Fully Differential Single-Stage Amplifier Design

- General Differential Analysis
- 5T Op Amp from simple quarter circuit
- Biasing with CMFB circuit
- Common-mode and differential-mode analysis
- Common Mode Gain
- Overall Transfer Characteristics
Basic Op Amp Design

- Fundamental Amplifier Design Issues
- Single-Stage Low Gain Op Amps
- Single-Stage High Gain Op Amps
- Two-Stage Op Amp
- Other Basic Gain Enhancement Approaches

Review from last lecture:
Where we are at:
Review from last lecture:

Where we are at:

Single-Stage Low-Gain Op Amps

- Single-ended input

- Differential Input

(Symbol does not distinguish between different amplifier types)
Review from last lecture:

Differential Input Low Gain Op Amps

Will Next Show That:

- Differential input op amps can be readily obtained from single-ended op amps

- Performance characteristics of differential op amps can be directly determined from those of the single-ended counterparts
Counterpart Networks

Definition: The counterpart network of a network is obtained by replacing all n-channel devices with p-channel devices, replacing all p-channel devices with n-channel devices, replacing $V_{SS}$ biases with $V_{DD}$ biases, and replacing all $V_{DD}$ biases with $V_{SS}$ biases.
Review from last lecture:

Counterpart Networks

Theorem: The parametric expressions for all small-signal characteristics, such as voltage gain, output impedance, and transconductance of a network and its counterpart network are the same.
Synthesis of fully-differential op amps from symmetric networks and counterpart networks

Theorem: If F is any network with a single input and P is its counterpart network, then the following circuits are fully differential circuits --- “op amps”.

\[ V_d = V_1 - V_2 \]
Synthesis of fully-differential op amps from symmetric networks and counterpart networks

Terminology

Review from last lecture:

\[ V_d = V_1 - V_2 \]
A fully differential op amp is derived from any quarter circuit by combining it with its counterpart to obtain a half-circuit, combining two half-circuits to form a differential symmetric circuit and then biasing the symmetric differential circuit on the axis of symmetry.

Further, most of the properties of the operational amplifier can be obtained by inspection, from those of the quarter circuit.
A fully differential op amp is derived from any quarter circuit by combining it with its counterpart to obtain a half-circuit, combining two half-circuits to form a differential symmetric circuit and then biasing the symmetric differential circuit on the axis of symmetry.

Further, most of the properties of the operational amplifier can be obtained by inspection, from those of the quarter circuit.

Implications: Much Op Amp design can be reduced to designing much simpler quarter-circuits where it is much easier to get insight into circuit performance.
• General Differential Analysis
  • 5T Op Amp from simple quarter circuit
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Characterization of Quarter Circuit

If the input impedance is infinite, the two-port network only has two characterizing parameters: $G_M$ and $G$.

\[ V_{\text{OUT}} (G + sC_L) + G_M V_1 = 0 \]
\[ V_{\text{IN}} = V_1 \]

\[ A_{VQC}(s) = \frac{-G_M}{sC_L + G} \]

\[ A_{VOQC} = -\frac{G_M}{G} \]
\[ BW = \frac{G}{C_L} \]
\[ GB = \frac{G_M}{C_L} \]
Characterization of Quarter Circuit (or Counterpart Circuit) with input port terminated in short circuit

If the input port of a two-port has an ac short, then the two-port reduces to a one-port characterized by the conductance G.
Determination of op amp characteristics from quarter circuit characteristics

-- The “differential” gain --

Small signal differential half-circuit

Derivation: from KCL and KVL:

\[
\begin{align*}
\psi_1^- (G_1 + G_2 + sC_L) + G_{M1} \psi_1 &= 0 \\
\psi_1 &= \frac{\psi_d}{2}
\end{align*}
\]

\[
A_V = \frac{\psi_o^-}{\psi_d} = \frac{-G_{M1}}{2} \frac{1}{sC_L + G_1 + G_2}
\]

Note: Factor of 2 reduction of gain since only half of the differential input is applied to the half-circuit

Note: More reduction of gain since denominator increases
Determination of op amp characteristics from quarter circuit characteristics

**Small signal differential half-circuit**

\[
A_V = \frac{v_o^-}{v_d} = \frac{-G_{M1}}{2sC_L + G_1 + G_2}
\]

\[
A_{V0} = \text{?}
\]

\[
\text{BW} = \frac{G_1 + G_2}{C_L}
\]

\[
\text{GB} = \frac{G_{M1}}{2C_L}
\]
Determination of op amp characteristics from quarter circuit characteristics

-- The “differential” gain --

Small signal Quarter Circuit

\[ A_{VQ} (s) = \frac{-G_M}{sC_L + G} \]

Small signal differential half-circuit (repeated from last slide)

\[ A_V = \frac{\nu_o}{\nu_d} = \frac{-G_{M1}}{2} \frac{sC_L + G_1 + G_2}{sC_L + G_1 + G_2} \]
Determination of op amp characteristics from quarter circuit characteristics

-- The “differential” gain --

Small signal Quarter Circuit

\[ A_{\text{vqc}}(s) = \frac{-G_M}{sC_L + G} \]

Small signal differential amplifier

\[ A_V = \frac{V^-}{V_d} = \frac{-G_{M1}}{2} \]

\[ \frac{1}{sC_L + G_1 + G_2} \]
Determination of op amp characteristics from quarter circuit characteristics
-- The “differential” gain --

Small signal Quarter Circuit

Small signal differential amplifier

\[ A_{\text{VOQC}} = -\frac{G_M}{G} \]

\[ BW = \frac{G}{C_L} \]

\[ GB = \frac{G_M}{C_L} \]

Note: Factor of 4 reduction of gain if \( G_1 = G_2 \) (this often occurs)

Note: Factor of 2 increase of BW if \( G_1 = G_2 \) (this often occurs)

Note: Factor of 2 reduction of GB if \( G_1 = G_2 \) (this often occurs)

Remember this is applicable to ANY quarter circuit!
Comparison of Tail Voltage and Tail Current Source Structures

-- The “differential” gain --

Small signal half-circuits are identical so voltage gains, BW, and GB are all the same
Biasing Issues for Differential Amplifier

- Tail voltage bias not suitable for large common-mode (CM) input range but does offer good output swing

- Tail current bias provides good CM input range but at the expense of a modest reduction in output signal swing
Differential Output Amplifiers

-- The “differential” gain --

**Theorem:** For a symmetric circuit with symmetric outputs and differential excitations:

- **Differential Voltage Gain Double that of Single-Ended Structure**
- **BW is the same**
- **GB Doubles for the Differential Output Structure**
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Applications of Quarter-Circuit Concept to Op Amp Design

consider initially the basic single-ended amplifier
Single-stage single-input low-gain op amp

Basic Structure

Quarter Circuit

Practical Implementation
Small signal model of half-circuit

Two-port model of half-circuit

\[ G = G_1 + G_2 \]

\[ G_M = G_{M1} \]
Single-stage low-gain differential op amp

-- The “differential” gain --

Quartier Circuit

Single-Ended Output : Differential Input Gain

\[
A(s) = \frac{v_{OUT}}{v_{i}} = \frac{-g_{m1}}{2sC_{L} + g_{o1} + g_{o3}}
\]

\[
A_{V0} = \frac{-g_{m1}}{2(g_{o1} + g_{o3})}
\]

\[
BW = \frac{g_{o1} + g_{o3}}{C_{L}}
\]

\[
GB = \frac{g_{m1}}{2C_{L}}
\]

Circuit is Very Sensitive to \( V_{B1} \) and \( V_{B2} \) !!

- Have synthesized fully differential op amp from quarter circuit!
- Have obtained analysis of fully differential op amp directly from quarter circuit!
- Still need to determine what happens if input is not differential!
• General Differential Analysis
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• Overall Transfer Characteristics
Single-stage low-gain differential op amp

-- The “differential” gain --

Need CMFB circuit to establish $V_{B1}$ or $V_{B2}$ !!

- CMFB circuit determines average value of the drain voltages
- Compares the average to the desired quiescent drain voltages
- Established a feedback signal $V_{B1}$ to set the right Q-point
- Shown for $V_{B1}$ but could alternately be applied to $V_{B2}$

Details about CMFB circuits will be discussed later
Single-stage low-gain differential op amp

-- The “differential” gain --
Need CMFB circuit

Have obtained differential gain of 5T Op Amp by inspection from quarter circuit
• General Differential Analysis
• 5T Op Amp from simple quarter circuit
• Biasing with CMFB circuit

Common-mode and differential-mode analysis
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• Overall Transfer Characteristics
Common-Mode and Differential-Mode Analysis

Consider an output voltage for any linear circuit with two inputs

By superposition

\[ v_{OUT} = A_1 v_1 + A_2 v_2 \]

where \( A_1 \) and \( A_2 \) are the gains (transfer functions) from inputs 1 and 2 to the output respectively.

Define the common-mode and difference-mode inputs by

\[ v_c = \frac{v_1 + v_2}{2} \]
\[ v_d = v_1 - v_2 \]

These two equations can be solved for \( v_1 \) and \( v_2 \) to obtain

\[ v_1 = v_c + \frac{v_d}{2} \]
\[ v_2 = v_c - \frac{v_d}{2} \]
Consider an output voltage for any linear circuit with two inputs

\[ V_{\text{OUT}} = A_1 V_1 + A_2 V_2 \]

Substituting into the expression for \( V_{\text{OUT}} \), we obtain

\[
V_{\text{OUT}} = A_1 \left( V_c + \frac{V_d}{2} \right) + A_2 \left( V_c - \frac{V_d}{2} \right)
\]

Rearranging terms we obtain

\[
V_{\text{OUT}} = V_c \left( A_1 + A_2 \right) + V_d \left( \frac{A_1 - A_2}{2} \right)
\]

If we define \( A_c \) and \( A_d \) by

\[
A_c = A_1 + A_2 \quad \quad A_d = \frac{A_1 - A_2}{2}
\]

Can express \( V_{\text{OUT}} \) as

\[
V_{\text{OUT}} = V_c A_c + V_d A_d
\]
Common-Mode and Differential-Mode Analysis

Consider any output voltage for any linear circuit with two inputs

\[ \text{Linear Circuit} \]

\[ \mathbf{V}_{\text{OUT}} \]

\[ \mathbf{V}_1 \]

\[ \mathbf{V}_2 \]

\[ \mathbf{A} \]

\[ \mathbf{B} \]

\[ \mathbf{V}_{\text{OUT}} = A_1 \mathbf{V}_1 + A_2 \mathbf{V}_2 \]

\[ A_c = A_1 + A_2 \]

\[ A_d = \frac{A_1 - A_2}{2} \]

\[ \mathbf{V}_{\text{OUT}} = \mathbf{V}_c A_c + \mathbf{V}_d A_d \]

\[ \mathbf{V}_{\text{OUT}} = \mathbf{V}_c \left( A_1 + A_2 \right) + \mathbf{V}_d \left( \frac{A_1 - A_2}{2} \right) \]

Implication: Can solve a linear two-input circuit by applying superposition with \( \mathbf{V}_1 \) and \( \mathbf{V}_2 \) as inputs or by applying \( \mathbf{V}_c \) and \( \mathbf{V}_d \) as inputs

Implication: In a circuit with \( A_2 = -A_1 \), \( A_c = 0 \) we obtain

\[ \mathbf{V}_{\text{OUT}} = \mathbf{V}_d A_d \]

Analysis of op amps up to this point have assumed differential excitation
Common-Mode and Differential-Mode Analysis

Depiction of single-ended inputs and common/difference mode inputs

\[ v_{\text{OUT}} = A_1 v_1 + A_2 v_2 \]

\[ v_{\text{OUT}} = v_c A_c + v_d A_d \]

- Applicable to any linear circuit with two inputs and a single output
- Op amps often have symmetry and this symmetry further simplifies analysis
Common-Mode and Differential-Mode Analysis

Extension to differential outputs and symmetric circuits

Theorem: The symmetric differential output voltage for any symmetric linear network excited at symmetric nodes can be expressed as

\[ V_{OUT} = A_d V_d \]

where \( A_d \) is the differential voltage gain and the voltage \( V_d = V_1 - V_2 \)

Theorem: The differential output for any linear network can be expressed equivalently as

\[ V_{OUT} = A_1 V_1 + A_2 V_2 \]

or as

\[ V_{OUT} = V_c A_c + V_d A_d \]

and superposition can be applied to either \( V_1 \) and \( V_2 \) to obtain \( A_1 \) and \( A_2 \) or to \( V_c \) and \( V_d \) to obtain \( A_c \) and \( A_d \).
Proof for Symmetric Circuit with Symmetric Differential Output:

By superposition, the single-ended outputs can be expressed as
\[
\begin{align*}
V_{\text{OUT}+} &= T_{\text{OPA}} V_1 + T_{\text{OPB}} V_2 \\
V_{\text{OUT}^-} &= T_{\text{ONA}} V_1 + T_{\text{ONB}} V_2
\end{align*}
\]

where \( T_{\text{OPA}}, T_{\text{OPB}}, T_{\text{ONA}} \) and \( T_{\text{ONB}} \) are the transfer functions from the A and B inputs to the single-ended + and - outputs.

Taking the difference of these two equations, we obtain
\[
V_{\text{OUT}} = V_{\text{OUT}+} - V_{\text{OUT}^-} = (T_{\text{OPA}} - T_{\text{ONA}}) V_1 + (T_{\text{OPB}} - T_{\text{ONB}}) V_2
\]

By symmetry, we have
\[
T_{\text{OPA}} = T_{\text{ONB}} \quad \text{and} \quad T_{\text{ONA}} = T_{\text{OPB}}
\]

Thus, can be express \( V_{\text{OUT}} \) as
\[
V_{\text{OUT}} = (T_{\text{OPA}} - T_{\text{ONA}}) (V_1 - V_2)
\]

or as
\[
V_{\text{OUT}} = A_d V_d
\]

where \( A_d = T_{\text{OPA}} - T_{\text{ONA}} \) and where \( V_d = V_1 - V_2 \)
End of Lecture 4