Integrated Filters and Amplifiers

- Integrators
- OTA-C Filters
- Switched-Capacitor Filters
- Voltage Amplifiers
Standard Approach to Building Voltage References

Pick $K$ so that at some temperature $T_0$, \[ \frac{\partial (X_N + KX_P)}{\partial T} \bigg|_{T=T_0} = 0 \]
First Bandgap Reference (and still widely used!)

\[ V_{REF} = a_1 + b_1 T + c_1 T \ln T \]

\[ V_{REF} = V_{G0} + \frac{kT_{\text{INF}}}{q} (m - 1) \]

- Review from last lecture.
Review from last lecture.
Bamba Bandgap Reference

\[ I_{R0} = \frac{\Delta V_{BE}}{R_0} \]
\[ I_{R1} = \frac{V_{BE1}}{R_1} \]
\[ I_{R2} = I_{R1} \]
\[ I_2 = I_{R0} + I_{R2} \]
\[ I_3 = K I_2 \quad K \text{ is the ratio of } I_3 \text{ to } I_2 \]
\[ V_{REF} = \theta I_3 R_4 \]

Substituting, we obtain

\[ V_{REF} = \theta K R_4 \left( \frac{V_{BE}}{R_1} + \frac{\Delta V_{BE}}{R_0} \right) \]

\[ V_{REF} = \theta K \frac{R_4}{R_1} \left( V_{BE} + \frac{R_1}{R_0} \Delta V_{BE} \right) \]

\[ V_{REF} = a_{11} + b_{11} T + c_{11} T \ln T \]
What is a filter?

A filter can be viewed as an amplifier with a frequency-dependent gain where the characteristics of the frequency-dependent gain are of interest.
What is a filter?

- Filter is always a time-domain device
- Often interested in the properties of a filter in the frequency domain

Typically one or more of the following often must be attained by the designer:

**Magnitude Response**

\[ |T(j\omega)| \]

\[ \omega \]

\[ \omega_0 \]

**Phase Response**

\[ \angle T(j\omega) \]

\[ -90^\circ \]

**Step Response**

\[ X_{IN}(t) \rightarrow X_{OUT}(t) \]

\[ t \]
Some standard types of filters

Requirements can be very specific
Requirements can be very stringent

\[ |T(j\omega)| \]

Band edges may need to be controlled to 0.1% or better in some applications
Example of a passive filter

\[ T(s) = \frac{1}{1+RCs} \]

\[ \omega_0 = \frac{1}{RC} \]
Typical Filter Implementation

Biquads often LP or BP
Typical Biquad Implementation (Two-Integrator Loop)

\[ T(s) = \frac{-l_0^2}{s^2 + \alpha l_0 s + l_0^2} \]

Accurate control of \( l_0 \) and \( \alpha \) is essential for building most filters!
Two-integrator Loop

\[ T(s) = \frac{-sI_0}{s^2 + \alpha I_0 s + I_0^2} \]

Accurate control of \( I_0 \) and \( \alpha \) is essential for building most filters!
Alternate two-integrator loop

\[
\begin{align*}
T(s) &= \frac{-l_0^2}{s^2 + \alpha l_0 s + l_0^2} \\
T_1(s) &= \frac{-s l_0}{s^2 + \alpha l_0 s + l_0^2}
\end{align*}
\]

Accurate control of \( l_0 \) and \( \alpha \) is essential for building most filters!
Alternate two-integrator loop

\[ T(s) = \frac{-l_0^2}{s^2 + \alpha l_0 s + l_0^2} \]

Accurate control of \( l_0 \) and \( \alpha \) is essential for building most filters!
Observation:

• The integrator is the key building block in most filters
• Accuracy of $I_0$ and $\alpha$ is important!
The Integrator

\[
V_{\text{OUT}}(s) = \frac{l_0}{s} \cdot V_{\text{IN}}(s)
\]

Frequency Domain Characterization

\[
V_{\text{OUT}}(s) = \frac{l_0}{s} \cdot V_{\text{IN}}(s)
\]

Time Domain Characterization

Integral Form:

\[
v_{\text{OUT}}(t) = l_0 \left( \int_{\tau=t_1}^{t} v_{\text{IN}}(\tau) \, d\tau + v_{\text{IN}}(t_1) \right)
\]

Differential Form:

\[
\frac{\partial v_{\text{OUT}}}{\partial t} = l_0 v_{\text{IN}}
\]

Key property: \( l_0 \)

Accurate control of \( l_0 \) is essential for building most filters!
The Lossy Integrator

Frequency Domain Characterization

\[ V_{\text{OUT}}(s) = \frac{l_0}{s + \alpha l_0} \cdot V_{\text{IN}}(s) \]

Time Domain Characterization

Integral Form:

\[ V_{\text{OUT}}(t) = l_0 \left( \int_{\tau=t_1}^{t} v_{\text{IN}}(\tau) d\tau + v_{\text{IN}}(t_1) \right) - \alpha l_0 V_{\text{OUT}}(t_1) \]

Differential Form:

\[ \frac{\partial V_{\text{OUT}}}{\partial t} + \alpha l_0 V_{\text{OUT}} = l_0 v_{\text{IN}} \]

Key properties: \( l_0, \alpha \)

Accurate control of \( l_0 \) and \( \alpha \) is essential for building most filters!
Active RC Integrator Implementations

\[ V_{\text{IN}} \xrightarrow{\frac{I_0}{s}} V_{\text{OUT}} \]

\[ V_{\text{IN}} \xrightarrow{\frac{I_0}{s}} V_{\text{OUT}} \]

\( V_{\text{OUT}} - V_{\text{IN}} = I(s) = -\frac{1}{sRC} \)

\( I_0 = \frac{1}{RC} \)

R_{1A} = R_{1B}

Accurate control of the RC product is essential for building most filters!

And accurate control of R_{1A}/R_{1B} is essential for noninverting integrator
Active RC Lossy Integrators

\[ \frac{V_{OUT}}{V_{IN}}(s) = I(s) = \frac{R_F}{R} \frac{1}{1+\alpha sCR_F} \]

Accurate control of the RC product is essential for building most filters!
And accurate control of \( R_{1A}/R_{1B} \) is essential for noninverting integrator.
Accurate control of the $g_m/C$ ratio is essential for building most filters!
Accurate control of the $g_m/C$ ratio is essential for building most filters!

And accurate control of $R_F g_m$ or $g_{mA}/g_m$ is essential for noninverting integrator.
RC Biquadratic Filter

\[ X_{IN} \xrightarrow{+} \frac{l_0}{s + a l_0} \xrightarrow{-} \frac{l_0}{s} \xrightarrow{-1} X_{OUT} \]

\[ V_{IN} \xrightarrow{R, R_Q, C} \xrightarrow{R, C} \xrightarrow{R_A, R_A} V_{OUT} \]
Observations:

Key transitions are determined by $I_0$ (and approx equal to $I_0$).
Features around transitions determined by loss.
How accurate must $I_0$ and loss terms be?

Depends upon application
Often to 1% or 0.1% or better
What happens if accuracy is not attained?

With process variations of +/- 20% in sheet resistance and another +/-15% variation in resistance with temperature, variability of $R$ is several orders of magnitude too large.

Process variations in $C$ are in the +/- 20% range as well.

Unacceptable performance (variability in $I_0$ orders of magnitude too large!)
No market opportunity!
Economic Implications

If $I_0 = 1 \text{ KRad/Sec}$ and $C = 1 \text{ pF}$, how large must $R$ be?

$I_0 = \frac{1}{RC}$
Economic Implications

\[ I_0 = \frac{1}{RC} \]

If \( I_0 = 1 \text{KRad/Sec} \) and \( C = 1 \text{pF} \), how large must \( R \) be?

\[ R = \frac{1}{I_0C} = 10^9 \]

If sheet resistance is 30 ohms/square, one resistor requires 33 million squares!