EE 435

Lecture 4

Fully Differential Single-Stage Amplifier Design
Parameter Domains for Characterizing Amplifier Performance

Degrees of Freedom: 2

Small signal parameter domain:

\[ A_{V0} = \frac{-g_m}{g_0} \]

Natural design parameter domain:

\[ A_{V0} = \left[ \frac{2\sqrt{2\mu C_{OX}}}{\lambda} \right] \frac{W}{L} \frac{1}{\sqrt{DQ}} \]
\[ GB = \frac{\sqrt{2\mu C_{OX}}}{C_L} \frac{W}{L} \sqrt{\frac{1}{bQ}} \]

Alternate parameter domain:

\[ A_{V0} = \left[ \frac{2}{\lambda} \right] \frac{1}{V_{EB}} \]
\[ GB = \left[ \frac{2}{V_{DD}C_L} \right] \frac{P}{V_{EB}} \]

Architecture Dependent

Review from last lecture:
Design With the Basic Amplifier Structure

Consider basic op amp structure

\[ \{P, V_{EB}\} \]

\[ A_{V0} = \lambda \left[ \frac{2}{V_{EB}} \right] \left[ \frac{1}{V_{EB}} \right] \]

\[ GB = \left[ \frac{2}{V_{DD}C_L} \right] \left[ \frac{P}{V_{EB}} \right] \]

So, what performance can the designer really get with this circuit?

\[ V_{EB} = -V_{SS} - V_T \]

Designer really has no control of \( V_{EB} \) with this circuit so

- Gain is fixed by the architecture
- \( P \) can be used to determine \( GB \)

If gain is adequate, designer got “lucky” but \( GB \) can be engineered
Review from last lecture:
Design With the Basic Amplifier Structure

Consider basic op amp structure

\[ V_{EB} = -V_{SS} - V_T \]

GB varies linearly with P!
GB is very costly!

If gain is adequate, designer got “lucky” but GB can be engineered
Review from last lecture:

Architectural Modification of the Basic Amplifier Structure

\( \{ P, V_{EB}, V_{XX} \} \)

Mathematical Degrees of Freedom : 3

\[ V_{EB} = V_{XX} - V_{SS} - V_T \]

Circuit Constraints: 1

Effective Design Degrees of Freedom: 2

\[ A_{V0} = \left[ \frac{2}{\lambda} \right] \left[ \frac{1}{V_{EB}} \right] \]

\[ GB = \left[ \frac{2}{V_{DDCL}} \right] \left[ \frac{P}{V_{EB}} \right] \]

- \( V_{EB} \) used to determine the gain (P does not affect gain!)
- \( P \) used to determine GB (but \( V_{EB} \) does affect P needed for a given GB)
Design Space Exploration

Question: How does the GB of the modified single-stage amplifier change with bias current?

\[ GB = \frac{g_m}{C_L} \]

GB increases linearly with \( I_{DQ} \)

GB = \( \left[ \frac{2}{C_L} \right] \left[ \frac{I_{DQ}}{V_{EB}} \right] \)
Design Space Exploration

Question: How does the GB of the modified single-stage amplifier change with bias current?

\[ GB = \left[ \frac{2}{C_L} \right] \left[ \frac{I_{DQ}}{V_{EB}} \right] \]

- Increases Linearly

\[ GB = \left[ \frac{\sqrt{2\mu C_{OX}}}{C_L} \right] \left[ \frac{W}{L} \sqrt{I_{DQ}} \right] \]

- Increases Quadratically

\[ GB = \left[ \frac{2}{V_{DD}C_L} \right] \left[ \frac{P}{V_{EB}} \right] \]

- Independent of \( I_{DQ} \)

\[ GB = \frac{1}{\sqrt{I_{DQ}C_L}} \left[ \frac{P}{L} \sqrt{2\mu C_{OX}W} \right] \]

- Decreases Quadratically

\[ GB = \left[ \frac{L V_{DD}}{I_{Q} C_L} \right] \]

- Decreases Linearly

It depends upon how the design space is explored !!!
Design Space Exploration

Different trajectories through a design space

Note: After a device has been fabricated, often restricted on how varying of parameters can occur!
• Design space is often a high-dimensional system with many local extrema (minimums or maximums)

• Be careful about drawing conclusions about how any parameter individually affects system performance because its affect will depend upon how the design space is explored
Design Space for Single-Stage Op Amp

\[
GB = \left[ \frac{2}{V_{DD}C_L} \right] \left[ \frac{P}{V_{EB}} \right]
\]

Plot of \( \frac{P}{V_{EB}} \)
Design Space Exploration

Issue becomes more involved for amplifiers or circuits with more than one transistor

Choice of design parameters can have major impact on insight into design

Size of parameter domain should agree with the number of degrees of freedom

Affects of any parameter on performance whether it be in the identified parameter domain or not is strongly dependent on how design space is explored

Small signal and natural parameter domains give little insight into design or performance
• Multiple parameter domains can be used to characterize and explore a design space
• Performance characteristics of interest take on many different forms depending upon how design space is characterized
• Critical to identify the real number of degrees of freedom in design space (mathematical degrees of freedom minus the number of constraints)
• Performance characteristics often can be expressed as product of a process dependent term and an architecture dependent term
  – Facilitates comparison of different architectures
• Choice of characterization parameters can make a major difference on how hard it is to explore a design space
Where we are at:

Basic Op Amp Design

• Fundamental Amplifier Design Issues

• Single-Stage Low Gain Op Amps

• Single-Stage High Gain Op Amps

• Two-Stage Op Amp

• Other Basic Gain Enhancement Approaches
Where we are at:

Single-Stage Low-Gain Op Amps

• Single-ended input

• Differential Input

(Symbol does not distinguish between different amplifier types)
Differential Input Low Gain Op Amps

Will Next Show That:

- Differential input op amps can be readily obtained from single-ended op amps

- Performance characteristics of differential op amps can be directly determined from those of the single-ended counterparts
Systematic strategies for designing and analyzing op amps

• Analytical expressions for even simple op amps can become very complicated if brute force analysis techniques are used.
• Considerable insight into both performance and design can be obtained from a systematic strategy for design and analysis of op amps.
• Most authors present operational amplifiers from an “appear and analyze” approach.

A systematic strategy for designing and analyzing op amps will now be developed.
Theorem: If a linear network is symmetric, then for all differential symmetric excitations, the small signal voltage is zero at all points on the axis of symmetry.
Symmetric Networks

Theorem: Symmetric outputs of a symmetric network excited differentially have no common-mode components if biased at the axis of symmetry with an ideal current source.

\[
\frac{v_d}{2} - \frac{v_d}{2} = v_x = 0
\]
Counterpart Networks

Definition: The counterpart network of a network is obtained by replacing all n-channel devices with p-channel devices, replacing all p-channel devices with n-channel devices, replacing $V_{SS}$ biases with $V_{DD}$ biases, and replacing all $V_{DD}$ biases with $V_{SS}$ biases.
Counterpart Networks

Example:
Counterpart Networks

the counterpart network is unique
the counterpart of the counterpart is
the original network
Counterpart Networks

Theorem: The parametric expressions for all small-signal characteristics, such as voltage gain, output impedance, and transconductance of a network and its counterpart network are the same.
Synthesis of fully-differential op amps from symmetric networks and counterpart networks

Theorem: If F is any network with a single input and P is its counterpart network, then the following circuits are fully differential circuits --- “op amps”.

\[ V_d = V_1 - V_2 \]
Synthesis of fully-differential op amps from symmetric networks and counterpart networks

What do we do with the extra output?
What do we do with the extra output?

Use it or ignore it!!
Synthesis of fully-differential op amps from symmetric networks and counterpart networks

Terminology

\[ V_d = V_1 - V_2 \]
Synthesis of fully-differential op amps from symmetric networks and counterpart networks

A fully differential op amp is derived from any quarter circuit by combining it with its counterpart to obtain a half-circuit, combining two half-circuits to form a differential symmetric circuit and then biasing the symmetric differential circuit on the axis of symmetry.

Further, most of the properties of the operational amplifier can be obtained by inspection, from those of the quarter circuit.
Synthesis of fully-differential op amps from symmetric networks and counterpart networks

A fully differential op amp is derived from any quarter circuit by combining it with its counterpart to obtain a half-circuit, combining two half-circuits to form a differential symmetric circuit and then biasing the symmetric differential circuit on the axis of symmetry.

Further, most of the properties of the operational amplifier can be obtained by inspection, from those of the quarter circuit.

Implications: Much Op Amp design can be reduced to designing much simpler quarter-circuits where it is much easier to get insight into circuit performance.

Quarter Circuit
Characterization of Quarter Circuit

If the input impedance is infinite, the two-port network only has two characterizing parameters: $G_M$ and $G$

\[
\begin{align*}
V_{OUT}(G + sC_L) + G_M V_1 &= 0 \\
V_{IN} &= V_1
\end{align*}
\]

\[
A_{VOQC}(s) = \frac{-G_M}{sC_L + G}
\]

\[
BW = \frac{G}{C_L} \quad GB = \frac{G_M}{C_L}
\]
Characterization of Quarter Circuit (or Counterpart Circuit) with input port terminated in short circuit

If the input port has an ac short, then the two-port reduces to a one-port characterized by the conductance $G$
Determination of op amp characteristics from quarter circuit characteristics

Small signal differential half-circuit

Derivation:
From KCL and KVL:

\[
\begin{align*}
V_0^-(G_1 + G_2 + sC_L) + G_{M1}V_1 &= 0 \\
V_1 &= \frac{V_d}{2}
\end{align*}
\]

\[A_V = \frac{V_0^+}{V_d} = \frac{-G_{M1}}{2} \frac{1}{sC_L + G_1 + G_2}\]

Note: Factor of 2 reduction of gain since only half of the differential input is applied to the half-circuit
Determination of op amp characteristics from quarter circuit characteristics

Small signal differential half-circuit

$$A_V = \frac{V_O}{V_d} = \frac{-G_{M1}}{2} \left( sC_L + G_1 + G_2 \right)$$

$$A_{VO} = \frac{-G_{M1}}{2(G_1 + G_2)}$$

$$BW = \frac{G_1 + G_2}{C_L}$$

$$GB = \frac{G_{M1}}{2C_L}$$
Determination of op amp characteristics from quarter circuit characteristics

Small signal Quarter Circuit

\[ A_{VQC}(s) = \frac{-G_M}{sC_L + G} \]

Small signal differential amplifier

\[ A_V = \frac{V_O}{V_d} = \frac{-G_M}{2} \frac{1}{sC_L + G_1 + G_2} \]
Determination of op amp characteristics from quarter circuit characteristics

Small signal Quarter Circuit

- $A_{\text{VOQC}} = -\frac{G_M}{G}$
- $BW = \frac{G}{C_L}$
- $GB = \frac{G_M}{C_L}$

Small signal differential amplifier

- $A_{\text{VO}} = \frac{-G_{M1}}{2(G_1 + G_2)}$
- $BW = \frac{G_1 + G_2}{C_L}$
- $GB = \frac{G_{M1}}{2C_L}$

Note: Factor of 4 reduction of gain
Comparison of Tail Voltage and Tail Current Source Structures

Small signal half-circuits are identical so voltage gains, BW, and GB are all the same.
Biasing Issues for Differential Amplifier

• Tail voltage bias not suitable for large common-mode (CM) input range but does offer good output swing

• Tail current bias provides good CM input range but at the expense of a modest reduction in output signal swing
Differential Output Amplifiers

- **Differential Voltage Gain Double that of Single-Ended Structure**
- **BW is the same**
- **GB Doubles for the Differential Output Structure**
Applications of Quarter-Circuit Concept to Op Amp Design

consider initially the basic single-ended amplifier
Single-stage single-input low-gain op amp

Basic Structure

Quarter Circuit

Counterpart Circuit

Practical Implementation
Small signal model of half-circuit

\[ G = G_1 + G_2 \]

\[ G_M = G_{M1} \]
Single-stage low-gain differential op amp

Quarter Circuit

Single-Ended Output : Differential Input Gain

\[
A(s) = \frac{-g_{m1}}{2} \frac{2}{sC_L + g_{o1} + g_{o3}}
\]

\[
A_0 = \frac{2}{g_{o1} + g_{o3}}
\]

\[
GB = \frac{g_{m1}}{2C_L}
\]

Need a CMFB circuit to establish \(V_{b1}\)
End of Lecture 4
Single-stage low-gain differential op amp

A(s) = \frac{-g_{m1}}{2sC_L + g_{o1} + g_{o3}}

A_o = \frac{g_{m1}}{2g_{o1} + g_{o3}}

GB = \frac{g_{m1}}{2C_L}

What are the number of degrees of freedom?
(assume V_{DD}, C_L fixed)

Natural Parameters:
\{ \frac{W_1}{L_1}, \frac{W_3}{L_3}, \frac{W_5}{L_5}, V_{B1}, V_{B3} \}

Constraints: \quad I_{D5} \approx 2I_{D3} \quad \text{Net Degrees of Freedom: 4}

Practical Parameters:
\{ V_{EB1}, V_{EB3}, V_{EB5}, P \}

Need a CMFB circuit to establish V_{b1}
Single-stage low-gain differential op amp

Quarter Circuit

Single-Ended Output : Differential Input Gain

\[ A(s) = \frac{-g_{m1}}{2} \times \frac{2}{sC_L + g_{o1} + g_{o3}} \]

\[ A_o = \frac{2}{g_{o1} + g_{o3}} \times \frac{g_{m1}}{2} \]

\[ GB = \frac{g_{m1}}{2C_L} \]

\[ A_o = \left[ \frac{1}{\lambda_1 + \lambda_3} \right] \left( \frac{1}{V_{EB1}} \right) \]

\[ GB = \left( \frac{P}{V_{DD}C_L} \right) \cdot \left[ \frac{1}{2V_{EB1}} \right] \]

Need a CMFB circuit to establish \( V_{b1} \)
Single-stage low-gain differential op amp

Quarter Circuit

\[ V_{OD} = V_o^+ - V_o^- \]

Differential Output : Differential Input Gain

\[ A(s) = \frac{g_{m1}}{sC_L + g_{o1} + g_{o3}} \]

\[ A_0 = \frac{g_{m1}}{g_{o1} + g_{o3}} \]

\[ GB = \frac{g_{m1}}{C_L} \]

\[ A_0 = \left[ \frac{1}{\lambda_1 + \lambda_3} \right] \left( \frac{2}{V_{EB1}} \right) \]

\[ GB = \left( \frac{P}{V_{DD} C_L} \right) \cdot \left[ \frac{1}{V_{EB1}} \right] \]

Need a CMFB circuit to establish \( V_{B1} \) or \( V_{B2} \)
Operational Amplifier Small Signal Characteristics in Terms of Quarter Circuit Performance

Assumptions: Bias current in quarter circuits same as in Op Amps and $C_L$ is load capacitance on each side of op amp

**Single-ended Output**

$$A(s) = \frac{-g_{MN}}{2sC_L + g_{ON} + g_{OP}}$$

$$A_0 = \frac{1}{2} \frac{g_{MN}}{g_{ON} + g_{OP}}$$

$$GB = \frac{1}{2} \frac{g_{MN}}{C_L}$$

**Differential Output**

$$A(s) = \frac{-g_{MN}}{sC_L + g_{ON} + g_{OP}}$$

$$A_0 = \frac{g_{MN}}{g_{ON} + g_{OP}}$$

$$GB = \frac{g_{MN}}{C_L}$$

Expressions valid for both tail-current and tail-voltage op amp
Expressions valid for both tail-current and tail-voltage op amp

So which one should be used?

• Common-mode input range large for tail current bias
• Improved rejection of common-mode signals for tail current bias
• Extra design degree of freedom for tail current bias
• Improved output signal swing for tail voltage bias (will show later)
Single-stage low-gain differential op amp

Consider single-ended output performance:

Will term this the **reference op amp**
Will make performance comparisons of other op amps relative to this

\[
A(s) = \frac{2}{sC_L + g_{o1} + g_{o3}}
\]

mixed parameters

\[
A_{VO} = \frac{1}{2} \frac{g_{m1}}{g_{o1} + g_{o3}}
\]

\[
GB = \frac{g_{m1}}{2C_L}
\]

\[
SR = \frac{I_T}{2C_L}
\]

practical parameters

\[
A_{V0} = \left[ \frac{1}{\lambda_1 + \lambda_3} \right] \left( \frac{1}{V_{EB1}} \right)
\]

\[
GB = \left( \frac{P}{2V_{dd}C_L} \right) \cdot \left[ \frac{1}{V_{EB1}} \right]
\]

\[
SR = \frac{P}{2V_{dd}C_L}
\]

Need a CMFB circuit to establish \( V_{b1} \)
Reference Op Amp

single-ended output

\[ A(s) = \frac{g_{m1}}{2sC_L + g_{o1} + g_{o3}} \]

\[ A_{v0} = \frac{1}{2} \frac{g_{m1}}{g_{o1} + g_{o3}} \]

\[ GB = \frac{g_{m1}}{2C_L} \]

\[ SR = \frac{I_T}{2C_L} \]

\[ A_{v0} = \left[ \frac{1}{\lambda_1 + \lambda_3} \right] \left[ \frac{1}{V_{EB1}} \right] \]

\[ GB = \left( \frac{P}{2V_{DD}C_L} \right) \cdot \left[ \frac{1}{V_{EB1}} \right] \]

\[ SR = \frac{P}{2V_{DD}C_L} \]

Need a CMFB circuit to establish \( V_{b1} \)
### Amplifier Structure Summary

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Single-stage low-gain differential op amp

Need a CMFB circuit to establish $V_{B1}$ or $V_{B2}$

CMFB amplifies difference between $V_{B1}$ and average of two signal inputs

Can apply to either $V_{B1}$ or $V_{B2}$ but not both
Single-stage low-gain differential op amp

- Can eliminate CMFB circuit if only single-ended output is needed by connecting counterpart circuits as a current mirror
- This will double the voltage gain and the GB as well
- Still uses counterpart circuits but terminated in different ways
- Although not symmetric, previous analysis results with specified modifications still nearly apply
Single-stage low-gain differential op amp
Current-Mirror Connected Counterpart Circuit

No CMFB Circuit Needed

\[ A(s) = \frac{g_{m1}}{sC_L + g_{o1} + g_{o3}} \]

\[ A_0 = \frac{g_{m1}}{g_{o1} + g_{o3}} \]

\[ GB = \frac{g_{m1}}{C_L} \quad SR = \frac{I_T}{C_L} \]

In terms of practical design space parameters

\[ A_0 = \left[ \frac{1}{\lambda_1 + \lambda_3} \right] \left( \frac{2}{V_{EB1}} \right) \quad GB = \left( \frac{P}{V_{DD}C_L} \right) \cdot \left[ \frac{1}{V_{EB1}} \right] \quad SR = \frac{P}{V_{DD}C_L} \]
Signal Swing of Single-Stage Op Amp

Constraining Equations:

To keep $M_2$ in Saturation:

$$V_{OUT} > V_{ic} - V_{T2}$$

To keep $M_4$ in Saturation:

$$V_{OUT} < V_{DD} - |V_{EB4}|$$

To keep $M_1$ in Saturation:

$$V_{ic} < V_{DD} + V_{T1} - |V_{T3}| - |V_{EB3}|$$

To keep $M_5$ in Saturation:

$$V_{ic} > V_{T1} + V_{EB1} + V_{EB5} + V_{SS}$$
Signal Swing of Single-Stage Op Amp

Constraining Equations:

\[ V_{\text{OUT}} < V_{\text{DD}} - |V_{\text{EB4}}| \]

\[ V_{\text{OUT}} > V_{\text{ic}} - V_{T2} \]

\[ V_{\text{ic}} < V_{\text{DD}} + V_{T1} - |V_{T3}| - |V_{\text{EB3}}| \]

\[ V_{\text{ic}} > V_{T1} + V_{\text{EB1}} + V_{\text{EB5}} + V_{\text{SS}} \]
Signal Swing of Single-Stage Op Amp

\[
\begin{align*}
V_{\text{OUT}} & = V_{\text{DD}} - |V_{\text{EB4}}| \\
V_{\text{SS}} & = V_{\text{DD}} - (|V_{\text{EB3}}| + |V_{\text{T3}}| - V_{\text{T1}}) \\
V_{\text{T1}} + V_{\text{EB1}} + V_{\text{EB5}} & = V_{\text{SS}} \\
V_{\text{T2}} & = \frac{V_{\text{OUT}}}{2}
\end{align*}
\]
Signal Swing of Single-Stage Op Amp

Constraining Equations:

\[ V_{\text{OUT}} < V_{\text{DD}} - |V_{\text{EB4}}| \]
\[ V_{\text{OUT}} > V_{\text{ic}} - V_{T2} \]
\[ V_{\text{ic}} < V_{\text{DD}} + V_{T1} - |V_{T3}| - |V_{\text{EB3}}| \]
\[ V_{\text{ic}} > V_{T1} + V_{\text{EB1}} + V_{\text{EB5}} + V_{\text{SS}} \]

Signal swings are Important Performance Parameters!!
Design space for single-stage op amp

Performance Parameters in Practical Parameter Domain \{ V_{EB1}, V_{EB2}, V_{EB5}, P \}:

\[
A_0 = \left[ \frac{1}{\lambda_1 + \lambda_3} \right] \left( \frac{2}{V_{EB1}} \right)
\]

\[
GB = \left( \frac{P}{V_{DD}C_L} \right) \left[ \frac{2}{V_{EB1}} \right]
\]

\[
SR = \frac{P}{V_{DD}C_L}
\]

\[
V_{OUT} < V_{DD} - |V_{EB3}|
\]

\[
V_{OUT} > V_i - V_{T2}
\]

\[
V_i < V_{DD} + V_{T1} - |V_{T3}| - |V_{EB3}|
\]

\[
V_i > V_{T1} + V_{EB1} + V_{EB5} + V_{SS}
\]

Simple Expressions in Practical Parameter Domain