![Graph showing frequency response with peaks and valleys, labeled with parameters: Rec Win N=65536 Np =2 Ns = 2978.90909 nres = 12 fCL/fsig = 11 fDFT/fsig = 32768.](image-url)
Spectral Characteristics of DAC

Consider the following example

- $f_{\text{SIG}}=50\text{Hz}$
- $k_1=230$
- $k_2=23$
- $N_P=1$
- $n_{\text{res}}=12\text{bits}$
- $X_{\text{in}}(t) = .95\sin(2\pi f_{\text{SIG}}t) \ (-.4455\text{dB})$

Thus

- $N_{P1}=23$
- $\theta_{\text{SR}}=10$
- $N_{P2}=23$
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab

![Graph showing frequency vs. magnitude in dB with labels: Rec Win N=65536 Np = 1 Nsam = 284.93913 nres = 12 fCL/fsig = 10 fDFT/fsig = 2849.3913]
DFT Simulation from Matlab

The diagram shows a plot of the magnitude of the Discrete Fourier Transform (DFT) for a signal. The parameters are as follows:

- **Rec Win**: 65536
- **Np = 1**
- **Nsam = 284.93913**
- **nres = 12**
- **fCL/fsig = 10**
- **fDFT/fsig = 2849.3913**

The x-axis represents the frequency, ranging from 0 to 5000, and the y-axis represents the magnitude in dB, ranging from -120 to -20 dB.
DFT Simulation from Matlab
DFT Simulation from Matlab

\( f_{\text{SIG}} = 50\text{Hz} \quad k_1 = 230 \quad k_2 = 23 \quad N_p = 1 \quad n_{\text{res}} = 12\text{bits} \quad X_{\text{in}(t)} = 0.95\sin(2\pi f_{\text{SIG}} t) \) (\(-0.4455\text{dB}\)) \( N_{p_1} = 23 \quad \theta_{\text{SR}} = 10 \quad N_{p_2} = 23 \)

Columns 1 through 7

\(-68.1646 \quad -94.7298 \quad -120.0000 \quad -90.8893 \quad -120.0000 \quad -75.8402 \quad -120.0000\)

Columns 8 through 14

\(-97.7128 \quad -120.0000 \quad -69.7549 \quad -120.0000 \quad -90.5257 \quad -120.0000 \quad -95.1113\)

Columns 15 through 21

\(-120.0000 \quad -94.3119 \quad -120.0000 \quad -91.2004 \quad -120.0000 \quad -79.4167 \quad -120.0000\)

Columns 22 through 28

\(-97.6931 \quad -120.0000 \quad -0.5886 \quad -120.0000 \quad -90.1044 \quad -120.0000 \quad -95.4585\)

Columns 29 through 35

\(-120.0000 \quad -93.8547 \quad -120.0000 \quad -91.4631 \quad -120.0000 \quad -81.9608 \quad -120.0000\)
### DFT Simulation from Matlab

<table>
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<th>Columns 36 through 42</th>
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<td>-91.6806</td>
<td>-80.7859</td>
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<td>-120.0000</td>
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DFT Simulation from Matlab

Columns 71 through 77

-120.0000  -92.2056  -120.0000  -91.9896  -120.0000  -86.9037  -120.0000

Columns 78 through 84

-97.4143  -120.0000  -81.3430  -120.0000  -87.6762  -120.0000  -96.6112

Columns 85 through 91

-120.0000  -91.5441  -120.0000  -92.0844  -120.0000  -88.0732  -120.0000

Columns 92 through 98

-97.2936  **-82.6264**  -83.1604  -120.0000  -86.8155  -120.0000  -96.8068

Columns 99 through 100

-120.0000  -90.8133
Spectral Characteristics of DAC

Consider the following example

- $f_{\text{SIG}}=50\text{Hz}$
- $k_1=230$
- $k_2=23.1$
- $N_P=1$
- $n_{\text{res}}=12\text{bits}$
- $X_{\text{in}}(t) = 0.95\sin(2\pi f_{\text{SIG}} t)$ ($-0.4455\text{dB}$)

Thus

- $N_{P1}=23.1$
- $\theta_{\text{SR}}=9.957$
- $N_{P2}=23.1$
DFT Simulation from Matlab
DFT Simulation from Matlab
DFT Simulation from Matlab
Spectral Characteristics of DAC

Consider the following example

- $f_{\text{SIG}}=50\text{Hz}$
- $k_1=230$
- $k_2=23$
- $N_P=1$
- $n_{\text{res}}=12\text{bits}$
- $X_{\text{in}}(t) = 0.88\sin(2\pi f_{\text{SIG}}t) + 0.1\sin(2\pi f_{\text{SIG}}t)$
- (-1.11db fundamental, -20dB 2\text{nd harmonic})

Thus

- $N_{P1}=23$
- $\theta_{SR}=10$
- $N_{P2}=23$
DFT Simulation from Matlab
DFT Simulation from Matlab
**DFT Simulation from Matlab**

\[ f_{\text{SIG}}=50\text{Hz} \quad k_1=230 \quad k_2=23 \quad N_p=1 \quad n_{res}=12\text{bits} \quad X_{in}(t)=0.88\sin(2\pi f_{\text{SIG}}t)+0.1\sin(2\pi f_{\text{SIG}}t) \quad (-1.11\text{db fundamental}, -20\text{dB 2nd harmonic}) \quad N_{p1}=23 \quad \theta_{SR}=10 \quad N_{p2}=23 \]

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<th>Columns 29 through 35</th>
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</table>
DFT Simulation from Matlab

Columns 36 through 42

-98.3035 -107.3983 -70.2715 -101.8108 -90.2829 -104.3909 -96.4685

Columns 43 through 49

-108.8814 -94.0244 -101.2937 -92.3447 \textcolor{red}{-20.5694} -84.6007 -108.2298

Columns 50 through 56

-98.2433 -107.2276 -76.1993 -103.7144 -89.7244 -104.7634 -96.7781

Columns 57 through 63

-108.8537 -93.4756 -100.5602 -92.5195 -83.3389 -86.2119 -108.3343

Columns 64 through 70

-98.1627 -107.0564 -79.6196 -105.4341 -89.0818 -105.1065 \textcolor{red}{-82.5417}
DFT Simulation from Matlab

Columns 71 through 77


Columns 78 through 84


Columns 85 through 91


Columns 92 through 98

-97.9383  -82.1713  -83.8248  -108.1091  -87.4797  -105.7091  -97.4305

Columns 99 through 105

Skip from Slide 25
Summary of time and amplitude quantization assessment

Time and amplitude quantization do not introduce harmonic distortion

Time and amplitude quantization do increase the noise floor
Quantization Noise

• DACs and ADCs generally quantize both amplitude and time
• If converting a continuous-time signal (ADC) or generating a desired continuous-time signal (DAC) these quantizations cause a difference in time and amplitude from the desired signal
• First a few comments about Noise
We will define “Noise” to be the difference between the actual output and the desired output of a system.

Types of noise:

- Random noise due to movement of electrons in electronic circuits
- Interfering signals generated by other systems
- Interfering signals generated by a circuit or system itself
- Error signals associated with imperfect signal processing algorithms or circuits
Noise

We will define “Noise” to be the difference between the actual output and the desired output of a system.

All of these types of noise are present in data converters and are of concern when designing most data converters.

Can not eliminate any of these noise types but with careful design can manage their effects to certain levels.

Noise (in particular the random noise) is often the major factor limiting the ultimate performance potential of many if not most data converters.
We will define “Noise” to be the difference between the actual output and the desired output of a system.

Types of noise:

• Random noise due to movement of electrons in electronic circuits
• Interfering signals generated by other systems
• Interfering signals generated by a circuit or system itself
• Error signals associated with imperfect signal processing algorithms or circuits

Quantization noise is a significant component of this noise in ADCs and DACs and is present even if the ADC or DAC is ideal.
Quantization Noise in ADC
(same concepts apply to DACs)

Consider an Ideal ADC with first transition point at \(0.5X_{\text{LSB}}\)

If the input is a low frequency sawtooth waveform of period \(T\) that goes from 0 to \(X_{\text{REF}}\), the error signal in the time domain will be:

\[ \varepsilon_Q \]

This time-domain waveform is termed the Quantization Noise for the ADC with a sawtooth (or triangular) input
Quantization Noise in ADC

For large n, this periodic waveform behaves much like a random noise source that is uncorrelated with the input and can be characterized by its RMS value which can be obtained by integrating over any interval of length $T_1$. For notational convenience, shift the waveform by $T_1/2$ units.

\[
E_{\text{RMS}} = \sqrt{\frac{1}{T_1/2} \int_{-T_1/2}^{T_1/2} \varepsilon_Q^2(t) dt}
\]
Quantization Noise in ADC

In this interval, $\epsilon_Q$ can be expressed as

$$\epsilon_Q(t) = -\left(\frac{X_{\text{LSB}}}{T_1}\right)t$$
Quantization Noise in ADC

\[ E_{\text{RMS}} = \sqrt{\frac{1}{T_1}} \int_{-T_1/2}^{T_1/2} \varepsilon_Q^2(t) \, dt \]

\[ E_{\text{RMS}} = \sqrt{\frac{1}{T_1}} \int_{-T_1/2}^{T_1/2} \left( -\frac{x_{\text{LSB}}}{T_1} \right)^2 t^2 \, dt \]

\[ E_{\text{RMS}} = x_{\text{LSB}} \sqrt{\frac{1}{T_1^3}} \int_{-T_1/2}^{T_1/2} t^3 \, dt \]

\[ E_{\text{RMS}} = \frac{x_{\text{LSB}}}{\sqrt{12}} \]
Quantization Noise in ADC

\[ E_{\text{RMS}} = \frac{x_{\text{LSB}}}{\sqrt{12}} \]

The signal to quantization noise ratio (SNR) can now be determined. Since the input signal is a sawtooth waveform of period \( T \) and amplitude \( X_{\text{REF}} \), it follows by the same analysis that it has an RMS value of

\[ x_{\text{RMS}} = \frac{x_{\text{REF}}}{\sqrt{12}} \]

Thus the SNR is given by

\[ \text{SNR} = \frac{x_{\text{RMS}}}{E_{\text{RMS}}} = \frac{x_{\text{RMS}}}{x_{\text{LSB}}} = 2^n \]

or, in dB,

\[ \text{SNR}_{\text{dB}} = 20(n \cdot \log_2) = 6.02n \]

Note: dB subscript often neglected when not concerned about confusion
Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $X_{\text{REF}}$ centered at $X_{\text{REF}}/2$?

SNR $= 20(n \cdot \log_2) = 6.02n$
Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $X_{\text{REF}}$ centered at $X_{\text{REF}}/2$?

Time and amplitude quantization points
Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $X_{\text{REF}}$ centered at $X_{\text{REF}}/2$?
How does the SNR change if the input is a sinusoid that goes from 0 to $X_{\text{REF}}$ centered at $X_{\text{REF}}/2$?
Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $X_{\text{REF}}$ centered at $X_{\text{REF}}/2$?

- Appears to be highly uncorrelated with input even though deterministic
- Mathematical expression for $\epsilon_Q$ very messy
- Excursions exceed $X_{\text{LSB}}$ (but will be smaller and bounded by $\pm X_{\text{LSB}}/2$ for lower frequency signal/frequency clock ratios)
- For lower frequency inputs and higher resolution, at any time, errors are approximately uniformly distributed between $-X_{\text{LSB}}/2$ and $X_{\text{LSB}}/2$
- Analytical form for $\epsilon_{QRMS}$ essentially impossible to obtain from $\epsilon_Q(t)$
Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $X_{\text{REF}}$ centered at $X_{\text{REF}}/2$?

For low $f_{\text{SIG}}/f_{\text{CL}}$ ratios, bounded by $\pm X_{\text{LB}}$ and at any point in time, behaves almost as if a uniformly distributed random variable

$$\epsilon_Q \sim U[-0.5X_{\text{LSB}}, 0.5X_{\text{LSB}}]$$
Quantization Noise in ADC

Recall:

If the random variable $f$ is uniformly distributed in the interval $[A,B]$ $f : U[A,B]$ then the mean and standard deviation of $f$ are given by

\[
\mu_f = \frac{A+B}{2} \quad \sigma_f = \frac{B-A}{\sqrt{12}}
\]

Theorem: If $n(t)$ is a random process, then for large $T$,

\[
V_{\text{RMS}} = \sqrt{\frac{1}{T} \int_{t_1}^{t_1+T} n^2(t) \, dt} = \sqrt{\sigma_n^2 + \mu_n^2}
\]
Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $X_{\text{REF}}$ centered at $X_{\text{REF}}/2$?

$\varepsilon_Q \sim U[-0.5X_{\text{LSB}}, 0.5X_{\text{LSB}}]

\mu_{\varepsilon_Q} = \frac{A+B}{2} = 0 \quad \sigma_f = \frac{B-A}{\sqrt{12}} = \frac{X_{\text{LSB}}}{\sqrt{12}}$

$V_{\text{RMS}} = \sqrt{\frac{1}{T} \int_{t_1}^{t_1+T} n^2(t) \, dt} = \sqrt{\sigma_n^2 + \mu_n^2}

V_{\text{RMS}} = \sigma_{\varepsilon_Q} = \frac{X_{\text{LSB}}}{\sqrt{12}}$

Note this is the same RMS noise that was present with a triangular input.
Quantization Noise in ADC

How does the SNR change if the input is a sinusoid that goes from 0 to $X_{REF}$ centered at $X_{REF}/2$?

$$V_{RMS} = \frac{X_{LSB}}{\sqrt{12}}$$

But

$$V_{INRMS} = \left(\frac{X_{REF}}{2}\right) \frac{1}{\sqrt{2}}$$

Thus obtain

$$SNR = \frac{X_{REF}}{\frac{2\sqrt{2}}{X_{LSB}}} = 2^n \sqrt{\frac{3}{2}}$$

Finally, in db,

$$SNR_{dB} = 20\log\left(2^n \sqrt{\frac{3}{2}}\right) = 6.02n + 1.76$$
ENOB based upon Quantization Noise

\[ \text{SNR} = 6.02 \, n + 1.76 \]

Solving for \( n \), obtain

\[ \text{ENOB} = \frac{\text{SNR}_{\text{dB}} - 1.76}{6.02} \]

Note: could have used the \( \text{SNR}_{\text{dB}} \) for a triangle input and would have obtained the expression

\[ \text{ENOB} = \frac{\text{SNR}_{\text{dB}}}{6.02} \]

But the earlier expression is more widely used when specifying the ENOB based upon the noise level present in a data converter.
ENOB based upon Quantization Noise

For very low resolution levels, the assumption that the quantization noise is uncorrelated with the signal is not valid and the ENOB expression will cause a modest error

$$\text{SNR}_{\text{corr}} \approx (2^n - 2 + \frac{4}{\pi}) \sqrt{\frac{3}{2}}$$

from van de Plassche (p13)

<table>
<thead>
<tr>
<th>Res (n)</th>
<th>SNR$_{\text{corr}}$</th>
<th>SNR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.86</td>
<td>7.78</td>
</tr>
<tr>
<td>2</td>
<td>12.06</td>
<td>13.8</td>
</tr>
<tr>
<td>3</td>
<td>19.0</td>
<td>19.82</td>
</tr>
<tr>
<td>4</td>
<td>25.44</td>
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<td>31.66</td>
<td>31.86</td>
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<tr>
<td>6</td>
<td>37.79</td>
<td>37.88</td>
</tr>
<tr>
<td>8</td>
<td>49.90</td>
<td>49.92</td>
</tr>
<tr>
<td>10</td>
<td>61.95</td>
<td>61.96</td>
</tr>
</tbody>
</table>

Table values in dB

Almost no difference for $n \geq 3$

$$\text{SNR} = 6.02 \times n + 1.76$$
End of Lecture 31
Quantization Noise

Effects of quantization noise can be very significant, even at high resolution, when signals are not of maximum magnitude.

Quantization noise remains constant but signal level is reduced. The desire to use a data converter at a small fraction of full range is one of the major reasons high resolution is required.
Quantization Noise

Effects of quantization noise can be very significant, even at high resolution, when signals are not of maximum magnitude

\[ x_{\text{REF}} \quad x_{\text{IN}} \]

\[ t \]
Example: If a 14-bit audio output is derived from a DAC designed for providing an output of 100W but the normal listening level is at 50mW, what is the SNR due to quantization noise at maximum output and at the normal listening level? What is the ENOB of the audio system when operating at 50mW?

At 100W output, SNR=6.02n+1.76 = 90.6dB

\[ \frac{V^2}{R_L} = 100W \quad \frac{V_1^2}{R_L} = 50mW \quad V_1 = \frac{V}{44.7} \]

\[ 20\log_{10} V_1 = 20\log_{10} V - 20\log_{10} 44.7 = 20\log_{10} V - 33dB \]

At 50mW output, SNR reduced by 33dB to 57.6dB

\[ \text{ENOBSNR} = \frac{\text{SNR}_{\text{dB}} - 1.76}{6.02} = \frac{57.6 - 1.76}{6.02} = -9.3 \]

Note the dramatic reduction in the effective resolution of the DAC when operated at only a small fraction of full-scale.
ENOB Summary

Resolution:

\[ ENOB = \frac{\log_{10} N_{ACT}}{\log_{10} 2} = \log_2 N_{ACT} \]

INL:

\[ ENOB = n_R \cdot \log_2 (\nu) - 1 \]

\[ ENOB = -\log_2 (INL_{REF}) - 1 \]

DNL:

HW problem

Quantization noise:

\[ ENOB = \frac{SNR_{dB}}{6.02} \]

\[ ENOB = \frac{SNR_{dB} - 1.76}{6.02} \]

rel to triangle/sawtooth

rel to sinusoid
Performance Characterization of Data Converters

• Static characteristics
  – Resolution
  – Least Significant Bit (LSB)
  – Offset and Gain Errors
  Absolute Accuracy  Relative Accuracy
  – Integral Nonlinearity (INL)
  – Differential Nonlinearity (DNL)
  – Monotonicity (DAC)
  – Missing Codes (ADC)
  – Quantization Noise
  – Low-f Spurious Free Dynamic Range (SFDR)
  – Low-f Total Harmonic Distortion (THD)
  – Effective Number of Bits (ENOB)
  – Power Dissipation
Absolute Accuracy

Absolute Accuracy is the difference between the actual output and the ideal or desired output of a data converter.

The ideal or desired output is in reference to an absolute standard (often maintained by the National Bureau of Standards) and could be volts, amps, time, weight, distance, or one of a large number of other physical quantities.

Absolute accuracy provides no tolerance to offset errors, gain errors, nonlinearity errors, quantization errors, or noise.

In many applications, absolute accuracy is not of a major concern.

But ... scales, meters, etc. may be more concerned about Absolute accuracy than any other parameter.
Relative Accuracy

In the context of data converters, pseudo-static Relative Accuracy is the difference between the actual output and an appropriate fit-line to overall output of the data converter.

INL is often used as a measure of the relative accuracy.

In many, if not most, applications, relative accuracy is of much more concern than absolute accuracy.

Some architectures with good relative accuracy will have very small deviations in the outputs for closely-spaced inputs whereas others may have relatively large deviations in outputs for closely-spaced inputs.

DNL provides some measure of how outputs for closely-spaced inputs compare.
End of Lecture 31