Consider Voltage References

Observation – Variables with units Volts needed to build any voltage reference
Voltage References

VBIAS \rightarrow \text{Voltage Reference Circuit} \rightarrow VREF

Observation – Variables with units Volts needed to build any voltage reference

What variables available in a process have units volts?

- $V_{DD}$, $V_T$, $V_{BE}$ (diode), $V_Z$, $V_{BE}$, $V_t$

What variables which have units volts satisfy the desired properties of a voltage reference?

How can a circuit be designed that “expresses” the desired variables?
Voltage References

Consider the Diode

\[ I_D = J_S A e^{\frac{V_D}{V_t}} \]

\[ J_S = \tilde{J}_{SX} \left[ T^m e^{\frac{-V_{G0}}{V_t}} \right] \]

\[ V_t = \frac{kT}{q} \]

\[ \frac{k}{q} = \frac{1.38 \times 10^{-23}}{1.602 \times 10^{-19}} \frac{V}{^0K} = 8.614 \times 10^{-5} \frac{V}{^0K} \]

\[ V_{G0} = 1.206V \]

termed the bandgap voltage

pn junction characteristics highly temperature dependent through both the exponent and \( J_S \)

\( V_{G0} \) is nearly independent of process and temperature
Voltage References

Observation – Variables with units Volts needed to build any voltage reference

What variables available in a process have units volts?

\[ V_{DD}, V_T, V_{BE \text{ (diode)}}, V_Z, V_{BE}, V_t, V_{G0} \]

What variables which have units volts satisfy the desired properties of a voltage reference? \( V_{G0} \) and ??

How can a circuit be designed that “expresses” the desired variables?

\( V_{G0} \) is deeply embedded in a device model with horrible temperature effects! Good diodes are not widely available in most MOS processes!
Standard Approach to Building Voltage References

Negative Temperature Coefficient (NTC)

Positive Temperature Coefficient (PTC)

\[ X_{\text{OUT}} = X_N + KX_P \]

Pick \( K \) so that at some temperature \( T_0 \),

\[ \frac{\partial (X_N + KX_P)}{\partial T} \bigg|_{T=T_0} = 0 \]
Standard Approach to Building Voltage References

\[ X_N + KX_P \]

\[ \frac{\partial}{\partial T} \left( X_N + KX_P \right) \bigg|_{T=T_0} \]
Bandgap Voltage References

Consider two BJTs (or diodes)

\[ V_{BE2} - V_{BE1} = \Delta V_{BE} = \left[ \frac{k}{q} \ln \left( \frac{l_{C2}}{l_{C1}} \right) \right] T \]

\[ \frac{\partial (V_{BE2} - V_{BE1})}{\partial T} = \frac{k}{q} \ln \left( \frac{l_{C2}}{l_{C1}} \right) \]

At room temperature

\[ V_{BE2} - V_{BE1} = [8.6 \times 10^{-5} \times 300] = 25.8 \text{mV} \]

If \( \ln(l_{C2}/l_{C1}) = 1 \)

\[ \left. \frac{\partial (V_{BE2} - V_{BE1})}{\partial T} \right|_{T=T_0=300 \text{K}} = 8.6 \times 10^{-5} = 86 \mu V/°C \]

The temperature coefficient of the PTAT voltage is rather small
Bandgap Voltage References

Consider two BJTs (or diodes)

\[ I_C(T) = \left( \tilde{I}_{SXE} \left[ T^m e^{-\frac{V_{G0}}{V_t}} \right] \right) e^{\frac{V_{BE}(T)}{V_t}} \]

\[ V_{BE} = V_t \ln(I_C) + [V_{G0} - V_t \ln(\tilde{I}_{SXE} A_E) + m \ln T] \]

If \( I_C \) is independent of temperature, it follows that

\[ \frac{\partial V_{BE}}{\partial T} = \frac{k}{q} \left[ -m + \left( \frac{V_{BE} - V_{G0}}{V_t} \right) \right] \]

\[ \frac{\partial V_{BE}}{\partial T} \bigg|_{T=T_0=300^\circ K} \approx 8.6 \times 10^{-5} \left[ -2.3 + \left( \frac{0.65 - 1.2}{25 \text{mV}} \right) \right] \approx -2.1 \text{mV/}^\circ \text{C} \]
$V_{BE}$ and $\Delta V_{BE}$ with constant $I_C$
First Bandgap Reference (and still widely used!)

First Bandgap Reference (and still widely used!)

Current ratios is $\frac{R_4}{R_3}$
Current not highly dependent upon $T$

$$V_{\text{REF}} = V_{\text{BE2}} + \frac{R_1}{R_2} [V_{\text{BE1}} - V_{\text{BE2}}]$$
Thanks to Damek for the following assessment!
Limitations:

**Impedance of MOS switch is not zero.**
Settling time issue

**Switch impedance is input-code dependent.**
The $V_{th}$ of the MOS transistor switches varies with position along the R string.

Possible solutions:

- Increase device width to reduce impedance.
  - Will increase capacitance.
- Manipulate $V_{th}$ of transistors by controlling the bulk voltage.
  - Not all processes have both n- and p-wells.
- Use p-channel transistors.
  - More area/capacitance for same impedance as n-channel.
  - Just flips $V_{th}$ problem from one end to the other.
- Use transmission gates.
  - Works, but...
  - There is a large capacitance penalty (4x, if pMOS is sized to account for lower mobility).
  - Area penalty (4x, again).
Increased complexity
- More devices.
- Inverters needed to provide inverted signal to the other transmission gate input.

- Use pMOS on $V_{DD}$ end and nMOS on $V_{SS}$ end.
  - Transition point between n and p will NOT be the midpoint of the R string if the difference in $V_{th}$ is taken into account.
  - Do we need to invert the signals to the pMOS switches?
- More...???

**Time constants are input-code dependent.**

**Transition times are previous-code dependent.**
The previous input/output state of the DAC determines which and how many switch capacitances must be charged/discharged during the transition to the new output. This means that the I/O characteristics can vary from one test/simulation to another. This is a problem that is shared among many DAC architectures.

**Possible solutions:**
- Pre-charge switch capacitances before each code change
  - [Control circuitry introduces?] increased capacitance, area, complexity
- Return-to-zero. (Same thing as pre-charge?)

**Large capacitance on output node.**

**Requires complicated binary n-to-$2^n$ address decoder.**

**Possible Solutions:**
- Tree decoder can reduce complexity and design burden.
Latch the Boolean signal can reduce/eliminate logic transients which cause distortion. This is applicable to many DAC designs.
Tree Decoder (for R-string DAC)

Tree decoder as digital address decoder for control of switch input:

Decoder acts as a regular address decoder, converting binary to the switch address, to control the switches along the R string.

Tree decoder as ANALOG MULTIPLEXER:
Decoder serves as the decoder AND the switches—ANALOG MULTIPLEXER—Connects nodes along R string to the output.

**Limitations:**

Same limitations due to switch impedances.

Similar solutions.
Segmented R-String DAC

Segmented DAC—AKA sub-divider, sub-range, or dual-string DAC

Two switches required to insert fine string at each point along the coarse string.

“Cartwheeling” the fine string reduces the number of switches needed in the coarse string. Notice the extra switch at the top of the fine string to permit the flipping.

Fine string is also referred to as the “interpolator”.
Properties:

Less area.

Fewer switches.
Requires $2^{n_1} + 2^{n_2}$ switches for a 2-segment design, where $n_1 + n_2 = N$.
Single-string design requires $2^N$ switches.

Limitations:

Requires 2 decoders (for 2-segment design).
However, area and complexity will be less than in the single string design. Area is reduced as well.

Two switches on each output node of coarse string.

Possible solutions:

- “Cartwheeling” the fine segment for every other point of insertion along the coarse string will allow the use of only one string per output node on the coarse string. Also note the extra switch at the top of the interpolator needed to perform the flipping. (see diagram above)

  With cartwheeling, the high and low voltage ends of the interpolator are swapped. The interpolator string is effectively flipped end-for-end for every other insertion point in the coarse string.

  Cartwheeling presents its own problems. Elaborate. Consider making cartwheeling its own section.

Voltage across coarse string resistors change depending on where the interpolator is inserted.

Possible solutions:

- Op amp buffering between coarse and fine segments (does this solution go with this problem?). (See below.)
- “Dummy” interpolator resistances across all coarse string resistors. (See below.)
- Current source biasing of fine string. (Does that describe it correctly?) (See below.)
Buffered Coarse String Output

Several issues with the use of buffers:
- Must contend with offset voltage of the op amps.
- Need a large common-mode input swing.
- More area and power.
- Slewing of the buffers becomes a concern for large jumps or high frequency operation.
- Use of p- or n-channel inputs to the buffer?

p- or n-channel inputs to buffers?
Use 2 interpolators. Need more info.
“Dummy” Interpolator Loads on Coarse String

NOTE: “N_2R_f” above should be written “n_2R_f”

Solves what problem?

When the interpolator is inserted in parallel with one of the coarse string resistors, dummy resistances equivalent to the total interpolator resistance are connected in parallel with the remaining coarse resistors.
Current sources supply current equal to the current that would naturally flow through the coarse string in the absence of the interpolator. The interpolator, then, does not draw any current from the coarse string since it is instead supplied by the current sources. This prevents the voltage across the coarse sting resistors from changing with the position of the interpolator.
Matrix Structure for R-String DAC

Reduced decoder complexity.
Dual-Ladder R-String DAC

The more significant the bit, the more dramatic the effect an error in that bit will have on the DAC performance.

**Single-string design:**
All of the resistors must be made physically large for good matching.

**Dual-ladder design:**
Coarse string resistors, for which matching is critical, can be made physically large, and small in value.
Fine string(s) resistors, which are not as matching-critical, can be made physically small and large in value. This saves area compared to a regular single-string design.

Making the fine string resistors large-valued minimizes the loading effect they have on the coarse string.

**Single-string design:**
Very great design burden (wire routing) for common-centroid layout of all of the resistors.

**Dual-ladder design:**
A common-centroid layout can easily be done for the relatively few resistors in the coarse string, which are matching-critical. The fine string resistors can be laid with much more flexibility in such a way that eases wire routing, since they represent the least-significant bits and have a smaller penalty for mismatches.
Following is a summary of the design problems encountered and their solutions in the paper A 10-b 50-MHz CMOS D/A Converter with 75-Ω Buffer by Marcel J. M. Pelgrom. The design is based on the dual-ladder structure discussed above.

**Problem: Resistor matching**
The burden for producing a common-centroid layout for a 10-bit single-R-string DAC is incredibly great, due to the difficulty in routing the high number of interconnects.

**Solution: Dual-ladder design w/ matrix formation**
The fine ladder resistor strings are weighted less significantly, and are therefore less matching-critical. The common centroid layout can then be skipped for the fine string resistors. There are relatively few coarse string resistors (16 in Pelgrom’s design) so the common-centroid layout and interconnection for these resistors is relatively simple.

The matrix layout also simplifies the decoding design by allowing the use of two small decoders, instead of one large one.

**Problem: Code-dependent time constants**
The resistance along the path from $V_{ref}$ to $V_{out}$ changes with the input code, which changes the output time constant and causes distortion.

**Solution: Resistive output rail**
Resistors were placed between the switches along the output rails to make the number of resistors along any path from $V_{ref}$ to $V_{out}$ the same. Note that Pelgrom uses a “folded ladder” for the fine strings, which means that two adjacent fine strings share an output rail. This cuts the number of output rail resistors in half.
Problem: Switch impedance is dependent on position along ladder
The source voltage of the switch transistors depends on its position in the voltage-dividing resistor string. Since the gate “on” voltage is fixed, the gate-to-source voltage, and therefore the switch impedance, varies as well.

Solution: Supply ladder to reduce the turn-on voltage variation
A voltage-dividing resistor string can be used to supply different “on” voltages to the switches depending on their location to (ideally) eliminate the $V_{gs}$ variation.

Problem: Gradient effects

Solution: Anti-parallel layout of the coarse string
Each resistor in the coarse string is split in two. Each half-resistor is connected to its counterpart in parallel and each pair is connected in series to make the coarse string. In layout, the resistors are arranged in two rows, with each half-resistor is symmetrically opposed to its partner. This is just a specific form of common-centroid layout.

Problem: (Difficulty in routing?) using digital decoder and analog mux in a matrix organization
I don’t know I wrote this down correctly...
Most Published Analysis of Bandgap Circuits

\[ V_{\text{REF}} = V_{G0} + \frac{T}{T_0} (V_{BE0} - V_{G0}) + (m-1) \frac{kT}{q} \ln \left( \frac{T_0}{T} \right) + K \frac{kT}{q} \ln \left( \frac{J_2}{J_1} \right) \]

where K is the gain of the PTAT signal
First Bandgap Reference (and still widely used!)

\[ I_{E1} R_2 + V_{BE1} = V_{BE2} \]

\[ V_{REF} = V_{BE2} + (I_{E1} + I_{E2}) R_1 \]

\[ I_{C1} = \frac{V_{DD} - V_{C2} - V_{OS}}{R_3} \]

\[ I_{C2} = \frac{V_{DD} - V_{C2}}{R_4} \]

\[ I_{C1} = \alpha_{1} I_{E1} \]

\[ I_{C2} = \alpha_{2} I_{E2} \]

\[ \alpha = \frac{\beta}{1 + \beta} \]

\[ I_{E1} = I_{E2} \left[ \frac{\alpha_{2} R_4}{\alpha_{1} R_3} \right] - \frac{V_{OS}}{\alpha_{1} R_3} \]

\[ V_{REF} = V_{BE2} + (V_{BE2} - V_{BE1}) \left[ \frac{R_1}{R_2} \left(1 + \frac{\alpha_{1} R_3}{\alpha_{2} R_4} \right) \right] - V_{OS} \left[ \frac{R_1}{\alpha_{1} R_3} \right] \]
First Bandgap Reference (and still widely used!)

\[ V_{\text{REF}} = V_{\text{BE}2} + (V_{\text{BE}2} - V_{\text{BE}1}) \left[ \frac{R_1}{R_2} \left( 1 + \frac{\alpha_1 R_3}{\alpha_2 R_4} \right) \right] \]

\[ V_{\text{BE}2} = V_t \ln I_{C2} + \left[ V_{G0} - V_t \left\{ \ln \left( A_{E2} \tilde{J}_{SX} \right) + m \ln T \right\} \right] \]

\[ V_{\text{BE}1} = V_t \ln I_{C1} + \left[ V_{G0} - V_t \left\{ \ln \left( A_{E1} \tilde{J}_{SX} \right) + m \ln T \right\} \right] \]

\[ I_{C1} = \alpha_1 I_{E1} \]

\[ I_{C2} = \alpha_2 I_{E2} \]

\[ I_{E1} = I_{E2} \left[ \frac{\alpha_2 R_4}{\alpha_1 R_3} \right] \]

\[ V_{\text{BE}2} - V_{\text{BE}1} = \Delta V_{\text{BE}} = \left[ k \ln \left( A_{E1} \left[ \frac{R_3}{R_4} \right] \right) \right] T \]
First Bandgap Reference (and still widely used!)

\[ V_{\text{REF}} = V_{\text{BE}2} + (V_{\text{BE}2} - V_{\text{BE}1}) \left[ \frac{R_1}{R_2} \left( 1 + \frac{\alpha_1 R_3}{\alpha_2 R_4} \right) \right] \]

\[ V_{\text{BE}2} = V_t \ln I_{C_2} + \left[ V_{G0} - V_t \left\{ \ln \left( A_{E2} J_{SX} \right) + m \ln T \right\} \right] \]

\[ V_{\text{BE}1} = V_t \ln I_{C_1} + \left[ V_{G0} - V_t \left\{ \ln \left( A_{E1} J_{SX} \right) + m \ln T \right\} \right] \]

\[ V_{\text{BE}2} - V_{\text{BE}1} = \Delta V_{\text{BE}} = \left[ \frac{k}{q} \ln \left( \frac{A_{E1}}{A_{E2}} \left[ \frac{R_3}{R_4} \right] \right) \right] T \]

From the expression for \( V_{\text{BE}2} \) and some routine but tedious manipulations it follows that

\[ V_{\text{BE}2} = V_{G0} + (1-m)V_t \ln T + V_t \ln \left( \frac{k}{q} \frac{\alpha_1}{R_2 A_{E2} J_{SX}} \frac{R_3}{R_4} \ln \left( \frac{A_{E1}}{A_{E2}} \right) \right) \]
First Bandgap Reference (and still widely used!)

\[ V_{\text{REF}} = V_{\text{BE}2} + (V_{\text{BE}2} - V_{\text{BE}1}) \left[ \frac{R_1}{R_2} \left( 1 + \frac{\alpha_1}{\alpha_2} \frac{R_3}{R_4} \right) \right] \]

\[ V_{\text{BE}2} - V_{\text{BE}1} = \Delta V_{\text{BE}} = \left[ \frac{k}{q} \ln \left( \frac{A_{E1}}{A_{E2}} \left( \frac{R_3}{R_4} \right) \right) \right] T \]

\[ V_{\text{BE}2} = V_{G0} + (1 - m)V_t \ln T + V_t \ln \left( \frac{k}{q R_2 A_{E2}} \ln \left( \frac{A_{E1}}{A_{E2}} \right) \right) \]

It thus follows that:

\[ V_{\text{REF}} = V_t \ln \left( \frac{\alpha_1}{\alpha_2} \frac{R_3}{R_4} \right) + V_{G0} - V_t (\ln (i_{Sx2}) + m \ln T) + \left[ \frac{k}{q} \ln \left( \frac{A_{E1}}{A_{E2}} \left( \frac{R_3}{R_4} \right) \right) \right] \left( \frac{R_1}{R_2} \left( 1 + \frac{\alpha_1}{\alpha_2} \frac{R_3}{R_4} \right) \right) T \]
First Bandgap Reference  (and still widely used!)

\[ V_{\text{REF}} = V_{\text{BE2}} + (V_{\text{BE2}} - V_{\text{BE1}}) \left( \frac{R_1}{R_2} \left( 1 + \frac{\alpha_1}{\alpha_2} \frac{R_3}{R_4} \right) \right) \]

\[ V_{\text{REF}} = V_t \ln \left( \frac{R_3 - R_2}{R_4} \right) + \frac{k}{q} \ln \left( \frac{A_{E1}}{A_{E2}} \right) + V_{\text{GO}} - V_t \left( \ln \left( \frac{I_{sX2}}{I_{sX2}} \right) + m \ln T \right) + \left[ \frac{k}{q} \ln \left( \frac{A_{E1}}{A_{E2}} \right) \right] + \left[ \frac{R_1}{R_2} \left( 1 + \frac{\alpha_1}{\alpha_2} \frac{R_3}{R_4} \right) \right] T \]

\[ V_{\text{REF}} = a_1 + b_1 T + c_1 T \ln T \]

\[ a_1 = V_{\text{GO}} \]

\[ b_1 = \frac{k}{q} \left( \frac{R_1}{R_2} \left( 1 + \frac{R_3 \alpha_1}{R_4 \alpha_2} \right) \ln \left( \frac{R_3}{R_4} \frac{A_{E1}}{A_{E2}} \right) + \ln \left( \frac{k}{q} \frac{R_3}{R_4} \frac{A_{E1}}{A_{E2}} \right) \right) \]

\[ c_1 = \frac{k}{q} (1 - m) \]
First Bandgap Reference (and still widely used!)

\[ V_{\text{REF}} = a_1 + b_1 T + c_1 T \ln T \]

\[ a_1 = V_{G0} \]

\[ b_1 = \frac{k}{q} \left( \frac{R_1}{R_2} \left( 1 + \frac{R_3 \alpha_1}{R_4 \alpha_2} \right) \ln \left( \frac{R_3 A_{E1}}{R_4 A_{E2}} \right) + \ln \left( \frac{k R_3}{q R_4} \alpha_1 \frac{\ln \left( \frac{R_3 A_{E1}}{R_1 A_{E2}} \right)}{I_{SK2} R_2} \right) \right) \]

\[ c_1 = \frac{k}{q} (1 - m) \]

\[ \frac{dV_{\text{REF}}}{dT} = b_1 + c_1 (1 + \ln T) = 0 \]

\[ T_{\text{INF}} = e^{-\left( \frac{1}{c_1} \frac{b_1}{1} \right)} \]

\[ b_1 = -c_1 (1 + \ln T_{\text{INF}}) \]

\[ V_{\text{REF}} = a_1 - c_1 T_{\text{INF}} \]

\[ V_{\text{REF}} = V_{G0} + \frac{k T_{\text{INF}}}{q} (m - 1) \]
First Bandgap Reference (and still widely used!)

\[
V_{\text{REF}} = a_1 + b_1 T + c_1 T \ln T
\]

\[
V_{\text{REF}} = V_{G0} + \frac{kT_{\text{INF}}}{q} (m - 1)
\]

Bandgap Voltage Source

<table>
<thead>
<tr>
<th>VGO</th>
<th>1.206</th>
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<tbody>
<tr>
<td>TO</td>
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<td>VBEO2</td>
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<tr>
<td>m-1</td>
<td>1.3</td>
</tr>
<tr>
<td>k/q</td>
<td>8.61E-05</td>
</tr>
</tbody>
</table>
Temperature Coefficient

\[
TC = \frac{V_{\text{MAX}} - V_{\text{MIN}}}{T_2 - T_1}
\]

\[
TC_{\text{ppm}} = \frac{V_{\text{MAX}} - V_{\text{MIN}}}{V_{\text{NOM}}(T_2 - T_1)} \times 10^6
\]
Bamba Bandgap Reference

Bamba Bandgap Reference

\[ I_{R0} = \frac{\Delta V_{BE}}{R_0} \]

\[ I_{R1} = \frac{V_{BE1}}{R_1} \]

\[ I_{R2} = I_{R1} \]

\[ I_2 = I_{R0} + I_{R2} \]

\[ I_3 = K I_2 \quad \text{K is the ratio of } I_3 \text{ to } I_2 \]

\[ V_{\text{REF}} = \theta I_3 R_4 \]

Substituting, we obtain

\[ V_{\text{REF}} = \theta K R_4 \left( \frac{V_{BE}}{R_1} + \frac{\Delta V_{BE}}{R_0} \right) \]

\[ V_{\text{REF}} = \theta K \frac{R_4}{R_1} \left( V_{BE} + \frac{R_1}{R_0} \Delta V_{BE} \right) \]

\[ V_{\text{REF}} = a_{11} + b_{11} T + c_{11} T \ln T \]
Kujik Bandgap Reference

Kujik Bandgap Reference

\[ I_{R0} = \frac{\Delta V_{BE}}{R_0} \]

\[ I_2 = I_{R0} \]

\[ V_{REF} = I_2 R_2 + V_{BE1} \]

solving, we obtain

\[ V_{REF} = \frac{R_2}{R_0} \Delta V_{BE} + V_{BE1} \]

\[ V_{REF} = a_{22} + b_{22} T + c_{22} T \ln T \]
End of Lecture 44