EE 435

Lecture 31

Switched-Capacitor Amplifiers and Filters
Some of the most basic and widely used analog circuit

Not practical to implement on silicon

- Area for R too big
- Area for C too big
- Accuracy of $I_0$ and $p$ too poor

But ratio accuracy can be very good (0.1% or better with good layout and appropriate area)
How bad is the problem?

**PROCESS PARAMETERS**

<table>
<thead>
<tr>
<th>Sheet Resistance</th>
<th>N+ACTV</th>
<th>P+ACTV</th>
<th>POLY2_HR</th>
<th>POLY2</th>
<th>MTL1</th>
<th>MTL2</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>82.7</td>
<td>103.2</td>
<td>21.7</td>
<td>984</td>
<td>39.7</td>
<td>0.09</td>
<td>0.09</td>
<td>ohms/sq</td>
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<tr>
<td>Contact Resistance</td>
<td>56.2</td>
<td>118.4</td>
<td>14.6</td>
<td>24.0</td>
<td></td>
<td>0.78</td>
<td>ohms</td>
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<tr>
<td>Gate Oxide Thickness</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td>angstrom</td>
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**PROCESS PARAMETERS**

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<thead>
<tr>
<th>Sheet Resistance</th>
<th>MTL3</th>
<th>N\PLY</th>
<th>N_WELL</th>
<th>UNITS</th>
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<tbody>
<tr>
<td>0.05</td>
<td>824</td>
<td>815</td>
<td></td>
<td>ohms/sq</td>
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<tr>
<td>Contact Resistance</td>
<td></td>
<td>0.78</td>
<td></td>
<td>ohms</td>
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**CAPACITANCE PARAMETERS**

<table>
<thead>
<tr>
<th>Area (substrate)</th>
<th>N+ACTV</th>
<th>P+ACTV</th>
<th>POLY</th>
<th>POLY2</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>N_WELL</th>
<th>UNITS</th>
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<tr>
<td>429</td>
<td>721</td>
<td>82</td>
<td>32</td>
<td>17</td>
<td>10</td>
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<td>aF/um^2</td>
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<td>Area (N+active)</td>
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<td>2401</td>
<td>36</td>
<td>16</td>
<td>12</td>
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<tr>
<td>Area (P+active)</td>
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<td></td>
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<td></td>
<td>aF/um^2</td>
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</tr>
<tr>
<td>Area (poly)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>aF/um^2</td>
<td></td>
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<tr>
<td>Area (poly2)</td>
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<td></td>
<td></td>
<td></td>
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<td>aF/um^2</td>
<td></td>
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<tr>
<td>Area (metal1)</td>
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<td></td>
<td></td>
<td></td>
<td>aF/um^2</td>
<td></td>
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<tr>
<td>Area (metal2)</td>
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<td></td>
<td></td>
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<td>aF/um^2</td>
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<td>Fringe (substrate)</td>
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<td>256</td>
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<td>aF/um</td>
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<tr>
<td>Fringe (poly)</td>
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<td>28</td>
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<td></td>
<td>aF/um</td>
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<td>Fringe (metal1)</td>
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<td></td>
<td>55</td>
<td>32</td>
<td></td>
<td></td>
<td></td>
<td>aF/um</td>
<td></td>
</tr>
<tr>
<td>Fringe (metal2)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>aF/um</td>
<td></td>
</tr>
<tr>
<td>Overlap (N+active)</td>
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<td>aF/um</td>
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</tr>
<tr>
<td>Overlap (P+active)</td>
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<td>278</td>
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<td></td>
<td></td>
<td></td>
<td>aF/um</td>
<td></td>
</tr>
</tbody>
</table>

R□=21.7Ω/□ and Cd=0.864pF/μ^2
How bad is the problem?

Low-pass Active Filter

\[ \frac{V_{OUT}}{V_{IN}} = -R/R_1 \frac{1}{1 + RC_s} \]
\[ p = -\frac{1}{RC} \]

Assume \( p = 2\pi \times 1K \) and pole accuracy needed is 0.1%

Process tolerance on \( R \) and \( C \) is about ±20%

\( R_\square = 20\Omega/\square \) and \( C_d = 1pF/\mu^2 \)

If \( R = 1K \), require \( 1000/20 = 50 \) squares

\[ \frac{1}{RC} = 2000\pi \]
\[ C = \frac{1}{R \times 2000\pi} \]
\[ C = \frac{1}{1000 \times 2000\pi} = 0.159 \mu F \]

\[ A_C = \frac{C}{C_d} = \frac{0.159 \mu F}{1fF/\mu^2} = 1.59 \times 10^8 \mu^2 \]

Pole tolerance ±40%

Both are orders of magnitude unacceptable!
An amplifier alternative?:

- Capacitor version is area effective and can have very good accuracy
- But if we get any charge on the intermediate node there is no way to get it off
An amplifier alternative ?:

During $\Phi_1$

$C_1$ is charged to $V_{IN}$ and stores charge $Q_1 = C_1 V_{IN}$

$C_F$ is discharged and $V_{OUT} = 0$

During $\Phi_2$

$C_1$ is discharged but charge is transferred to $C_F$

$Q_2 = -Q_1$ and $V_{OUT} = Q_2 / C_2$

Substituting for $Q_1$ we obtain $V_{OUT} = -\frac{C_1}{C_2} V_{IN}$

Serves as a voltage amplifier during $\Phi_2$
An amplifier alternative!

\[ V_{\text{OUT}} = \frac{C_1}{C_2} V_{\text{IN}} \]

- Many applications only need amplifier output at discrete points in time
- Accuracy can be very good
- Area can be very small

But, what about the switches?
Switches for SC Circuits

- Often a single MOS transistor is adequate (either n-ch or p-ch)
- Sometimes need transmission-gate switch (parallel n-ch and p-ch)
- Switches work very well and can be very small but must manage their $R_{\text{ON}}$
Stray Insensitive SC Amplifiers

Noninverting

Ininverting
Summing amplifier inputs either inverting or noninverting can be easily obtained.
Consider the Basic Integrator

Key performance of integrator (and integrator-based filter) is determined by the integrator time constant $I_0$

Precision of time constants of a filter invariably determined by precision of $I_0$
Consider the Basic Integrator

1. Accuracy of R and C difficult to accurately control – particularly in integrated applications (often 2 or 3 orders of magnitude to variable)

2. Size of R and C unacceptably large if $I_0$ is in audio frequency range (2 or 3 orders of magnitude too large)

3. Amplifier GB limits performance

Incredible Challenge to Building Filters on Silicon!
Integrator Design Issues

Consider:

Assume $T_{CLK} \ll T_{SIG}$

$\Phi_1$ and $\Phi_2$ are complimentary nonoverlapping clocks

Termed a switched-capacitor circuit
Consider the Switched-Capacitor Circuit

\[ V_{IN} \quad C_1 \quad \Phi_1 \quad \Phi_2 \quad C \quad V_{OUT} \]

Assume \( T_{CLK} \ll T_{SIG} \)

\[ \Phi_1 \text{ and } \Phi_2 \text{ are complimentary nonoverlapping clocks} \]
Consider the Switched-Capacitor Circuit

\[ \text{V}_{\text{IN}} \quad \Phi_1 \quad \Phi_2 \quad \text{C} \quad \text{V}_{\text{OUT}} \]

Assume \( T_{\text{CLK}} \ll T_{\text{SIG}} \)

\( \Phi_1 \) and \( \Phi_2 \) are complimentary nonoverlapping clocks

\[ T = T_{\text{CLK}} \]

\[ V(nT) \quad V((n+1)T) \]

\[ \Phi_1 \quad \Phi_2 \quad T_{\text{CLK}} \quad (n+1)T_{\text{CLK}} \]

\[ nT_{\text{CLK}} \quad T = T_{\text{CLK}} \quad (n+1)T \]
Compare the performance of the following two circuits

\[ T(s) = -\frac{1}{RC_s} \]

\[ I_0 = \frac{1}{RC} \]
Consider the charge transferred to the feedback capacitor for both circuits in an interval of length $T_{CLK}$ at time $t_1$.

For the RC circuit:

$$Q_{RC} = \int_{t_1}^{t_1+T_{CLK}} i_{in}(t) \, dt$$

$$Q_{RC} = \int_{t_1}^{t_1+T_{CLK}} \frac{V_{in}(t)}{R} \, dt$$

Since $V_{in}$ changes slowly

$$Q_{RC} \approx \int_{t_1}^{t_1+T_{CLK}} \frac{V_{in}(t_1)}{R} \, dt$$

$$Q_{RC} \approx \left[ \frac{V_{in}(t_1)}{R} \right] \int_{t_1}^{t_1+T_{CLK}} 1 \, dt$$

$$Q_{RC} \approx \left[ \frac{V_{in}(t_1)}{R} \right] T_{CLK}$$
Consider the charge transferred to the feedback capacitor for both circuits in an interval of length $T_{CLK}$ at time $t_1$.

For the RC circuit:

$$Q_{RC} \approx \left[\frac{V_{in}(t_1)}{R}\right]T_{CLK}$$

Observe that a resistor “transfers” charge proportional to $V_{in}$ in a short interval of $T_{CLK}$. 
For the SC circuit

\[ Q_{C1} = C_1 V_{in}\left(t_1 + \frac{T_{CLK}}{2} - \varepsilon\right) \]

Since \( V_{in}(t) \) is slowly varying

\[ Q_{C1} \approx C_1 V_{in}(t_1) \]

But this is the charge that will be transferred to \( C \) during phase \( \Phi_2 \)

\[ Q_{SC} \approx C_1 V_{in}(t_1) \]

Observe that the SC circuit also transfers charge proportional to \( V_{in} \) in short intervals of length \( T_{CLK} \).
Comparing the two circuits

\[ T(s) = -\frac{1}{RC_s} \]

\[ I_0 = \frac{1}{RC} \]

Equating charges since both proportional to \( V_{in}(t_1) \)

\[ Q_{RC} \approx \left[ \frac{V_{in}(t_1)}{R} \right] T_{CLK} \]

\[ Q_{SC} \approx C_1 V_{in}(t_1) \]

\[ C_1 \approx \left[ \frac{1}{R} \right] T_{CLK} \]

\[ R_{EQ} \approx \frac{1}{f_{CLK} C_1} \]
Observe that a switched-capacitor behaves as a resistor!

This is an interesting observation that was made by Maxwell over 100 years ago but in and of itself was of almost no consequence

Note that large resistors require small capacitors!

This offers potential for overcoming one of the critical challenges for Implementing integrators on silicon at audio frequencies!
Equivalence Between Rapidly Switched Capacitor and Resistor

\[ R_{EQ} \approx \frac{1}{f_{CLK} C_1} \]
Consider again the SC integrator

\[ T_{SC}(s) = \frac{-1}{R_{EQ} C s} \]

\[ I_{0eq} = \frac{1}{R_{EQ} C} = \frac{C_1 f_{CLK}}{C} \]

\[ I_{0eq} = \left[ \frac{C_1}{C} \right] f_{CLK} \]

This is a frequency referenced filter!
Consider again the SC integrator

\[ T_{SC}(s) = \frac{-1}{R_{EQ}C_s} \]

\[ I_{0eq} = \left[ \frac{C_1}{C} \right] f_{CLK} \]

The expressions \( S^t_C \) and \( S^t_{C_1} \) have the same magnitude as for the RC integrator

- But the ratio of capacitors can be accurately controlled in IC processes (1% to .01% is achievable with careful layout)
- \( f_{CLK} \) can be VERY accurately controlled with a crystal (1 part in \( 10^6 \) or better)
- Variability of \( I_{0eq} \) is very small

The SC integrator can dramatically reduce the second main concern for building integrated integrators
Consider again the SC integrator

\[ T(s) = -\frac{1}{RC_s} \quad l_0 = \frac{1}{RC} \]

1. Accuracy of \( R \) and \( C \) difficult to accurately control (often 2 or 3 orders of magnitude to variable)
2. Area of \( R \) and \( C \) too large in audio frequency range (2 or 3 orders of magnitude too large)
3. Amplifier GB limits performance

\[ l_{0eq} = \left[ \frac{C_1}{C} \right] f_{CLK} \]

1. Accuracy of cap ratio and \( f_{CLK} \) very good
2. Area of \( C_1 \) and \( C \) not too large
3. Amplifier GB limits performance less
sC integrator with summing inputs
sC low-pass filter with summing inputs
Consider again the SC integrator

\[ \text{Observe this circuit has considerable parasitics} \]

\[ C_{1EQ} = C_1 + C_{s1} + C_{d2} + C_{T1} \]

Parasitic capacitors \( C_{s1} + C_{d2} + C_{T1} \) difficult to accurately match

• Parasitic capacitors of THIS SC integrator limit performance
• Other SC integrators (discussed later) offer same benefits but are not affected by parasitic capacitors
Stray insensitive Inverting and Noninverting SC integrators

(a) Noninverting

(b) Inverting
Arbitrary number of inverting and ioninverting Inputs can be added.
Switched-Capacitor Filter Issues

What if $T_{CLK}$ is not much-much smaller than $T_{SIG}$?

For $T_{CLK} \ll T_{SIG}$
Switched-Capacitor Filter Issues

What if $T_{CLK}$ is not much much smaller than $T_{SIG}$?

For $T_{CLK} < T_{SIG}$
Switched-Capacitor Filter Issues

What if $T_{CLK}$ is not much-much smaller than $T_{SIG}$?

For $T_{CLK} \ll T_{SIG}$
Switched-Capacitor Filter Issues

What if $T_{CLK}$ is not much much smaller than $T_{SIG}$?

For $T_{CLK} < T_{SIG}$
Switched-Capacitor Filter Issues

What if $T_{CLK}$ is not much-much smaller than $T_{SIG}$?

For $T_{CLK} \ll T_{SIG}$

- $V(nT)$ to $V((n+1)T)$

For $T_{CLK} < T_{SIG}$

Considerable change in $V(t)$ in clock period
Switched-Capacitor Filter Issues

What if $T_{CLK}$ is not much-much smaller than $T_{SIG}$?

For $T_{CLK} < T_{SIG}$

$V(nT) \rightarrow V((n+1)T)$

\[ V_0(nT+T) = V_0(nT) + \frac{\Delta Q}{C} \]

but $-uQ$ is the charge on $C_1$ and the time $\phi_1$ opens

\[ -uQ \approx C_1 V_{IN}(nT+T/2) \]

\[ \therefore V_{OUT}(nT+T) = V_{OUT}(nT) - (C_1/C)V_{IN}(nT+T/2) \]

If an input S/H, $V_{IN}$ constant over periods of length $T$

thus, assume $V_{IN}(nT+T/2) \approx V_{IN}(nT)$

So obtain

\[ V_{OUT}(nT+T) = V_{OUT}(nT) - (C_1/C)V_{IN}(nT) \]
Switched-Capacitor Filter Issues

What if $T_{CLK}$ is not much smaller than $T_{SIG}$?

$$V_{OUT}(nT+T)=V_{OUT}(nT)-(C_1/C)V_{IN}(nT)$$

for any $T_{CLK}$, characterized in time domain by difference equation

or in frequency domain characterized by transfer function obtained by taking z-transform of the difference equation

$$H(z) = -\frac{C_1}{C} \frac{1}{z-1}$$
What is really required for building a filter that has high-performance features?

Frequency domain:

Transfer function

\[ T(s) = \frac{1}{RCs} \]

Time domain:

Differential Equation

\[ V_{OUT}(t) = V_{OUT}(t_0) + \frac{1}{RC} \int_{t_0}^{t} V_{IN}(\tau) d\tau \]

Accurate control of polynomial coefficients in transfer function or accurate control of coefficients in the differential equation
What is really required for building a filter that has high-performance features?

Frequency domain:
Transfer function

\[ T(s) = \frac{1}{RCs} \]

Time domain:
Differential Equation

\[ V_{OUT}(t) = V_{OUT}(t_0) + \frac{1}{RC} \int_{t_0}^{t} V_{IN}(\tau) d\tau \]

Difference Equation

\[ V_{OUT}(nT + T) = V_{OUT}(nT) - (C_1/C)V_{IN}(nT) \]

Accurate control of polynomial coefficients in transfer function or accurate control of coefficients in the differential/difference equation.
Switched-Capacitor Filter Issues

What if $T_{CLK}$ is not much smaller than $T_{SIG}$?

\[ V_{OUT}(nT+T) = V_{OUT}(nT) - \left( \frac{C_1}{C} \right) V_{IN}(nT) \]

\[ H(z) = -\frac{C_1}{C} \frac{1}{z-1} \]

Switched-capacitor circuits have potential for good accuracy and attractive area irrespective of how $T_{CLK}$ relates to $T_{SIG}$.

But good layout techniques and appropriate area need to be allocated to realize this potential!
Consider the following circuit
Consider the following circuit

During $\Phi_1$

During $\Phi_2$
Consider the following circuit

\[ Q_1 = C_1 (V_{IN} - V^+) \]
\[ Q_2 = C_2 (V_{IN} - V^+) \]
Consider the following circuit

During $\Phi_2$

```
  C2
 / \
|   |
Vx---C1---Vin
     |     |
     |     |
     |     |
     Vx--Phi2
      +--Phi1

  C2
 / \
|   |
Vx---C1---Vin
     |     |
     |     |
     |     |
     Vx--Phi2
      +--Phi1

during $\Phi_2$
```
Consider the following circuit

During $\Phi_2$

$$Q_1 = C_1(V_{IN} - V^+)$$
$$Q_2 = C_2(V_{IN} - V^+)$$

$$Q_{1T} = C_1(V_{IN} - V^+) - C_1(V_X - V^+) = C_1(V_{IN} - V_X)$$

$$Q_{2F} = Q_2 + Q_{1T} = C_2(V_{IN} - V^+) + C_1(V_{IN} - V_X) = (C_1 + C_2)V_{IN} - C_2V^+ - C_1V_X$$
Consider the following circuit

During $\Phi_2$

$$Q_{2F} = Q_2 + Q_{1T} = C_2 (V_{IN} - V^+) + C_1 (V_{IN} - V_X) = (C_1 + C_2)V_{IN} - C_2 V^+ - C_1 V_X$$

$$V_{C2F} = \frac{Q_{2F}}{C_2} = \left(1 + \frac{C_1}{C_2}\right)V_{IN} - V^+ - \frac{C_1}{C_2} V_X$$

$$V_{OUTF} = V_{C2F} + V^+ = \left(1 + \frac{C_1}{C_2}\right)V_{IN} - \frac{C_1}{C_2} V_X$$
END of LECTURE 31