Integrated Filters and Amplifiers

- Integrators
- OTA-C Filters
- Switched-Capacitor Filters
- Voltage Amplifiers
Temperature Sensors

MOS-Based Temperature Sensors

\[ V_{Tn} = V_{Tn0} + \gamma T \]

\[ V_{01} = V_{Tn} \left( 1 - \frac{W_2L_1}{\sqrt{MW_1L_2}} \right) \]

\[ V_{02} = V_{Tn} \left( 1 + \frac{W_2L_3}{\sqrt{MW_1L_2}} - \frac{W_2L_1}{\sqrt{MW_1L_2}} \right) \]
Some standard types of filters

- Lowpass
- Bandpass
- Highpass

Requirements can be very specific.
Requirements can be very stringent

\[ |T(j\omega)| \]

Band edges may need to be controlled to 0.1% or better in some applications
Typical Filter Implementation

$$V_{IN} \rightarrow T_0(s) \text{ Biquad} \rightarrow T_1(s) \text{ Biquad} \rightarrow T_2(s) \text{ Biquad} \rightarrow \cdots \rightarrow T_k(s) \text{ Biquad} \rightarrow T_{m}(s) \text{ Biquad} \rightarrow V_{OUT}$$

Biquads often LP or BP

Review from last lecture
Accurate control of $I_0$ and $\alpha$ is essential for building most filters!
Accurate control of $l_0$ and $\alpha$ is essential for building most filters!
Alternate two-integrator loop

Accurate control of \( I_0 \) and \( \alpha \) is essential for building most filters!
Alternate two-integrator loop

\[ T(s) = \frac{-I_0^2}{s^2 + \alpha I_0 s + I_0^2} \]

Accurate control of \( I_0 \) and \( \alpha \) is essential for building most filters!
Observation:

- The integrator is the key building block in most filters
- Accuracy of $I_0$ and $\alpha$ is important!
The Integrator

\[ V_{\text{IN}} \xrightarrow{\frac{l_0}{s}} V_{\text{OUT}} \]

Frequency Domain Characterization

\[ V_{\text{OUT}}(s) = \frac{l_0}{s} \cdot V_{\text{IN}}(s) \]

Time Domain Characterization

Integral Form:

\[ V_{\text{OUT}}(t) = l_0 \left( \int_{\tau=t_1}^{t} V_{\text{IN}}(\tau) \, d\tau + V_{\text{IN}}(t_1) \right) \]

Differential Form:

\[ \frac{\partial V_{\text{OUT}}}{\partial t} = l_0 V_{\text{IN}} \]

Key property: \( l_0 \)

Accurate control of \( l_0 \) is essential for building most filters!
The Lossy Integrator

\[
V_{\text{IN}} \xrightarrow{\frac{l_0}{s+\alpha l_0}} V_{\text{OUT}}
\]

Frequency Domain Characterization

\[
V_{\text{OUT}}(s) = \frac{l_0}{s + \alpha l_0} \cdot V_{\text{IN}}(s)
\]

Time Domain Characterization

Integral Form:

\[
V_{\text{OUT}}(t) = l_0 \left( \int_{\tau=t_1}^{t} V_{\text{IN}}(\tau) \, d\tau + V_{\text{IN}}(t_1) \right) - \alpha l_0 V_{\text{OUT}}(t_1)
\]

Differential Form:

\[
\frac{\partial V_{\text{OUT}}}{\partial t} + \alpha l_0 V_{\text{OUT}} = l_0 V_{\text{IN}}
\]

Key properties: \( l_0 \), \( \alpha \)

Accurate control of \( l_0 \) and \( \alpha \) is essential for building most filters!
Active RC Integrator Implementations

Inverting Integrator
\[ \frac{V_{OUT}(s)}{V_{IN}(s)} = I(s) = -\frac{1}{sRC} \]

Noninverting Integrator
\[ \frac{V_{OUT}(s)}{V_{IN}(s)} = I(s) = \frac{1}{sRC} \left[ \frac{R_{1B}}{R_{1A}} \right] \]

Accurate control of the RC product is essential for building most filters!
And accurate control of \( R_{1A}/R_{1B} \) is essential for noninverting integrator
Active RC Lossy Integrators

\[ \frac{V_{OUT}(s)}{V_{IN}(s)} = I(s) = \frac{R_F}{1+sCR_F} \]

Accurate control of the RC product is essential for building most filters!
And accurate control of \( R_{1A}/R_{1B} \) is essential for noninverting integrator.
OTA-C Implementation

\[
\frac{V_{OUT}(s)}{V_{IN}(s)} = I(s) = \frac{g_m}{sC}
\]

Accurate control of the \(g_m/C\) ratio is essential for building most filters!
Accurate control of the $g_m/C$ ratio is essential for building most filters!

And accurate control of $R_F g_m$ or $g_{mA}/g_m$ is essential for noninverting integrator
RC Biquadratic Filter

\[ X_{IN} \xrightarrow{+} \frac{-l_0}{s+\alpha l_0} \xrightarrow{-l_0/s} -1 \xrightarrow{} X_{OUT} \]

- Input: \( X_{IN} \)
- Output: \( X_{OUT} \)
- Transistor Circuit Diagram:
  - Input: \( V_{IN} \)
  - Output: \( V_{OUT} \)
Observations:

Key transitions are determined by $I_0$ (and approx equal to $I_0$)
Features around transitions determined by loss
How accurate must $I_0$ and loss terms be?

Depends upon application
Often to 1% or 0.1% or better
What happens if accuracy is not attained?

With process variations of +/- 20% in sheet resistance and another +/-15% variation in resistance with temperature, variability of R is several orders of magnitude too large.

Process variations in C are in the +/- 20% range as well.

Unacceptable performance (variability in I₀ orders of magnitude too large!) No market opportunity!

\[ I₀ = \frac{1}{RC} \]
Economic Implications

If $I_0 = 1 \text{KRad/Sec}$ and $C = 1 \text{pF}$, how large must $R$ be?

$I_0 = \frac{1}{RC}$
If \( I_0 = 1 \text{ KRad/Sec} \) and \( C = 1 \text{ pF} \), how large must \( R \) be?

\[
R = \frac{1}{I_0 C} = 10^9
\]

If sheet resistance is 30 ohms/square, one resistor requires 33 million squares!
Challenges in Integrated Filter Design

• Accuracy of components is not good enough (orders of magnitude)
• Area too large for audio frequencies (orders of magnitude)
Challenges in Integrated Filter / Integrated Integrator Design

![Diagram of integrator circuit]

\[ T(s) = -\frac{1}{RCs} \]

\[ I_0 = \frac{1}{RC} \]

- Accuracy of R and C difficult to accurately control – particularly in integrated applications
- Size of R and C unacceptably large if \( I_0 \) is in audio frequency range
- Amplifier GB limits performance
Consider the following circuit

\[ Q_{RC} = - \frac{1}{R} \int_{t=t_1}^{t_1+T} v_{IN}(t) \, dt \]

\[ v_{OUT}(t_1+T) = v_{OUT}(t_1) - \frac{Q_{RC}}{C} \]

If \( T << T_{SIG} \), then \( v_{IN}(t) \approx v_{IN}(t_1) \) for \( t_1 < t < t_1 + T \)
Consider the following circuit

If $T << T_{\text{SIG}}$

$$Q_{RC} = \int_{t=t_1}^{t_1+T} i(t) \, dt = -\frac{1}{R} \int_{t=t_1}^{t_1+T} v_{\text{IN}}(t) \, dt \approx -\frac{v_{\text{IN}}(t_1)}{R} \int_{t=t_1}^{t_1+T} 1 \, dt = -\frac{v_{\text{IN}}(t_1)}{R} T$$

$$v_{\text{OUT}}(t_1+T) = v_{\text{OUT}}(t_1) - \frac{Q_{RC}}{C} = v_{\text{OUT}}(t_1) - \frac{v_{\text{IN}}(t_1)}{RC} T$$
Consider the following circuit

If $T \ll T_{SIG}$

$$Q_{SC} = -C_1 \cdot v_{IN}(t_1)$$

$$v_{OUT}(t_1 + T) = v_{OUT}(t_1) - \frac{Q_{SC}}{C} = v_{OUT}(t_1) - v_{IN}(t_1) \frac{C_1}{C}$$
Consider the following two circuits

Both transfer charge proportional to $V_{IN}(t_1)$
Consider the following two circuits

\[ Q_{RC} = -\frac{v_{IN}(t_1)}{R} T \]

\[ Q_{SC} = -C_1 \cdot v_{IN}(t_1) \]

Thus equating charges

\[ Q_{RC} \approx Q_{SC} \]

Obtain the equivalent resistance of the switched-capacitor circuit

\[ R_{EQ} \approx \frac{T}{C_1} \]
Thus, if clocked with a clock period of $T$, for $T << T_{SIG}$, we have the following equivalence:

$$R_{EQ} \approx \frac{T}{C_1}$$

Equivalence of a switched capacitor and a resistor has been known for over 100 years. Up until 1977, this knowledge was of little practical significance.
End of Lecture 43