EE 435

Lecture 41

References
Voltage References

Observation – Variables with units Volts needed to build any voltage reference

What variables available in a process have units volts?

$V_{DD}$, $V_T$, $V_{BE}$ (diode), $V_Z$, $V_{BE}$, $V_t$, $V_{G0}$

What variables which have units volts satisfy the desired properties of a voltage reference? $V_{G0}$ and ??

How can a circuit be designed that “expresses” the desired variables?

$V_{G0}$ is deeply embedded in a device model with horrible temperature effects! Good diodes are not widely available in most MOS processes!
Good diodes are not widely available in most MOS processes!
Voltage References

\[ I_C = J_S A e^{\frac{V_{BE}}{V_t}} \]

\[ J_S = \tilde{J}_{SX} \left[ T^m e^{\frac{-V_{G0}}{V_t}} \right] \]

\[ I_C(T) = \left( \tilde{J}_{SX} A \left[ T^m e^{\frac{-V_{G0}}{V_t}} \right] \right) e^{\frac{V_{BE}(T)}{V_t}} \]

Bandgap Voltage Appears in BJT Model Equation as well.

Review from last lecture.
Standard Approach to Building Voltage References

Pick $K$ so that at some temperature $T_0$, \[ \frac{\partial (X_N + KX_P)}{\partial T} \bigg|_{T=T_0} = 0 \]
Bandgap Voltage References

Consider two BJTs (or diodes)

If $I_C$ is independent of temperature, it follows that

$$\frac{\partial V_{BE}}{\partial T} \bigg|_{T=T_0=300^\circ K} \approx 8.6 \times 10^{-5} \left[ -2.3 + \left( \frac{0.65 - 1.2}{25 \text{mV}} \right) \right] \approx -2.1 \text{mV/}^\circ \text{C}$$

If $\ln(I_{C2}/I_{C1})=1$

$$\frac{\partial (V_{BE2} - V_{BE1})}{\partial T} \bigg|_{T=T_0=300^\circ K} = 8.6 \times 10^{-5} = 86 \mu \text{V/}^\circ \text{C}$$

Magnitude of TC of PTAT source is much smaller than that of $V_{BE}$ source

If

$$\frac{\partial (X_N + KX_P)}{\partial T} \bigg|_{T=T_0} = 0$$

$K$ will be large

$$X_{OUT} = X_N + KX_P$$
$V_{BE}$ and $\Delta V_{BE}$ with constant $I_C$
Comparison of $V_{BE}$ with constant current and PTAT current

Review from last lecture.
First Bandgap Reference  (and still widely used!)

Thanks to Ziyue

<table>
<thead>
<tr>
<th>Problem</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mismatch of resistors due to gradient effects</td>
<td>Use common centroid layout, such as anti-parallel layout (be careful of too many interconnects)</td>
</tr>
<tr>
<td>Mismatch of resistors due to local random variations</td>
<td>Make coarse resistors physically large and make its impedance small</td>
</tr>
<tr>
<td>Switch impedance is code dependent</td>
<td>Add an upper string DAC (to make all Vgs same)</td>
</tr>
<tr>
<td>Settling time is code dependent</td>
<td>Add series impedance in all lines to make R same or add a differential pair on the bottom to compensate</td>
</tr>
<tr>
<td>Too many routing is required to get the switch control signal into R string</td>
<td>Use logic cells physically located internal to the resistor array</td>
</tr>
<tr>
<td>β is code dependent</td>
<td>Add a dummy resistor in series with a switch on the bottom.</td>
</tr>
<tr>
<td>How to determine the impedance of the resistor and switch</td>
<td>It depends on the matching characteristics of resistors relative to switches</td>
</tr>
<tr>
<td>Use NMOS switch on the bottom</td>
<td>Save area, improve speed (larger Vgs for NMOS) and reduce signal dependent delay when switch is turned on.</td>
</tr>
<tr>
<td>Switches makes current not constant</td>
<td>Use differential pairs on the bottom to steering current to ground or dump instead of switching current. This keeps the current constant</td>
</tr>
<tr>
<td>For R-2R DAC, switch impedance cause mismatch in impedance</td>
<td>Let the switches exclude from resistors</td>
</tr>
<tr>
<td>During switching, changing voltage introduce nonlinearity</td>
<td>Use self cascode or even double cascode to reduce the nonlinearity</td>
</tr>
<tr>
<td>Dynamic current source used in DAC requires refreshing.</td>
<td>Use latch and decoder to do the current correction rather than capacitors. But a temperature sensor is needed.</td>
</tr>
<tr>
<td>Dynamic current source need refreshing</td>
<td>Use 2 transistors with large size ratio. And use latch and decoder to do the current correction rather than capacitor. (including a temperature sensor)</td>
</tr>
<tr>
<td>--------------------------------------</td>
<td>--------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Invalid bullin level occurs</td>
<td>Use regenerate comparator to make a decision</td>
</tr>
<tr>
<td>Offset compensation</td>
<td>Use a switch and a cap which stores the offset voltage. When phi1 close, use Vdac to have the same voltage as Cap.</td>
</tr>
<tr>
<td>Number of comparator goes with resolution</td>
<td>Split ADC into multi stages. i.e, using two step flash ADC to reduce the # of comparators.</td>
</tr>
</tbody>
</table>

Interpolating ADC reduces offset requirement for comparators.

Cyclic (Algorithmic) ADC can not take new samples until the whole conversion finishes. We can play with bias current and compensation cap to decrease the power and effective thru-put.
First Bandgap Reference (and still widely used!)

Current ratios is $R_4/R_3$
Current not highly dependent upon $T$

$$V_{REF} = V_{BE2} + \frac{R_1}{R_2} \left[ V_{BE1} - V_{BE2} \right]$$
First Bandgap Reference (and still widely used!)

\[ I_{E1} R_2 + V_{BE1} = V_{BE2} \]

\[ V_{REF} = V_{BE2} + (I_{E1} + I_{E2}) R_1 \]

\[ I_{C1} = \frac{V_{DD} - V_{C2} - V_{OS}}{R_3} \]

\[ I_{C2} = \frac{V_{DD} - V_{C2}}{R_4} \]

\[ I_{C1} = \alpha_1 I_{E1} \]

\[ I_{C2} = \alpha_2 I_{E2} \]

\[ I_{E1} = I_{E2} \left[ \frac{\alpha_2 R_4}{\alpha_1 R_3} \right] - \frac{V_{OS}}{\alpha_1 R_3} \]

\[ V_{REF} = V_{BE2} + (V_{BE2} - V_{BE1}) \left[ \frac{R_1}{R_2} \left( 1 + \frac{\alpha_1 R_3}{\alpha_2 R_4} \right) \right] - V_{OS} \left[ \frac{R_1}{\alpha_1 R_3} \right] \]

\[ \alpha = \frac{\beta}{1+\beta} \]
First Bandgap Reference (and still widely used!)

\[
V_{\text{REF}} = V_{\text{BE2}} + (V_{\text{BE2}} - V_{\text{BE1}}) \left[ \frac{R_1}{R_2} \left( 1 + \frac{\alpha_1 R_3}{\alpha_2 R_4} \right) \right]
\]

\[
V_{\text{BE2}} = V_t \ln I_C + \left[ V_{G0} - V_t \left\{ \ln \left( A_{E2} J_{SX} \right) + m \ln T \right\} \right]
\]

\[
V_{\text{BE1}} = V_t \ln I_C + \left[ V_{G0} - V_t \left\{ \ln \left( A_{E1} J_{SX} \right) + m \ln T \right\} \right]
\]

\[
I_{C1} = \alpha_1 I_{E1}
\]

\[
I_{C2} = \alpha_2 I_{E2}
\]

\[
I_{E1} = I_{E2} \left[ \frac{\alpha_2 R_4}{\alpha_1 R_3} \right]
\]

\[
V_{\text{BE2}} - V_{\text{BE1}} = \Delta V_{\text{BE}} = \left[ \frac{k}{q} \ln \left( \frac{A_{E1}}{A_{E2}} \left[ \frac{R_3}{R_4} \right] \right) \right] T
\]
First Bandgap Reference (and still widely used!)

\[ V_{REF} = V_{BE2} + (V_{BE2} - V_{BE1}) \left( \frac{R_1}{R_2} \left( 1 + \frac{\alpha_1 R_3}{\alpha_2 R_4} \right) \right) \]

\[ V_{BE2} = V_t \ln I_{C2} + \left[ V_{G0} - V_t \left\{ \ln \left( A_{E2} \tilde{J}_{SX} \right) + m \ln T \right\} \right] \]

\[ V_{BE1} = V_t \ln I_{C1} + \left[ V_{G0} - V_t \left\{ \ln \left( A_{E1} \tilde{J}_{SX} \right) + m \ln T \right\} \right] \]

\[ V_{BE2} - V_{BE1} = \Delta V_{BE} = \frac{k}{q} \ln \left( \frac{A_{E1}}{A_{E2}} \left[ \frac{R_3}{R_4} \right] \right) T \]

From the expression for \( V_{BE2} \) and some routine but tedious manipulations it follows that

\[ V_{BE2} = V_{G0} + (1-m)V_t \ln T + V_t \ln \left( \frac{k}{q} R_2 \frac{\alpha_1}{R_2 A_{E2} \tilde{J}_{SX} A_{E1} R_3} \ln \left( \frac{A_{E1} R_3}{A_{E2} R_4} \right) \right) \]
First Bandgap Reference (and still widely used!)

\[
V_{REF} = V_{BE2} + (V_{BE2} - V_{BE1}) \left[ \frac{R_1}{R_2} \left( 1 + \frac{\alpha_1 R_3}{\alpha_2 R_4} \right) \right]
\]

\[
V_{BE2} - V_{BE1} = \Delta V_{BE} = \left[ \frac{k}{q} \ln \left( \frac{A_{E1}}{A_{E2}} \frac{R_3}{R_4} \right) \right] T
\]

\[
V_{BE2} = V_{G0} + (1-m)V_t \ln T + V_t \ln \left( \frac{k \frac{\alpha_1}{q R_2 A_{E2} J_{S X}}}{R_3} \ln \left( \frac{A_{E1} R_3}{A_{E2} R_4} \right) \right)
\]

It thus follows that:

\[
V_{REF} = V_t \ln \left[ \frac{\alpha_1 R_3}{R_2 R_4} \right] \frac{k}{q} \ln \left( \frac{A_{E1} R_3}{A_{E2} R_4} \right) + V_{G0} - V_t \ln \left( \ln \left( \frac{\alpha_1}{q R_2 A_{E2} J_{S X}} \right) + m \ln T \right) + \left[ \frac{k}{q} \ln \left( \frac{A_{E1}}{A_{E2}} \frac{R_3}{R_4} \right) \right] \left[ \frac{R_1}{R_2} \left( 1 + \frac{\alpha_1 R_3}{\alpha_2 R_4} \right) \right] T
\]
First Bandgap Reference (and still widely used!)

\[ V_{REF} = V_{BE2} + \left( V_{BE2} - V_{BE1} \right) \frac{R_1}{R_2} \left( 1 + \frac{\alpha_1 R_3}{\alpha_2 R_4} \right) \]

\[ V_{REF} = V_t \ln \left( \frac{\alpha_1 R_3}{R_2 R_4} \right) T \frac{k}{q} \ln \left( \frac{A_{E1} R_3}{A_{E2} R_4} \right) + V_{G0} - V_t \ln \left( \frac{I_{Sx2}}{T_{ln}} \right) + m \ln T + \left[ \frac{k}{q} \ln \left( \frac{A_{E1}}{A_{E2} R_4} \right) \right] \left( \frac{R_1}{R_2} \left( 1 + \frac{\alpha_1 R_3}{\alpha_2 R_4} \right) \right) T \]

\[ V_{REF} = a_1 + b_1 T + c_1 T \ln T \]

\[ a_1 = V_{GO} \]

\[ b_1 = \frac{k}{q} \frac{R_1}{R_2} \left( 1 + \frac{\alpha_1 R_3}{R_4 \alpha_2} \right) \ln \left( \frac{R_3 A_{E1}}{R_4 A_{E2}} \right) + \ln \left( \frac{k R_3}{q R_4} \alpha_1 \frac{\ln \left( \frac{R_3 A_{E1}}{R_1 A_{E2}} \right)}{T_{SK2} R_2} \right) \]

\[ c_1 = \frac{k}{q} (1 - m) \]
First Bandgap Reference (and still widely used!)

\[ V_{\text{REF}} = a_1 + b_1 T + c_1 T \ln T \]

\[ a_1 = V_{\text{GO}} \]

\[ b_1 = \frac{k}{q} \left( \frac{R_1}{R_2} \left(1 + \frac{R_3 a_1}{R_4 a_2}\right) \ln \left(\frac{R_3 A_{E1}}{R_4 A_{E2}}\right) \right) + \ln \left(\frac{k R_3 a_1}{q R_4} \frac{\ln \left(\frac{R_3 A_{E1}}{R_1 A_{E2}}\right)}{I_{\text{SK2}} R_2}\right) \]

\[ c_1 = \frac{k}{q} (1 - m) \]

\[ \frac{dV_{\text{REF}}}{dT} = b_1 + c_1 (1 + \ln T) = 0 \]

\[ T_{\text{INF}} = e^{-\frac{1 + b_1}{c_1}} \]

\[ b_1 = -c_1 (1 + \ln T_{\text{INF}}) \]

\[ V_{\text{REF}} = a_1 - c_1 T_{\text{INF}} \]

\[ V_{\text{REF}} = V_{\text{G0}} + \frac{k T_{\text{INF}}}{q} (m - 1) \]
First Bandgap Reference (and still widely used!)

\[ V_{\text{REF}} = a_1 + b_1 T + c_1 T \ln T \]

\[ V_{\text{REF}} = V_{g0} + \frac{kT_{\text{INF}}}{q} (m - 1) \]

- **VGO** = 1.206
- **TO** = 300
- **VBE02** = 0.65
- **m-1** = 1.3
- **k/q** = 8.61E-05

**Bandgap Voltage Source**

![Bandgap Voltage Source](image-url)
Temperature Coefficient

\[ TC = \frac{V_{\text{MAX}} - V_{\text{MIN}}}{T_2 - T_1} \]

\[ TC_{\text{ppm}} = \frac{V_{\text{MAX}} - V_{\text{MIN}}}{V_{\text{NOM}}(T_2 - T_1)} 10^6 \]
Bamba Bandgap Reference

Bamba Bandgap Reference

\[
I_{R0} = \frac{\Delta V_{BE}}{R_0}
\]

\[
I_{R1} = \frac{V_{BE1}}{R_1}
\]

\[
I_{R2} = I_{R1}
\]

\[
I_2 = I_{R0} + I_{R2}
\]

\[
I_3 = K I_2 \quad \text{K is the ratio of } I_3 \text{ to } I_2
\]

\[
V_{REF} = \frac{3}{4} R_4
\]

Substituting, we obtain

\[
V_{REF} = \theta K R \left( \frac{V_{AV}}{R_1} \right) \left( \frac{V_{BE}}{R_0} \right) + \theta \frac{V_{AV}}{R_1} \left( \frac{R_4}{R_0} \right) \left( \frac{V_{BE}}{R_1} \right)
\]

\[
V_{REF} = a_{11} + b_{11} T + c_{11} T \ln T
\]
\[ I_{R0} = \frac{\Delta V_{BE}}{R_0} \]

\[ I_2 = I_{R0} \]

\[ V_{REF} = I_2 R_2 + V_{BE1} \]

Solving, we obtain

\[ V_{REF} = \frac{R_2}{R_0} \Delta V_{BE} + V_{BE1} \]

\[ V_{REF} = a_{22} + b_{22} T + c_{22} T \ln T \]
Voltage References

Consider the Diode

\[ I_D = J_S A e^{\frac{V_D}{V_t}} \]

\[ J_S = \tilde{J}_{Sx} \left[ T^m e^{-\frac{V_{G0}}{V_t}} \right] \]

\[ V_t = \frac{kT}{q} \]

\[ k = \frac{1.38 \times 10^{-23}}{1.602 \times 10^{-19}} \text{ V K} = 8.614 \times 10^{-5} \text{ V K} \]

\[ V_{G0} = 1.206 \text{V} \]

Termed the bandgap voltage

Pn junction characteristics highly temperature dependent through both the exponent and \( J_S \)

\( V_{G0} \) is nearly independent of process and temperature
Voltage References

Observation – Variables with units Volts needed to build any voltage reference

What variables available in a process have units volts?

V_{DD}, V_T, V_{BE} (diode), V_Z, V_{BE}, V_t, V_{G0} ???

What variables which have units volts satisfy the desired properties of a voltage reference? V_{G0} and ??

How can a circuit be designed that “expresses” the desired variables?

V_{G0} is deeply embedded in a device model with horrible temperature effects! Good diodes are not widely available in most MOS processes!
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These diodes interact and actually form substrate pnp transistor

Not practical to forward bias junction
Good diodes are not widely available in most MOS processes!
*Voltage References*

\[ I_C = J_S A e^{\frac{V_{BE}}{V_t}} \]

\[ J_S = \tilde{J}_{sx} \left[ T^m e^{\frac{-V_{G0}}{V_t}} \right] \]

Bandgap Voltage Appears in BJT Model Equation as well

\[ I_C(T) = \left( \tilde{J}_{sx} A \left[ T^m e^{\frac{-V_{G0}}{V_t}} \right] \right) e^{\frac{V_{BE}(T)}{V_t}} \]
Voltage references that “express” the bandgap voltage are termed “Bandgap References”.

$V_{G0}$ is deeply embedded in a device model with horrible temperature effects!

Good BJTs are not widely available in most MOS processes but the substrate pnp is available!
Standard Approach to Building Voltage References

Negative Temperature Coefficient (NTC)

Positive Temperature Coefficient (PTC)

Pick $K$ so that at some temperature $T_0$, \[
\left. \frac{\partial (X_N + KX_P)}{\partial T} \right|_{T=T_0} = 0
\]
Standard Approach to Building Voltage References

\[ V = V_0 + \alpha (T - T_0) \]

\( V_0 \) is the voltage at temperature \( T_0 \), \( \alpha \) is the slope of the curve, and \( T \) is the temperature. The graph shows the relationship between voltage and temperature for both positive and negative temperature coefficients.
Standard Approach to Building Voltage References

\[ V = \frac{\partial (X_N + KX_P)}{\partial T} \bigg|_{T=T_0} \]

\[ X_N + KX_P \]

\[ T_0 \]

\[ T \]
Bandgap Voltage References

Consider two BJTs (or diodes)

\[
I_C(T) = \left( \tilde{I}_{Sx} \left[ T^m e^{-\frac{\Delta V_0}{V_t}} \right] \right) e^{\frac{V_{BE}(T)}{V_t}}
\]

\[
V_{BE} = V_t \ln(I_c) + \left[ V_{G0} - V_t \left( \ln(\tilde{J}_{sx} A_e) + m \ln T \right) \right]
\]

\[
V_{BE2} - V_{BE1} = \Delta V_{BE} = \left[ \frac{k \ln \left( \frac{I_{C2}}{I_{C1}} \right)}{q} \right] T
\]

If the \( I_{C2}/I_{C1} \) ratio is constant, the TC of \( \Delta V_{BE} \) is positive

\( \Delta V_{BE} \) is termed a PTAT voltage (Proportional to Absolute Temperature)

This relationship applies irrespective of how temperature dependent \( I_{C1} \) and \( I_{C2} \) may be provided the ratio is constant!!
Bandgap Voltage References

Consider two BJTs (or diodes)

\[ V_{BE2} - V_{BE1} = \Delta V_{BE} = \left[ \frac{k}{q} \ln \left( \frac{I_{C2}}{I_{C1}} \right) \right] T \]

\[ \frac{\partial (V_{BE2} - V_{BE1})}{\partial T} = \frac{k}{q} \ln \left( \frac{I_{C2}}{I_{C1}} \right) \]

At room temperature

\[ V_{BE2} - V_{BE1} = [8.6 \times 10^{-5} \times 300] = 25.8 \text{mV} \]

If \( \ln(I_{C2}/I_{C1})=1 \)

\[ \left. \frac{\partial (V_{BE2} - V_{BE1})}{\partial T} \right|_{T=T_0=300^\circ K} = 8.6 \times 10^{-5} = 86 \mu V/\circ C \]

The temperature coefficient of the PTAT voltage is rather small
Consider two BJTs (or diodes):

\[
\frac{\partial (V_{BE2} - V_{BE1})}{\partial T} = \frac{k}{q} \ln \left( \frac{I_{C2}}{I_{C1}} \right)
\]

At room temperature, the temperature coefficient of the PTAT voltage is rather small even if large collector current ratios are used.
Bandgap Voltage References

Consider two BJTs (or diodes)

\[I_C(T) = \left( \tilde{I}_{sx} \left[ T^m e^{\left(-\frac{V_G}{V_t}\right)} \right] \right) e^{\left(\frac{V_{BE}(T)}{V_t}\right)}\]

\[V_{BE} = V_t \ln(I_C) + \left[ V_{G0} - V_t \left( \ln\left(\tilde{J}_{sx}A_E\right) + m \ln T \right) \right]\]

If \(I_C\) is independent of temperature, it follows that

\[\frac{\partial V_{BE}}{\partial T} = \frac{k}{q} \left[ -m + \left( \frac{V_{BE} - V_{G0}}{V_t} \right) \right]\]

\[\left. \frac{\partial V_{BE}}{\partial T} \right|_{T=T_0=300^\circ K} \approx 8.6 \times 10^{-5} \left[ -2.3 + \left( \frac{0.65 - 1.2}{25 mV} \right) \right] \approx -2.1 mV/^\circ C\]
Bandgap Voltage References

Consider two BJTs (or diodes)

If \( I_C \) is independent of temperature, it follows that

\[
\frac{\partial V_{BE}}{\partial T} \bigg|_{T=T_0=300^\circ K} \approx 8.6 \times 10^{-5} \left[ -2.3 + \left( \frac{0.65 - 1.2}{25 \text{mV}} \right) \right] \approx -2.1 \text{mV/}^\circ \text{C}
\]

If \( \ln(I_{C2}/I_{C1})=1 \)

\[
\left. \frac{\partial (V_{BE2} - V_{BE1})}{\partial T} \right|_{T=T_0=300^\circ K} = 8.6 \times 10^{-5} = 86 \mu \text{V/}^\circ \text{C}
\]

Magnitude of TC of PTAT source is much smaller than that of \( V_{BE} \) source

If

\[
\left. \frac{\partial (X_N + KX_P)}{\partial T} \right|_{T=T_0} = 0 \quad \text{K will be large}\]

\[
X_{OUT} = X_N + KX_P
\]
End of Lecture 41